Chapter 0
overview of the whole course

Complements

LGV

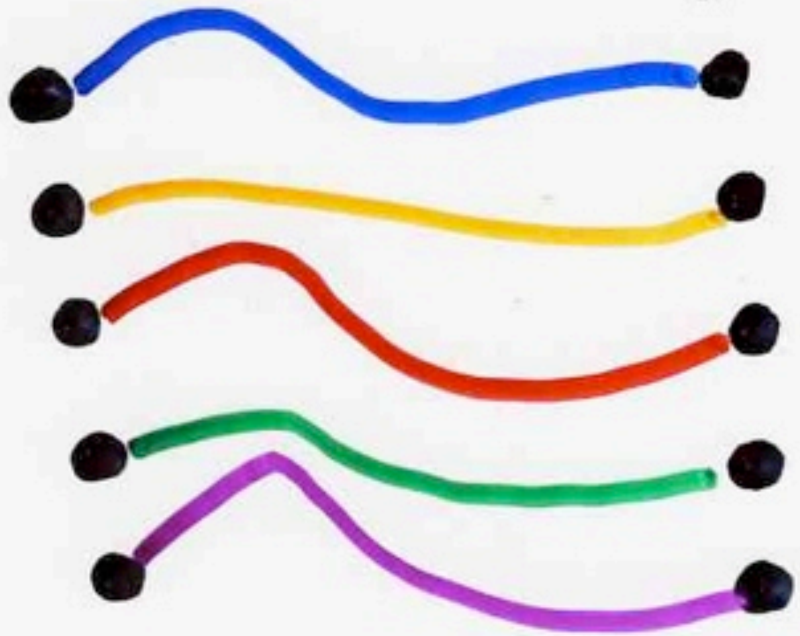
Karlin-McGregor, Lindström,
Gessel-Viennot, Fisher, ...

Gessel-Viennot
lemma

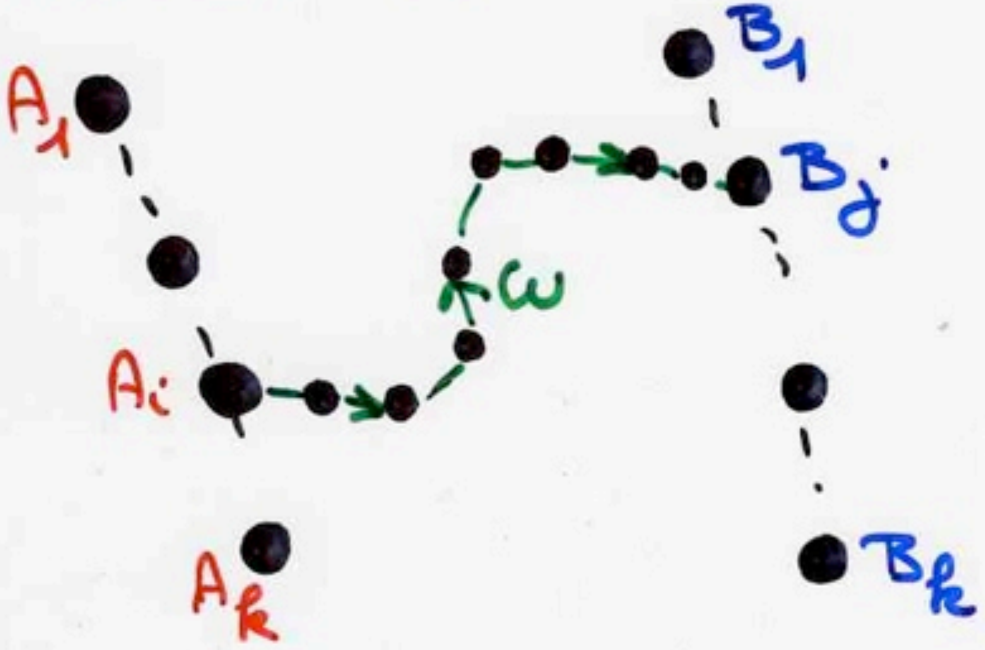
det =

Prop. (C)

$$\det(a_{ij}) = \sum_{\substack{\Omega=(\omega_1,\dots,\omega_k) \\ \omega_i : A_i \rightsquigarrow B_i \\ \text{2 by 2 disjoints}}} v(\omega_1)\dots v(\omega_k)$$



