Algebraic Combinatorics and interactions (with physics)

The cellular Ansatz

Talca

December 2010

January 2011

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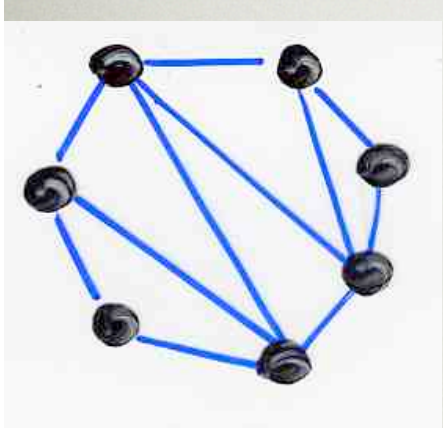
Introduction

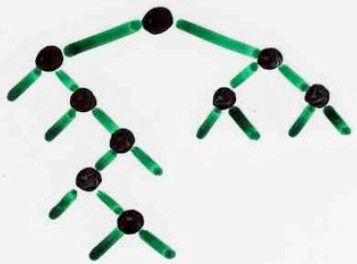
1 - Introduction to enumerative, algebraic and bijective combinatorics.

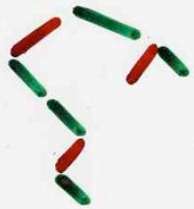
- ordinary generating functions

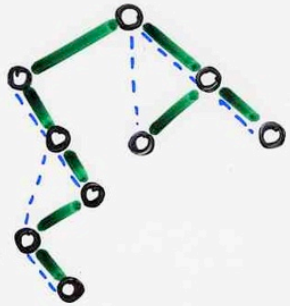
  the Catalan garden: trees, paths, triangulations, ...

