Chapter 4a

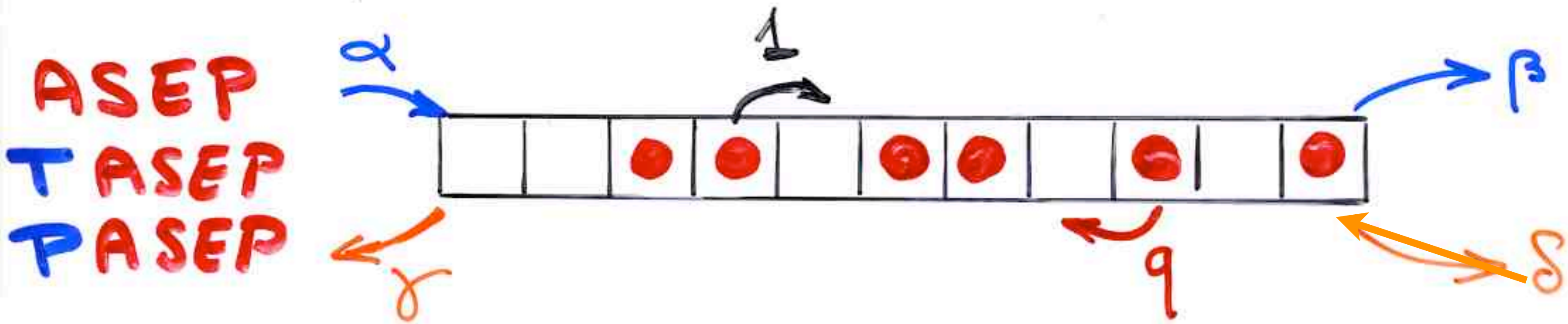
Alternative tableaux and the PASEP

(Partially ASymmetric Exclusion Process)

13 january 2011

Talca

The PASEP

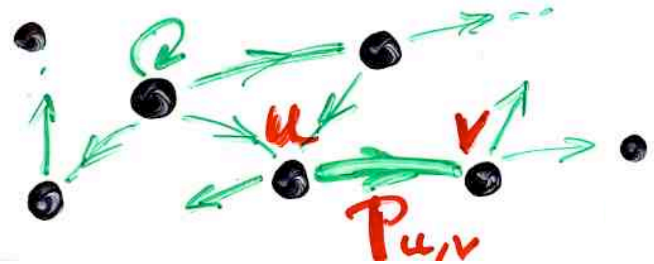

ASEP
TASEP
PASEP
α
1
β
γ
q
δ

Markov chains

2^n states — $w = (\tau_1, \dots, \tau_n)$

$\tau_i = \begin{cases} 0 \\ 1 \end{cases}$

graph



probability

S : states

$M = (p_{u,v})_{u,v \in S}$ probabilities matrix (stochastic)

$\pi = (p_u, \dots)$ vector (time t)

$\pi \cdot M$ vector (time t+1)