Gessel-Viennot methodology



path

$\omega = (s_0, \dots, s_n) \quad s_i \in \Pi$

valuation

$v : \Pi \times \Pi \to \mathbb{K}$ ring

$v(\omega) = v(s_0, s_1) \cdots v(s_{n-1}, s_n)$

$A_1, \dots, A_k$

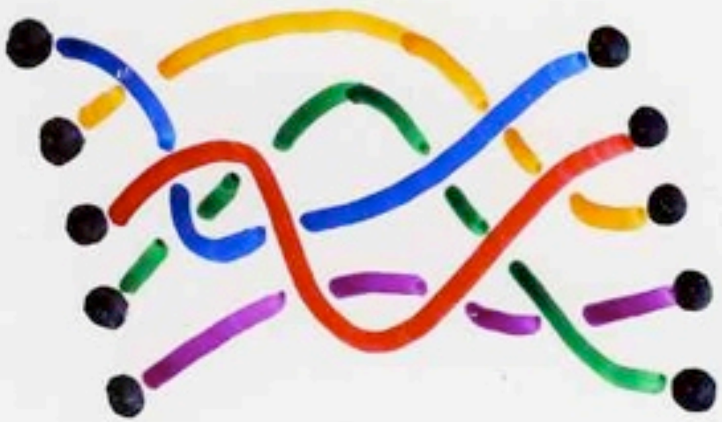
$B_1, \dots, B_k$

$$a_{ij} = \sum_{A_i \rightsquigarrow B_j} v(\omega)$$

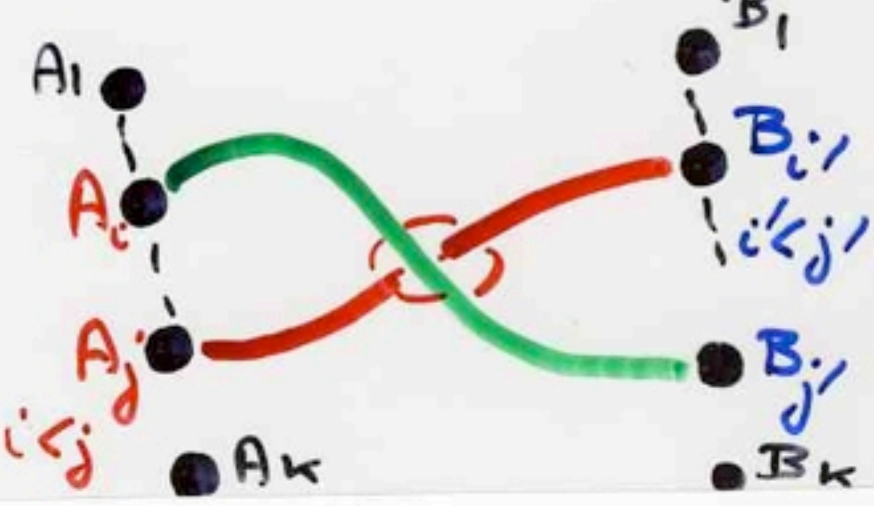
suppose finite sum

$$\det(a_{ij}) = \sum_{(\sigma;\, \omega_1, \dots, \omega_k)} (-1)^{\mathrm{Inv}(\sigma)}\, v(\omega_1) \cdots v(\omega_k)$$

$\omega_i : A_i \rightsquigarrow B_{\sigma(i)}$



(C) crossing condition



Schur functions

fonction de Schur

$$s_\lambda(x_1, x_2, \dots, x_n) = \frac{\det\left(x_j^{\,n-i+\lambda_i}\right)_{1\le i,j\le n}}{\det\left(x_j^{\,n-i}\right)_{1\le i,j\le n}}$$

$\lambda = (\lambda_1, \dots, \lambda_n)$

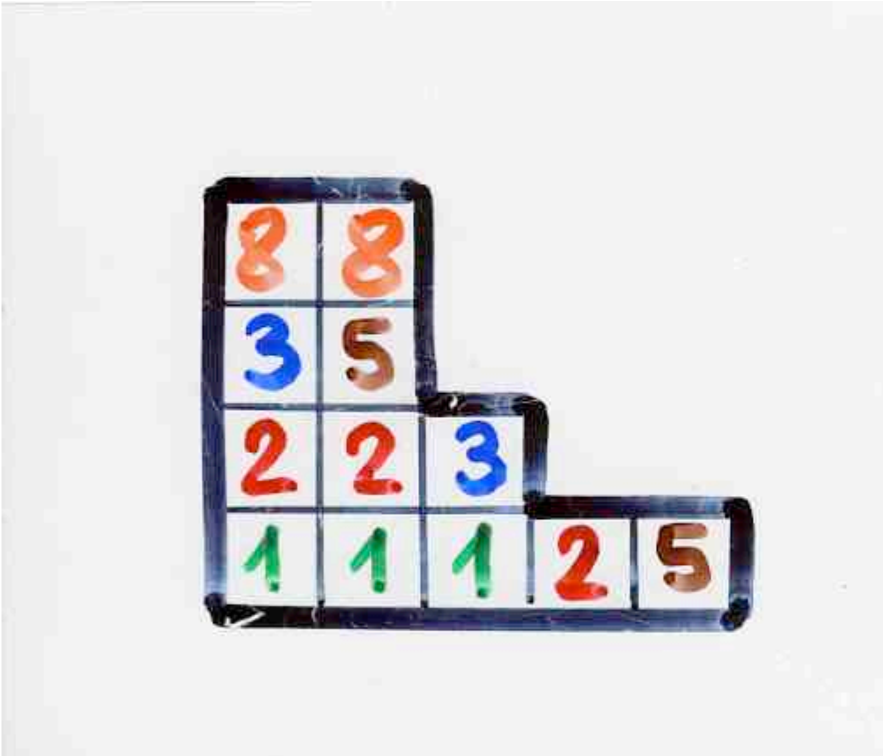
Issai Schur 1875–1941

théorie des invariants

Cauchy 1812

Jacobi 1841 det (homogène)

N. Trudi 1864


8 8
3 5
2 2 3
1 1 1 2 5

Schur Functions

$$S_\lambda(x_1, x_2, \ldots, x_m) = \sum_{T} v(T)$$

Young tableau
shape λ
entries $1, 2, \ldots, m$

Jacobi (1841)
Schur (1901)

Littlewood-Richardson (1934)