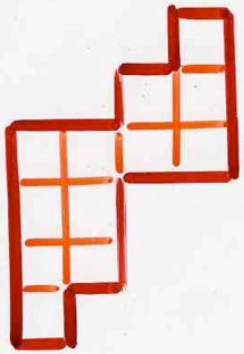
the Catalan garden

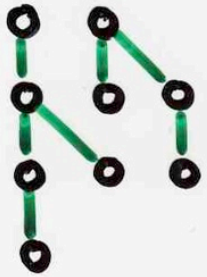


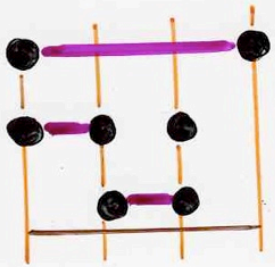


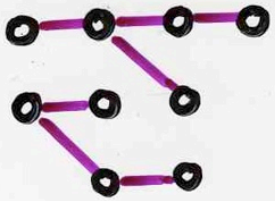


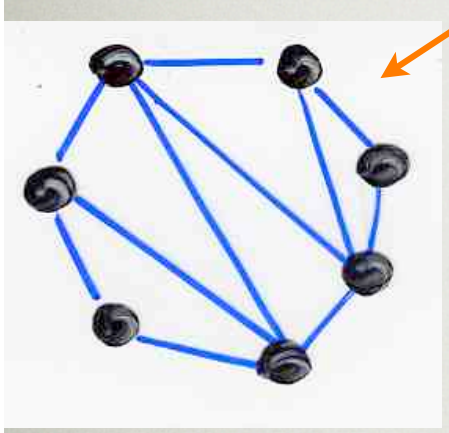


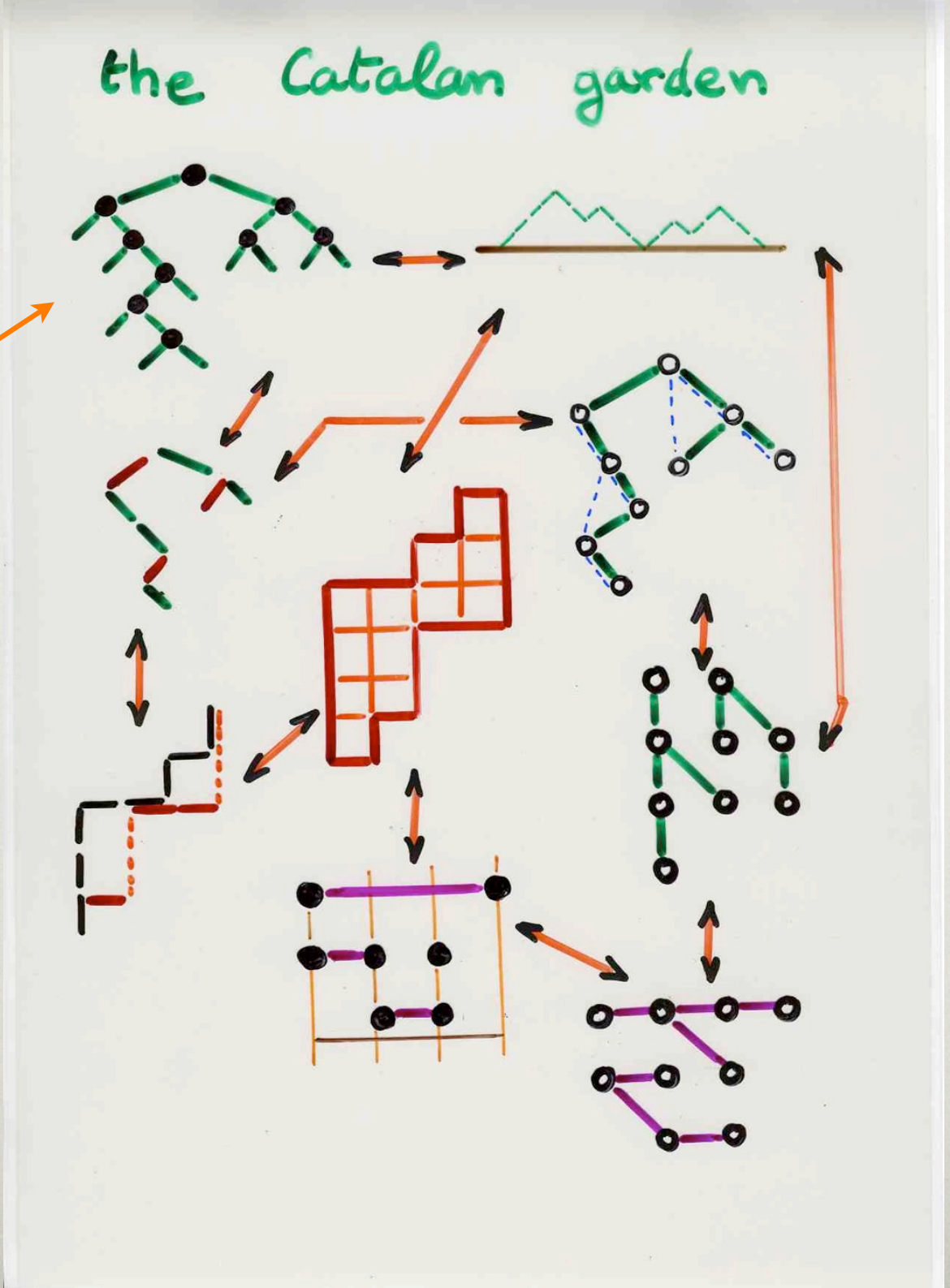











the Catalan garden

1 - Introduction to enumerative, algebraic and bijective combinatorics.

- ordinary generating functions

  the Catalan garden: trees, paths, triangulations, ...