$$p_v^{(t+1)} = \sum_u p_u^{(t)} \, p_{u,v}$$

$t+1$ time t

stationary probabilities

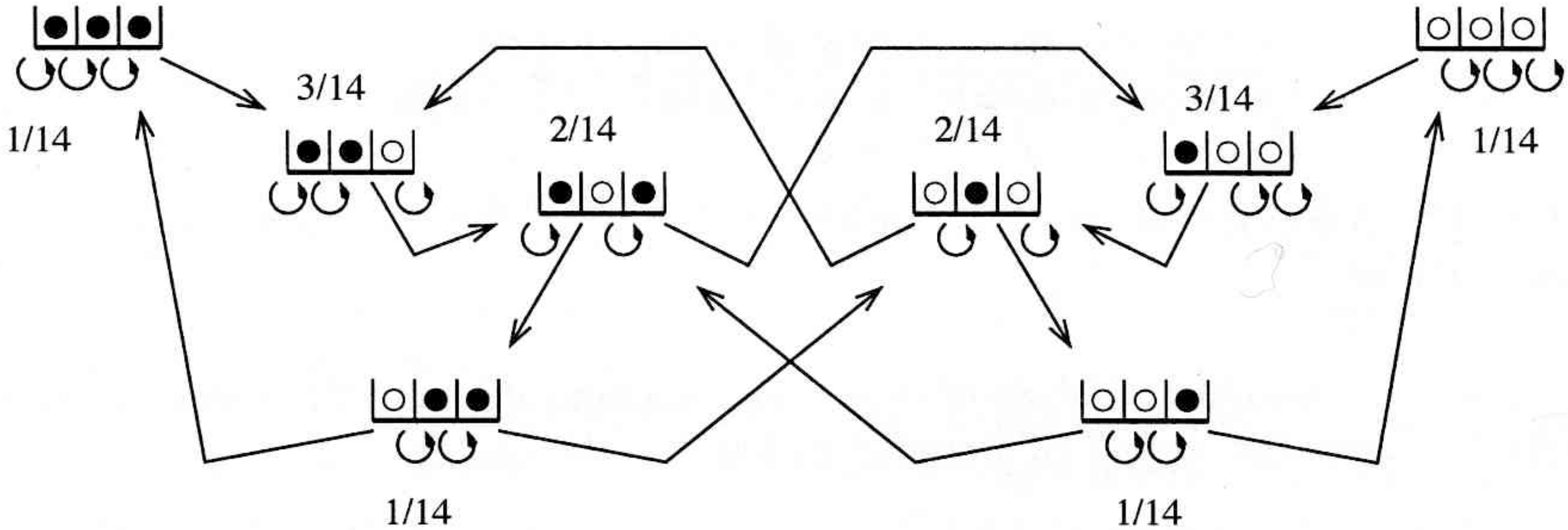
$$\pi \cdot M = \pi$$

unicity

eigenvector $\qquad M^T$ eigenvalue 1

time $\longrightarrow \infty$

$$p_v = \sum_u p_u \, p_{u,v}$$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non-equilibrium statistical mechanics

... relaxation → stationary state

states $\tau = (\tau_1, \tau_2, \ldots, \tau_n)$

$$\tau_i = \begin{cases} 1 & \text{site } i \text{ occupied} \\ 0 & \text{site } i \text{ empty} \end{cases}$$

unique stationary state $\quad \frac{d}{dt} P_n(\tau_1, \ldots, \tau_n) = 0$

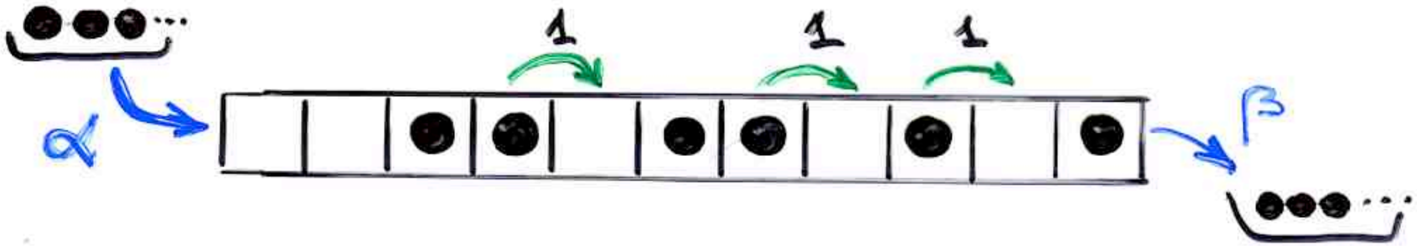
Derrida, Evans, Hakim, Pasquier (1993)

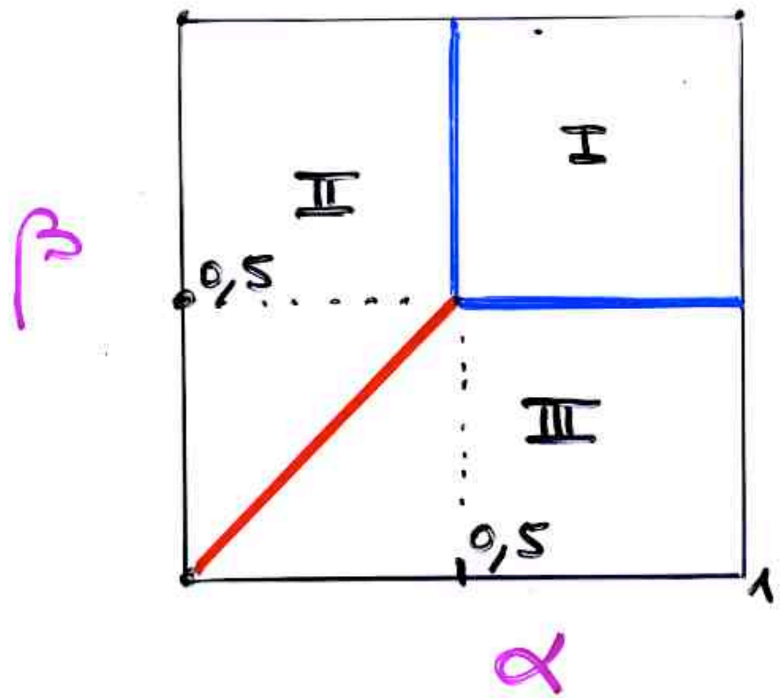
boundary induced phase transitions

molecular diffusion
linear array of enzymes
biopolymers
traffic flow
------ formation of shocks

TASEP

"totally asymmetric exclusion process"