basis of **symmetric** functions

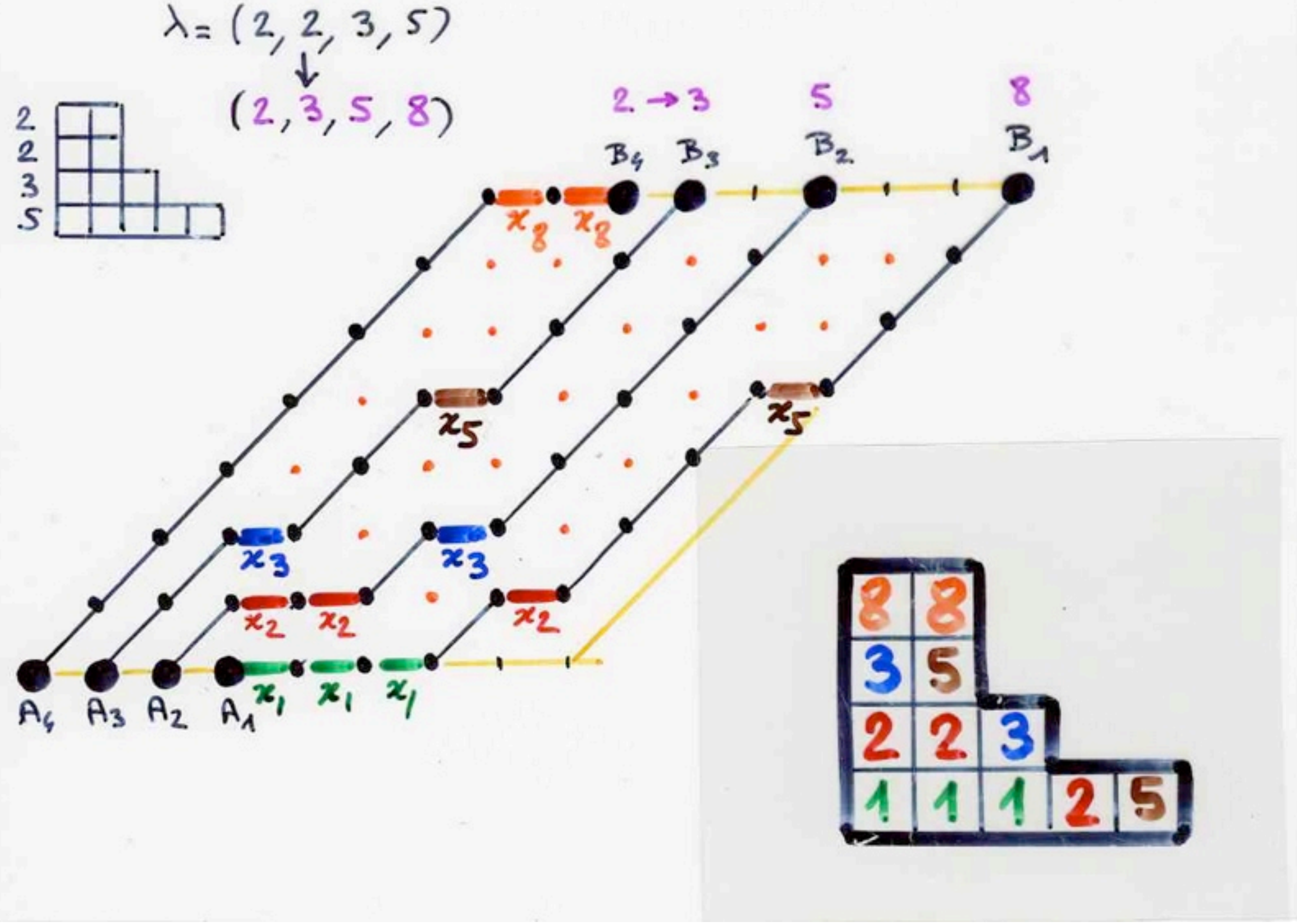
Jacobi - Trudi

$$\det\left(h_{\lambda_i - i + j}\right)_{1 \le i, j \le r} = S_\lambda(x_1, \dots, x_m)$$

Schur

$$\begin{vmatrix} h_5 & h_6 & h_7 & h_8 \\ h_2 & h_3 & h_4 & h_5 \\ h_0 & h_1 & h_2 & h_3 \\ h_{-1} & h_0 & h_1 & h_2 \end{vmatrix}$$

transpose



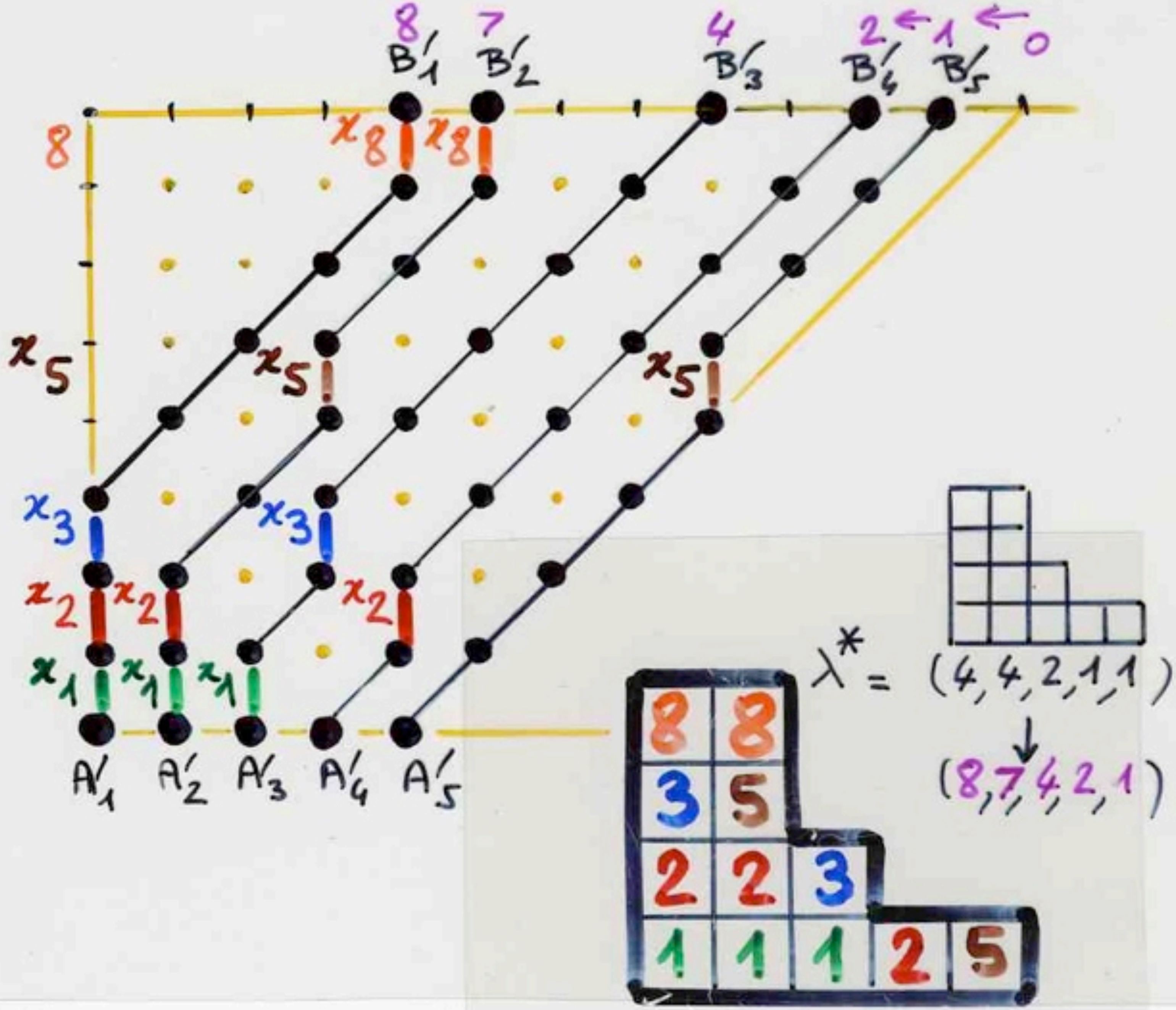
λ = (2, 2, 3, 5)
(2, 3, 5, 8)
2
2
3
5
2 → 3
5
8
B4 B3 B2 B1
x8 x8
x5
x5
x3
x3
x2 x2
x2
x1 x1 x1
A4 A3 A2 A1
8 8
3 5
2 2 3
1 1 1 2 5