- q-series and q-analogues

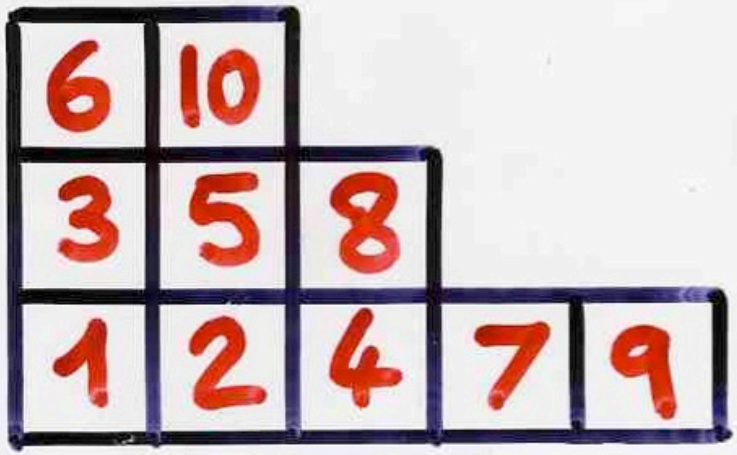
- exponential structures and generating functions

  permutations

  bijections: inversion tables, «Laguerre histories» (=weighted paths),

  RSK (Robinson-Schensted-Knuth correspondance)

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 1 & 6 & 10 & 2 & 5 & 8 & 4 & 9 & 7 \end{pmatrix}$$



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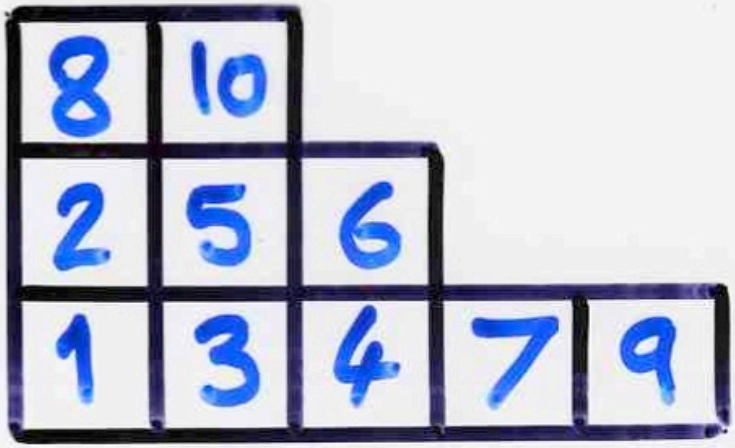


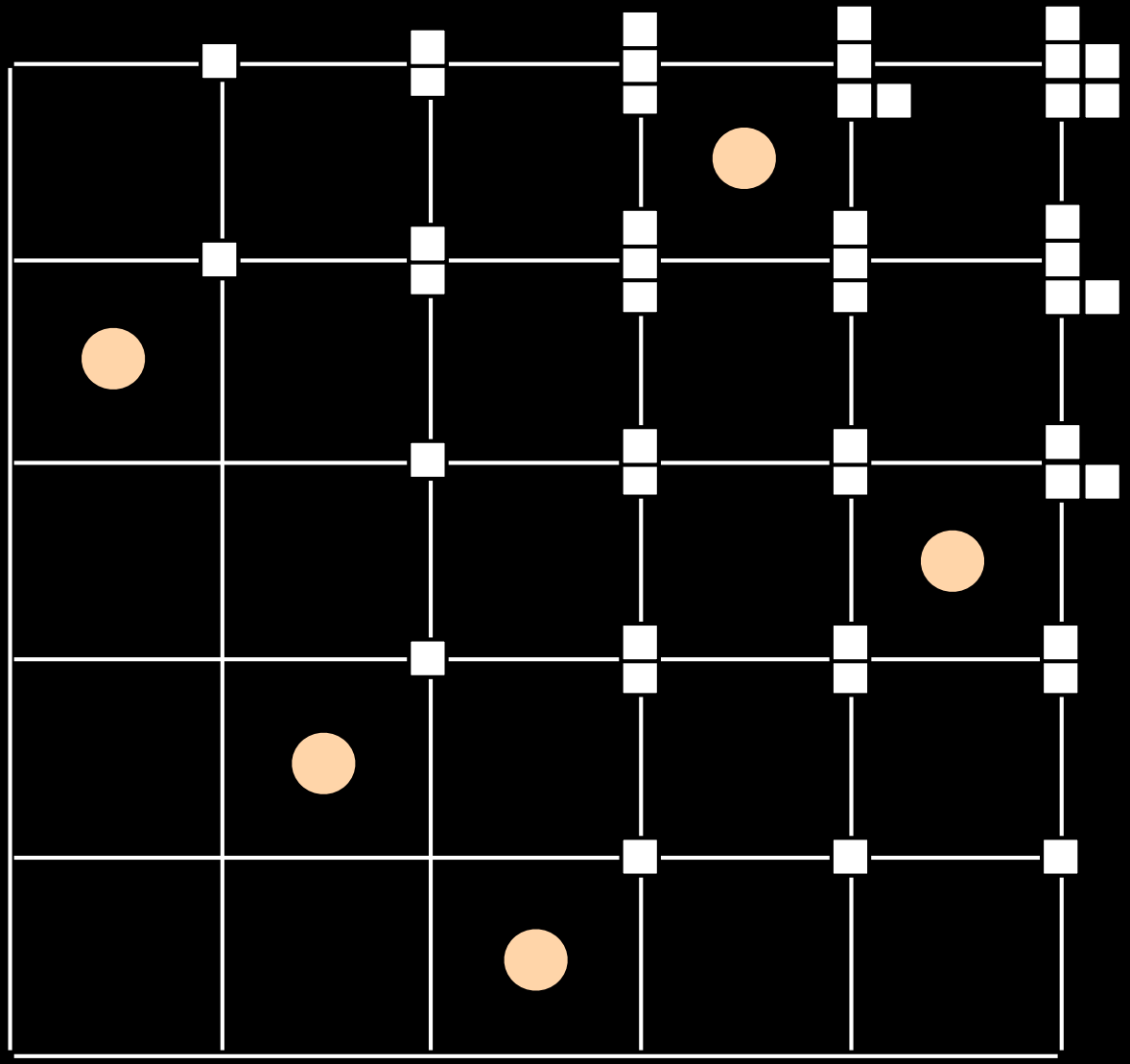
Q

2 - Some topics in algebraic combinatorics

- Paths, determinants and Young tableaux

- RSK «local» from the algebra $UD=DU+Id$

- Combinatorial theory of orthogonal polynomials and continued fractions

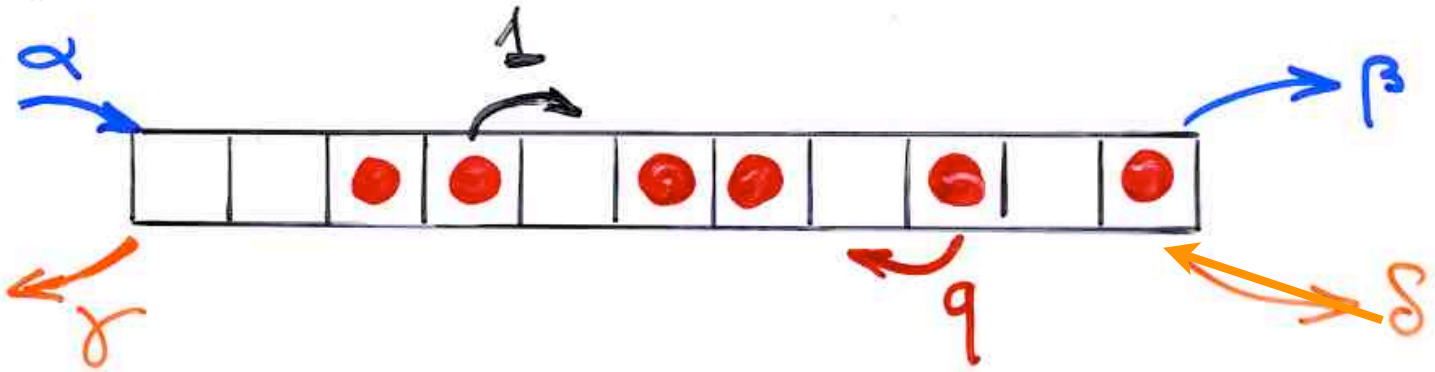


3 - Combinatorics and physics: the cellular Ansatz

- PASEP (partially asymetric exclusion process)
  the PASEP algebra $DE = qED + E + D$