$$\det\left(e_{\lambda'_i - i + j}\right) = s_\lambda(x_1, \ldots, x_m)$$

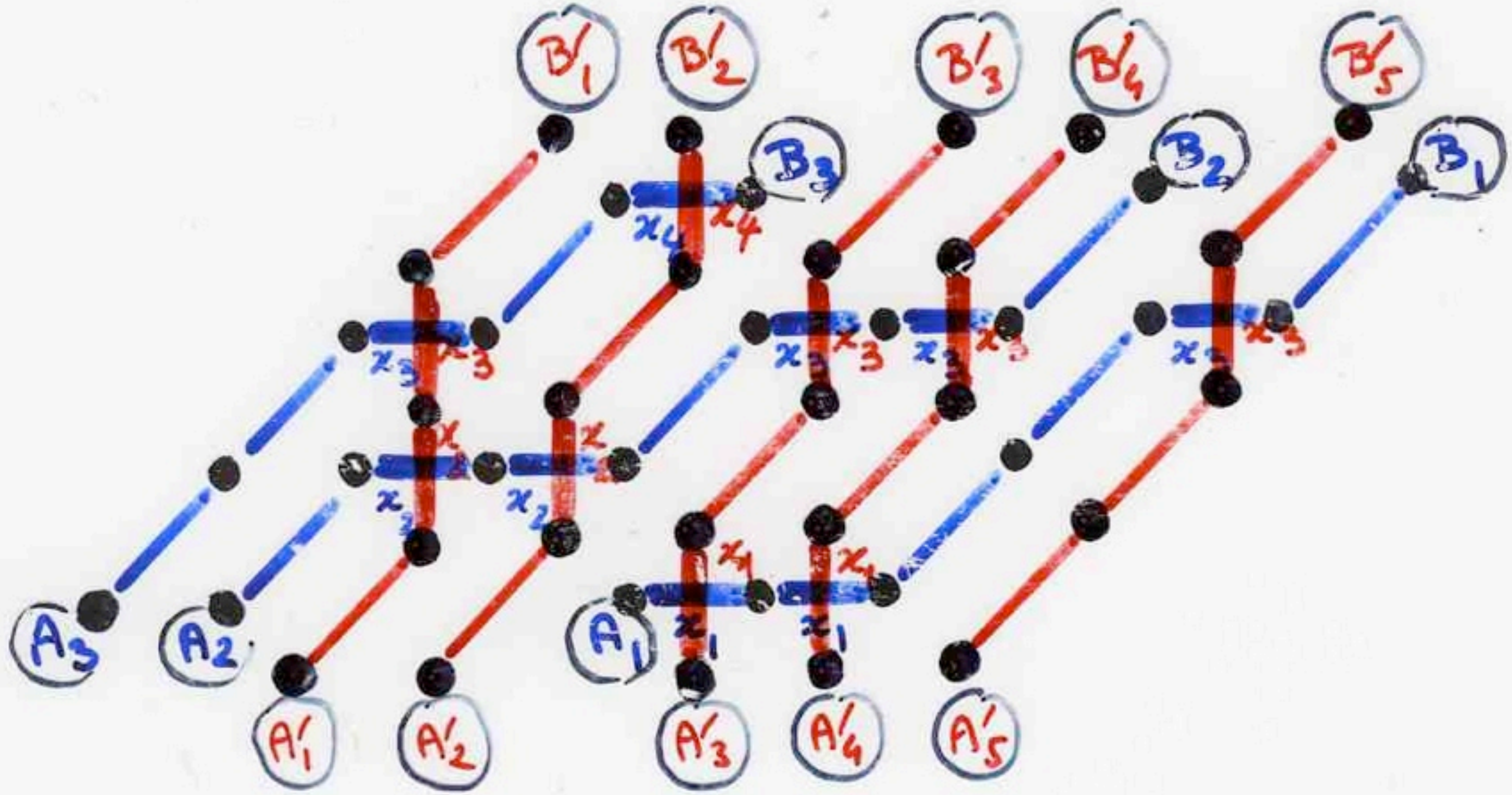
Schur

$$\begin{vmatrix} e_4 & e_5 & e_6 & e_7 & e_8 \\ e_3 & e_4 & e_5 & e_6 & e_7 \\ e_0 & e_1 & e_2 & e_3 & e_4 \\ e_{-2} & e_{-1} & e_0 & e_1 & e_2 \\ e_{-3} & e_{-2} & e_{-1} & e_0 & e_1 \end{vmatrix}$$

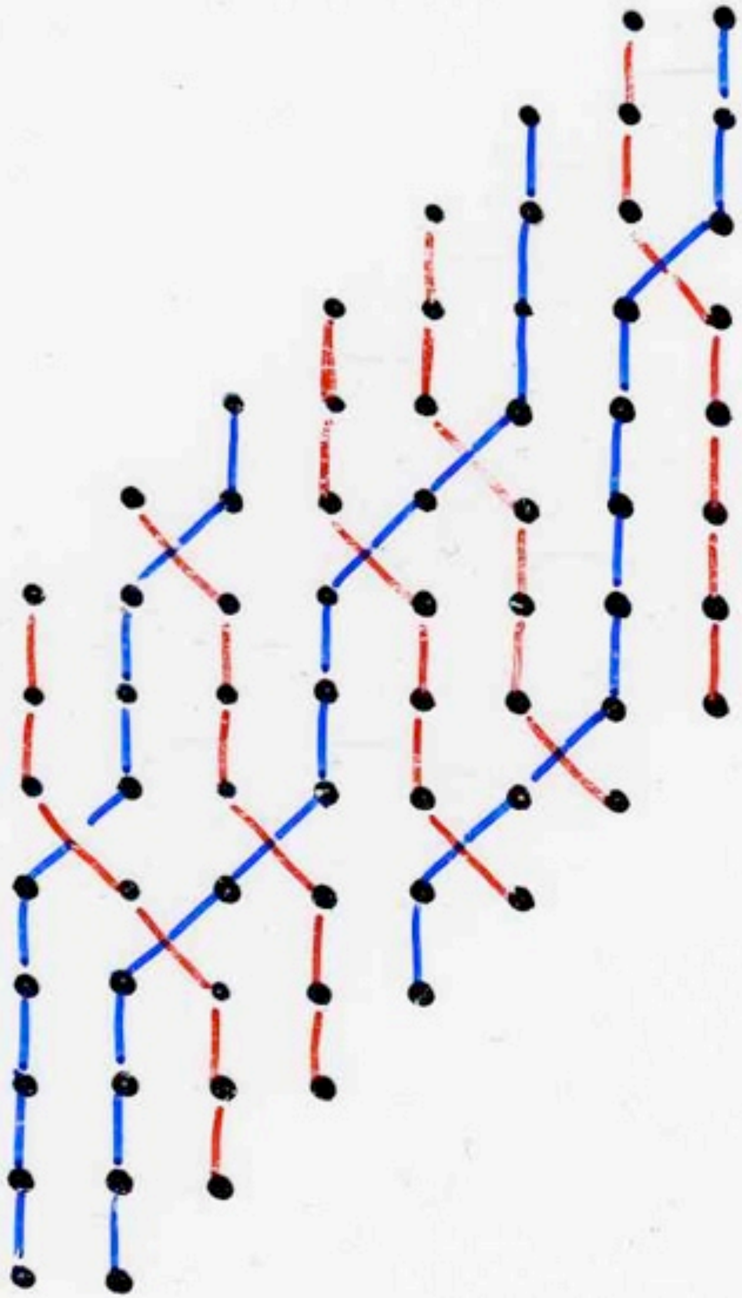
transpose

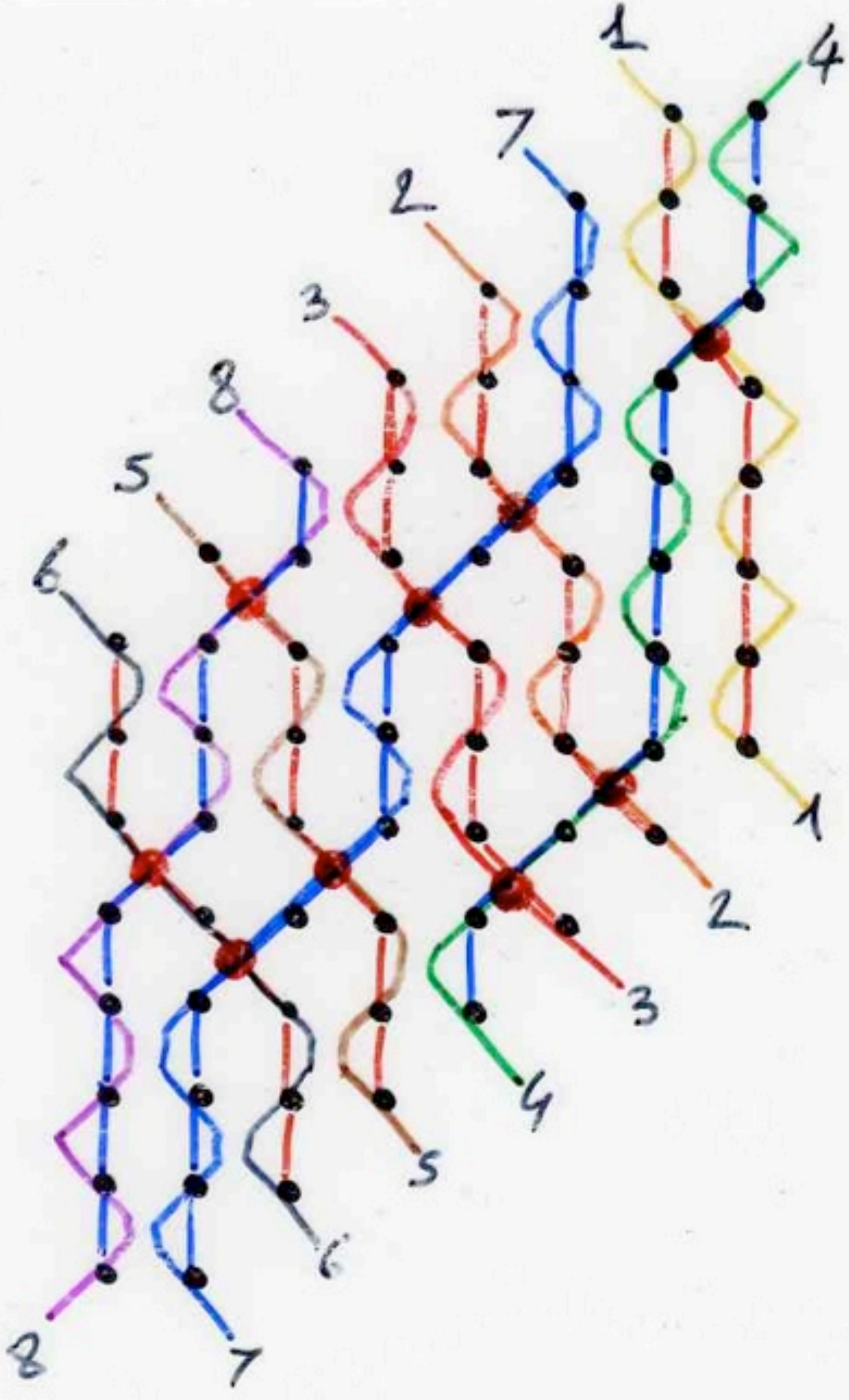


8 7 4 2 1 0
B'_1 B'_2 B'_3 B'_4 B'_5
8
x_8 x_8
x_5 x_5 x_5
x_3 x_3
x_2 x_2 x_2
x_1 x_1 x_1
A'_1 A'_2 A'_3 A'_4 A'_5
$\lambda^* = (4,4,2,1,1)$
$(8,7,4,2,1)$
8 8
3 5
2 2 3
1 1 1 2 5