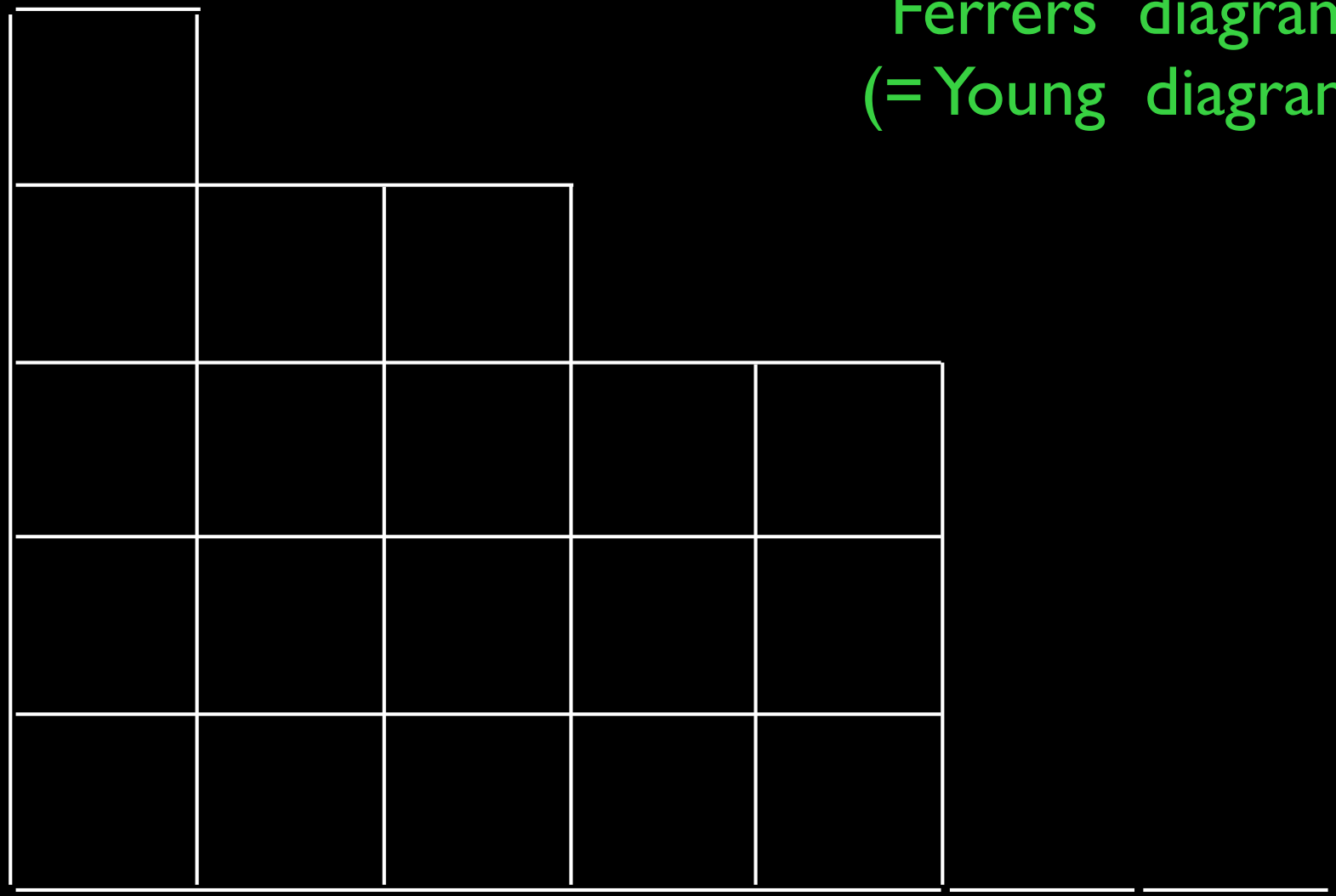
- Alternative tableaux, permutation tableaux

ASEP
TASEP
PASEP

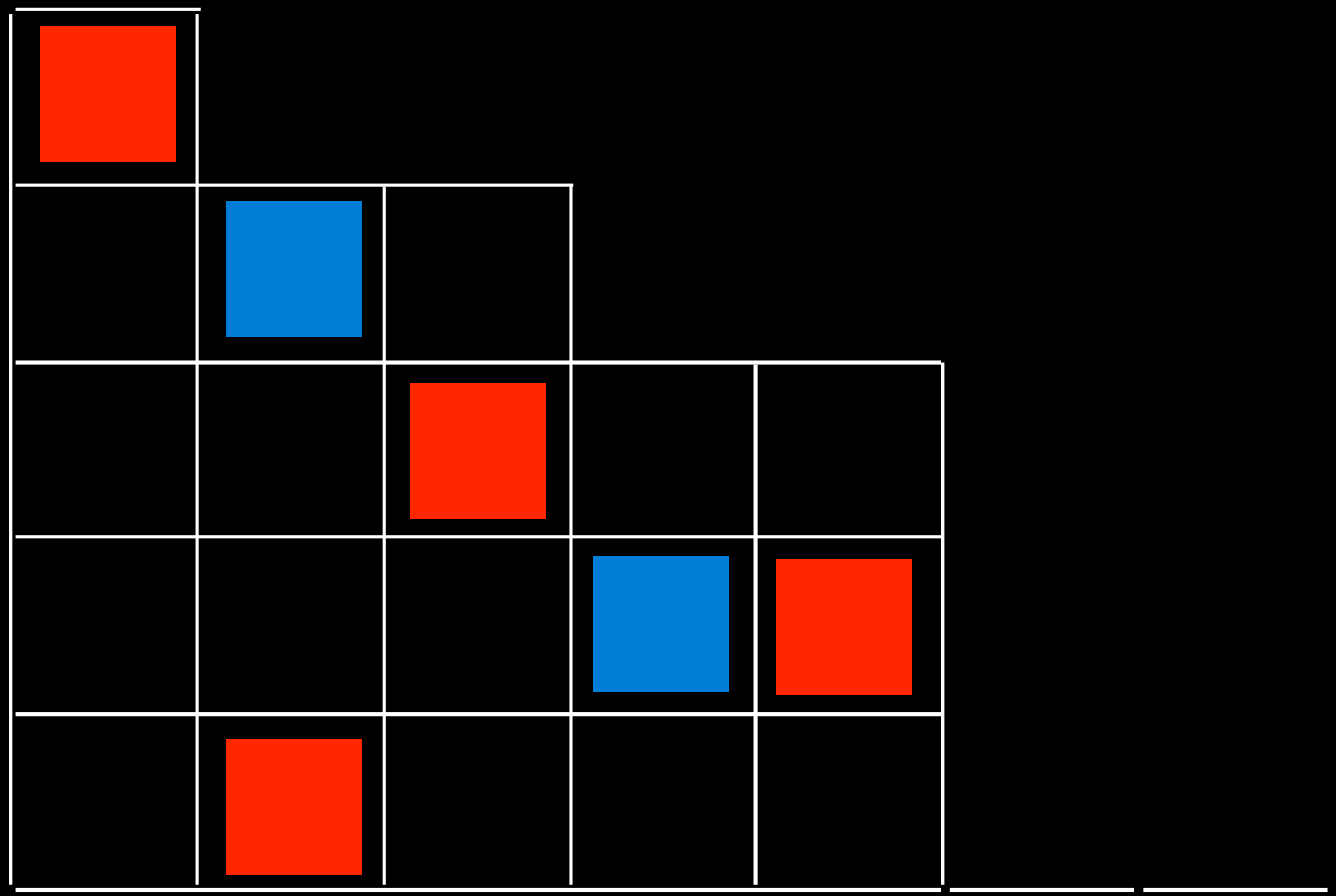


alternative tableau

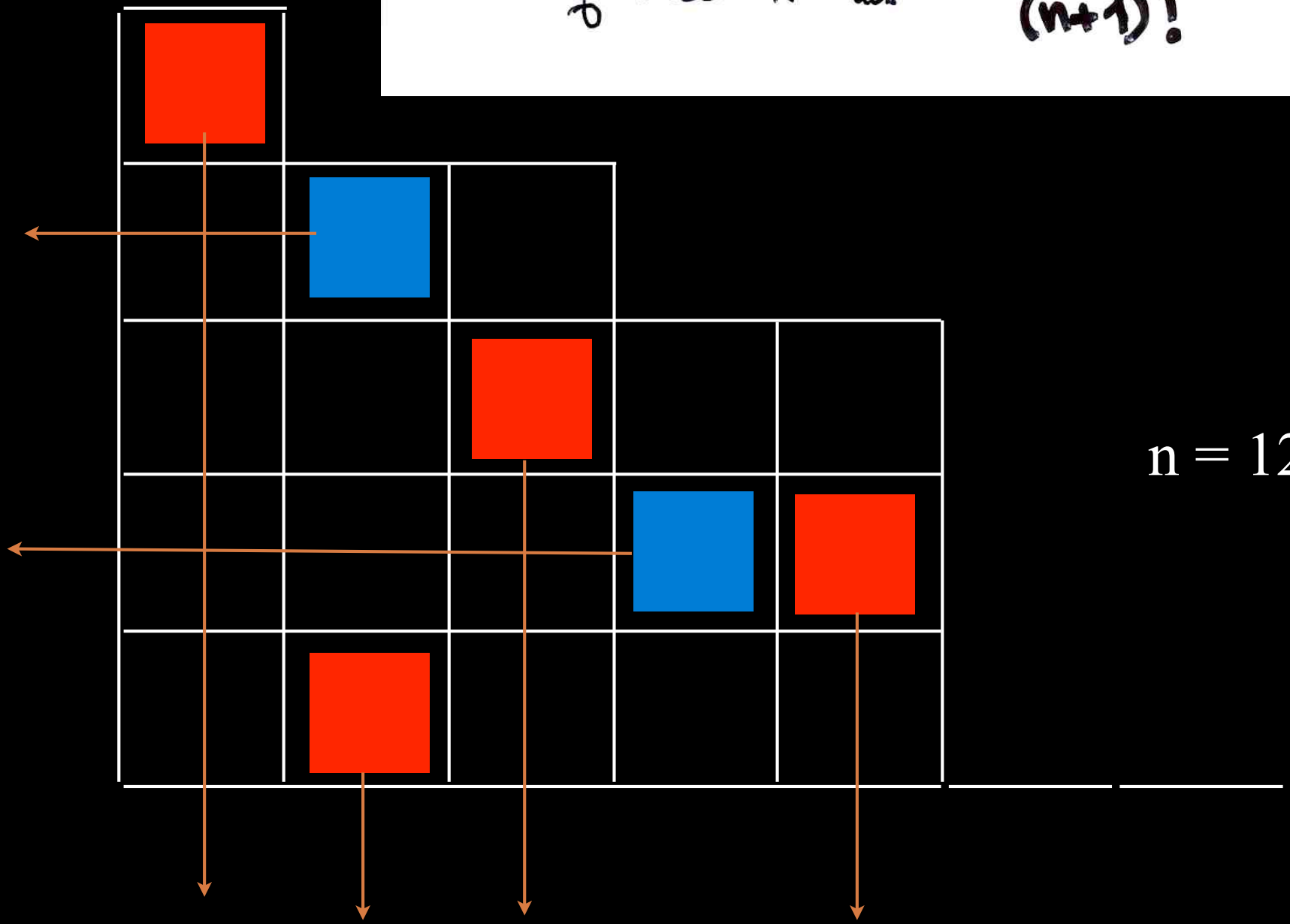
Ferrers diagram
(= Young diagram)



alternative tableau



Prop. The number of alternative tableaux of size n is $(n+1)!$



$n = 12$

3 - Combinatorics and physics: the cellular Ansatz

- PASEP (partially asymetric exclusion process)
  the PASEP algebra DE = qED + E + D