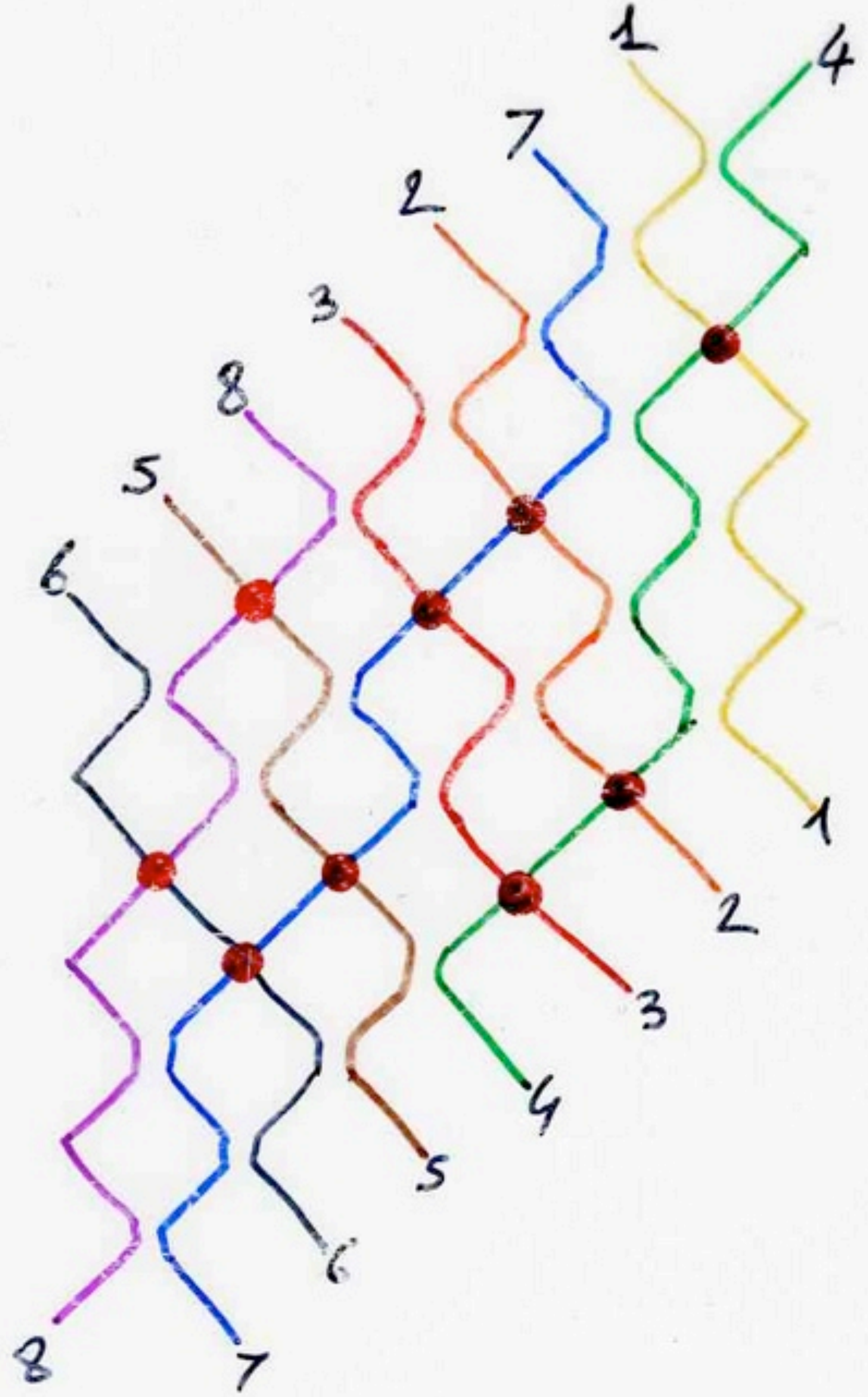


B'1
B'2
B'3
B'4
B'5
B3
B2
B1
A3
A2
A1
A'1
A'2
A'3
A'4
A'5





1
4
7
2
3
8
5
6
1
2
3
4
5
6
8
7


1
4
7
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1
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4
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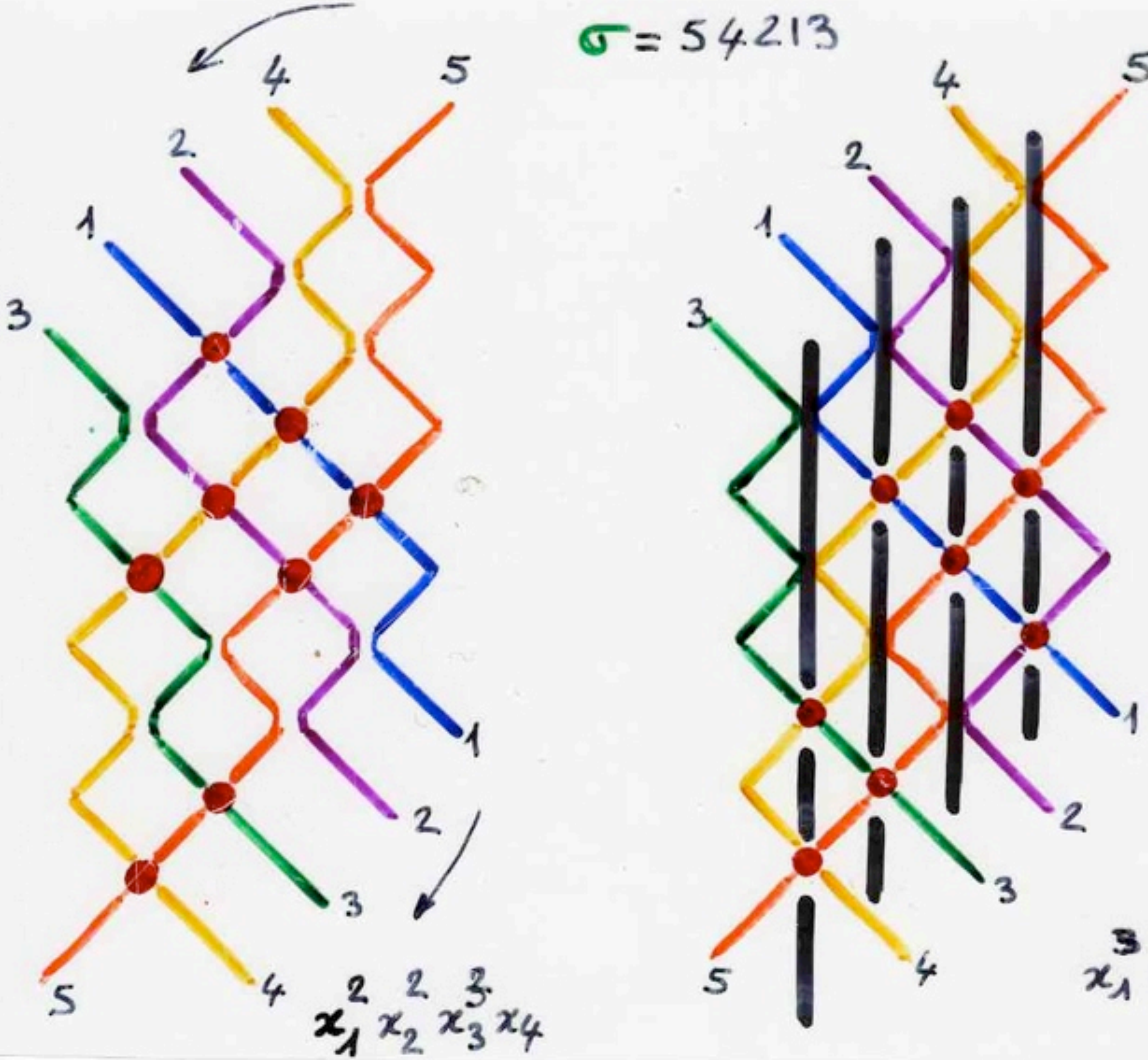
8
7

$\sigma = 54213$



$x_1^2 x_2^2 x_3^3 x_4$

$x_1^3 x_2^3 x_3^2$

Cours Spécialisés

COLLECTION SMF

Fonctions symétriques, polynômes de Schubert et lieux de dégénérescence

Numéro 3

Laurent Manivel

SOCIÉTÉ MATHÉMATIQUE DE FRANCE

Publié avec le concours du ministère de l'éducation nationale, de la recherche et de la technologie

Schubert polynomials

geometric construction

Fomin-Kirillov

algèbre des différences divisées

Newton

$$\partial_i f = \frac{f(x_1, \dots, x_i, x_{i+1}, \dots) - f(\dots, x_{i+1}, x_i \dots)}{x_i - x_{i+1}}$$

$$\partial_i \partial_{i+1} \partial_i = \partial_{i+1} \partial_i \partial_{i+1}$$

géométrie algébrique

calcul de Schubert

Lascoux - Schützenberger 1982 ...

Schubert polynomials

$$X_\sigma = \partial_{\omega\sigma}\left(x_1^{n-1}\, x_2^{n-2} \cdots x_{n-1}^{1}\, x_n^{0}\right)$$

σ permutation

$$\omega = \begin{pmatrix} 1 & 2 & \cdots & n \\ n & (n-1) & \cdots\ 2 & 1 \end{pmatrix}$$

Mac Donald (1991) book

basis

Publi du LACIM UQAM
Montréal

non linear control theory

differential equations
with forced terms

équations différentielles
en régime forcé

$$y' = f(y, t) + u(t)$$

M. Fliess

non commutative
variables

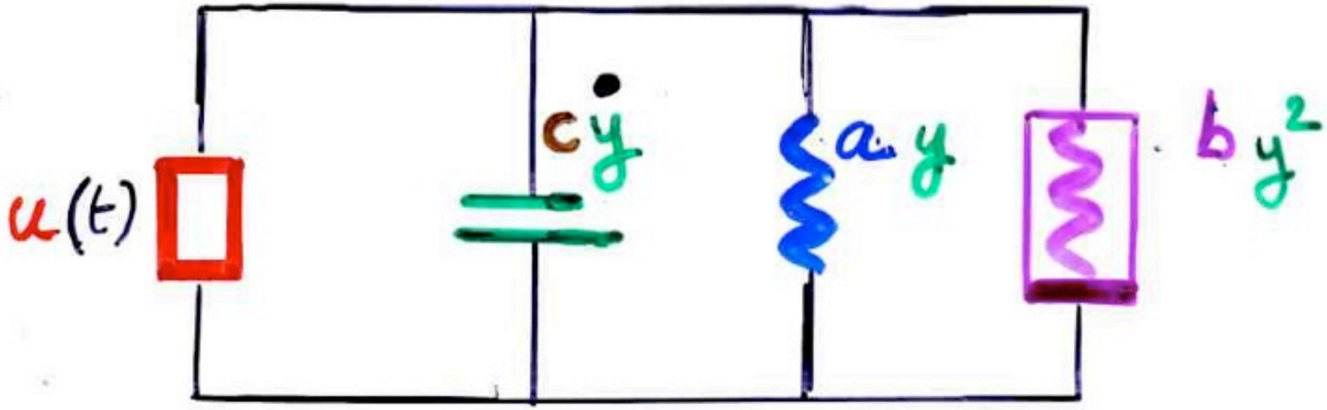
Volterra kernels

Chern iterated
integrals

$$\int_0^t d\tau_5 \int_0^{\tau_5} u(\tau_4)\,d\tau_4 \int_0^{\tau_4} d\tau_3 \int_0^{\tau_3} d\tau_2 \int_0^{\tau_2} u(\tau_1)\,d\tau_1$$

$$x_0 \quad x_1 \quad x_0 \quad x_0 \quad x_1$$

A simple nonlinear circuit



$$\frac{dy}{dt} = \alpha y + \beta y^2 + u(t)$$

$$\alpha = -\frac{a}{c} \qquad \beta = -\frac{b}{c}$$

J. Bussgang, L. Ehrman, J. Graham (1974)

M. Lamnabhi, F. Lamnabhi-Lagarrigue (1980, 1982)

M. Fliess, ″, ″ (1983)

IEEE Trans. Circuits & Systems

$$\frac{dy}{dt} = \alpha y + \beta y^2 + u(t)$$

$$y(t) = \alpha \int_0^t y(\tau)\,d\tau + \beta \int_0^t y^2(\tau)\,d\tau + \int_0^t u(\tau)\,d\tau + \gamma$$