- Alternative tableaux, permutation tableaux

- TASEP and the Catalan garden
  Loday-Ronco algebra

- ASM (alternating sign matrices), FPL (Fully packed loops model),
  6-vertex model, RS (the ex-Razumov-Stroganov conjecture)

- Q-tableaux and planar automata
  8-vertex model

web site: www .labri.fr/perso /viennot

.... ici (for research, classes, ..)

"Cont'Science" (for vulgarisation of science)

web.mac.com/xgviennot/Xavier_Viennot

Nouveautés / New:

- videos with violin

1. ***-The cellular Ansatz,*** *Deux exposés au séminaire Algo, INRIA Rocquencourt,* *see the page* ***exposés***

2 Mars 2009

Bienvenue recherches exposés publications livres

petite école cours vulgarisation videos photos

web.mac.com/xgviennot/Xavier_Viennot

- see page "cours"

- cours combinatoire (ENS, 1988)
(in french)

- **Ch0** Séries formelles (7p)
- **Ch1** Séries génératrices rationnelles (25p)
- **Ch2** Séries génératrices algébriques (49p)
- **Ch3** Séries génératrices exponentielles (70p)

Cours au Service de Physique Théorique du CEA, Saclay
Septembre- Octobre 2007
"Eléments de combinatoire algébrique"

(partly in french, many slides in english)
diaporamas du cours

Ch1 - (14,2 Mo)(Mardi 11 Septembre) introduction à la combinatoire énumérative, séries génératrices ordinaires

Ch2 - (7,7 Mo)(Mardi 18 Septembre) séries génératrices et structures exponentielles

Ch3 - (15,6 Mo) commutations et empilements de pièces

Ch3 (suite) (15,6 Mo) empilements et physique statistique

Ch 4 - (9,4 Mo) théorie combinatoire des polynômes orthogonaux et fractions continues

Ch 5 - (11,3 Mo) chemins, déterminants et pavages,
tableaux de Young et fonctions de Schur

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XAVIER VIENNOT (2)

Summer School, Mathematical Physics

Frutillar, Chile, 7-11 December 2009

Combinatorics and statistical Mechanics

1. Introduction to enumerative combinatorics
   (pdf 20,2 Mo)
2. Commutation and heaps of pieces
   (pdf 26 Mo)
3. Paths, determinants and plane partitions
   (pdf 10 Mo)
4. The cellular Ansatz

   Alternating sign matrices and the Razumov-Stroganov conjecture
   Partially asymetric exclusion process (PASEP) (pdf 12,3 Mo)

web.mac.com/xgviennot/Xavier_Viennot2

XAVIER VIENNOT (2)

Cours TIFR Mumbai, February 2010

Algebraic Combinatorics and interactions

Chapter 0 overview of the whole course
- Part 1 (pdf 13,4 Mo)
- Part 2 (pdf 14,7 Mo)
- Part 3 (pdf 4,2 Mo)

Chapter 1 ordinary generating functions
- Part 1 (pdf 11,6 Mo)
- Complements (pdf 7,3 Mo)

Chapter 2 exponential structures and generating functions (pdf 11,5 Mo)

Chapter 3 commutations and heaps of pieces
- Part 1 (pdf 11,8 Mo)
- Part 2 (pdf 13,6 Mo)
- Part 3 (pdf 9,6 Mo)

Chapter 5-6 Paths, determinants and tilings
Young tableaux and Schur functions (pdf 17,2 Mo)

RSK