P. Leroux, X. V.

combinatorial theory

for

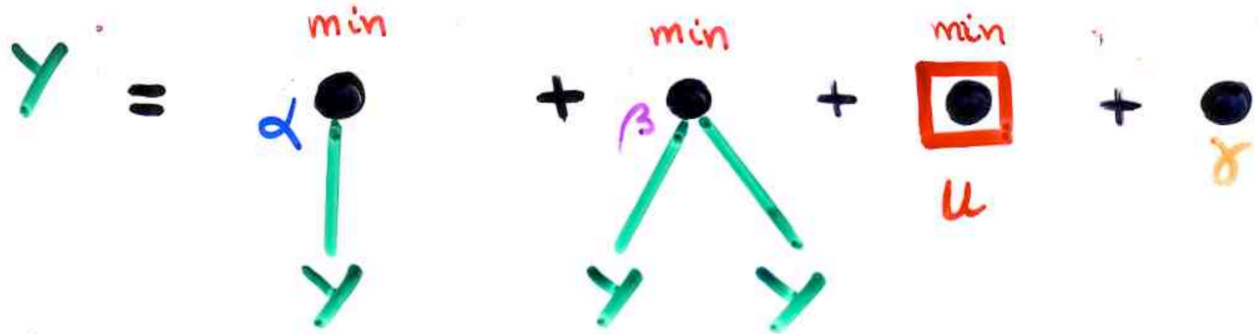
differential equations,

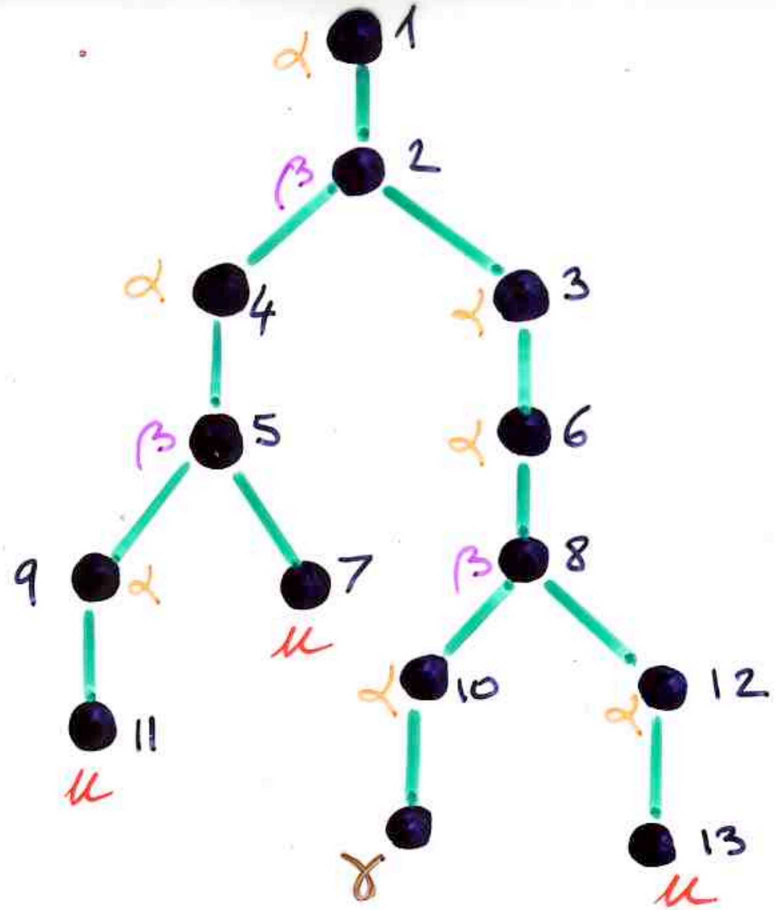
integral calculus,

.....

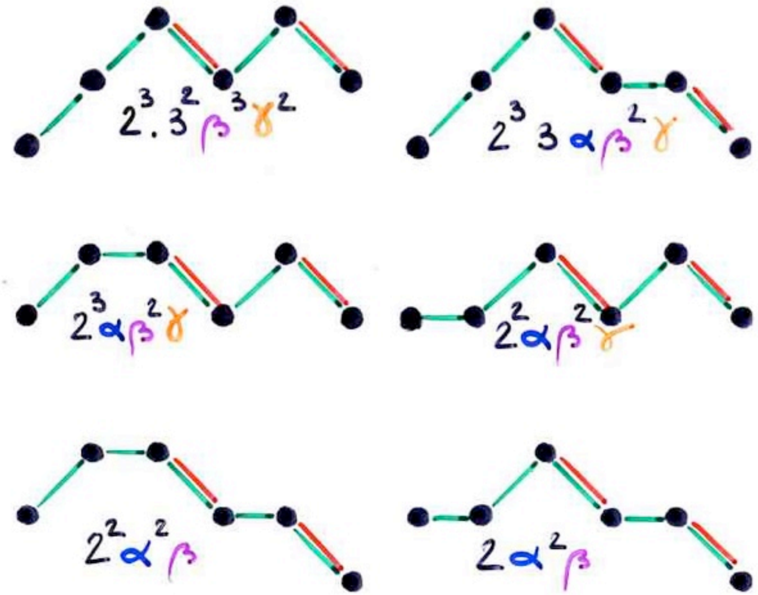
$$\frac{dy}{dt} = \alpha y + \beta y^2 + u(t)$$

$$y(t) = \alpha\int_0^t y(\tau)\,d\tau + \beta\int_0^t y^2(\tau)\,d\tau + \int_0^t u(\tau)\,d\tau + \gamma$$





$$w = x_0\, x_0\, x_1\, x_0\, x_1$$



$$c(w) = 36\, \beta^3 \gamma^2 + 36\, \alpha \beta^2 \gamma + 6\, \alpha^2 \beta$$

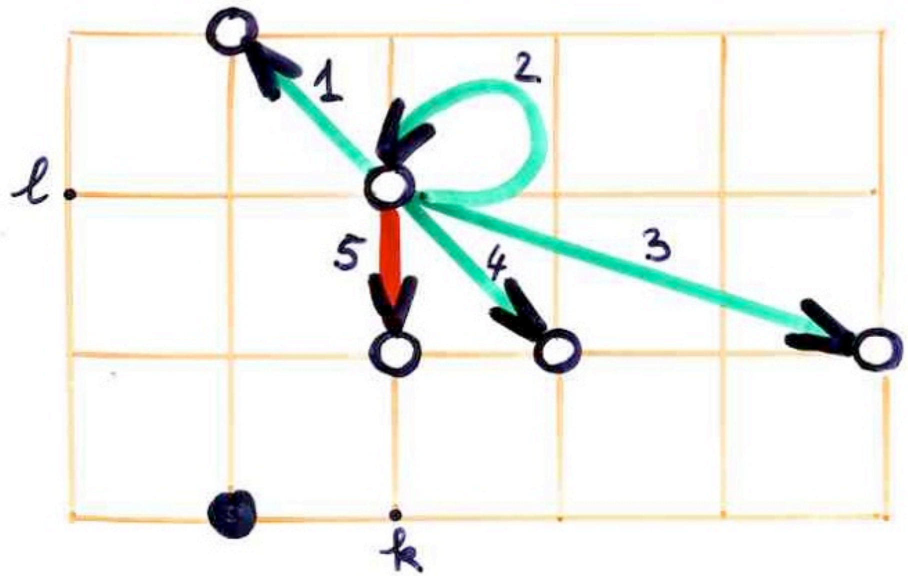
$$\begin{array}{ccccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & & \\ x_0 & x_0 & x_0 & x_0 & x_0 & x_0 & x_1 & x_0 & x_0 & x_0 & x_1 & x_0 & x_1 & = & w \end{array}$$

word w

weight $\alpha^7 \beta^3 \gamma$

Equation de Duffing

$$y'' = a\,y' + y + b\,y^3 + u(t)$$



Duffing equation