$n \to \infty$

$\rho = \langle \tau_i \rangle$ = taux moyen d'occupation

i loin des bords

(I) $\rho = 1/2$

(II) $\rho = \alpha$

(III) $\rho = 1-\beta$

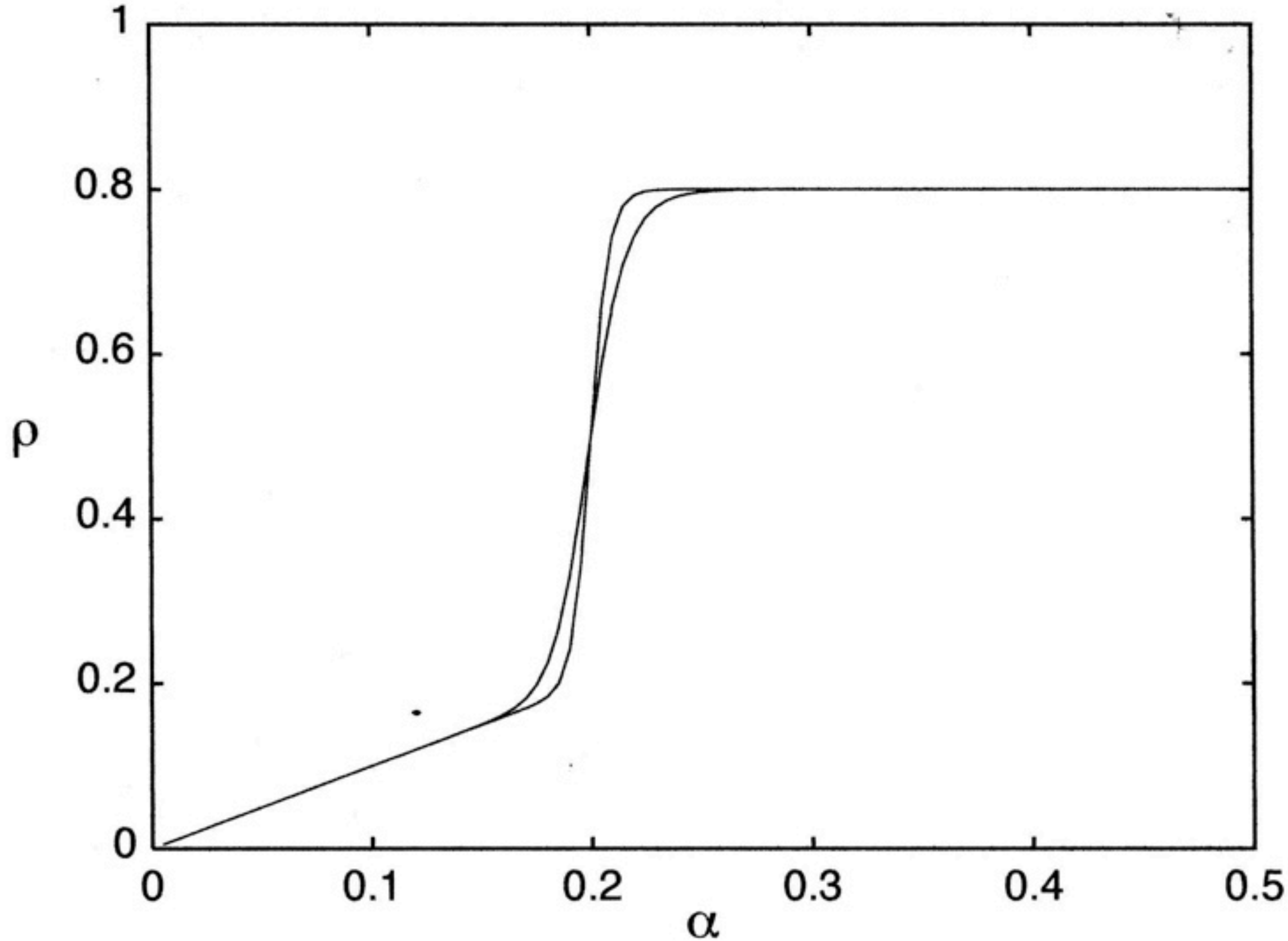
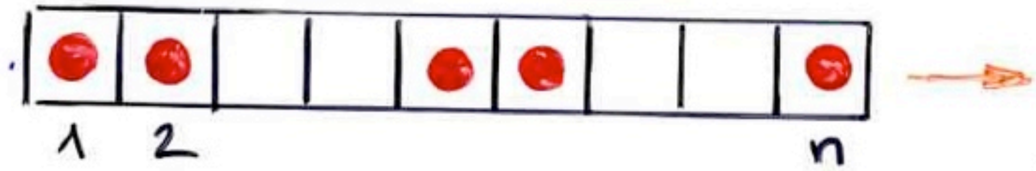
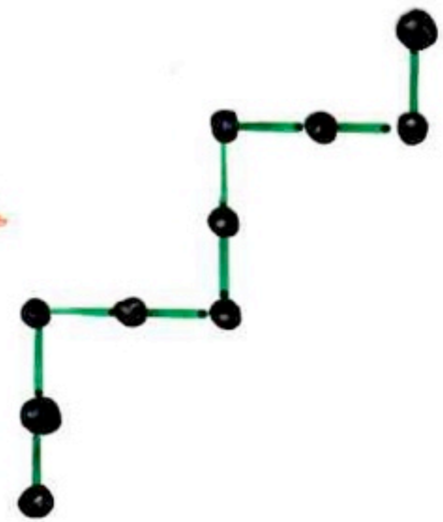


Figure 2: The average occupation $\rho = \langle \tau_{(N+1)/2} \rangle$ of the central site versus α for $N = 61$ and $N = 121$ when $\beta = .2$.

state $s = (\tau_1, \dots, \tau_n)$

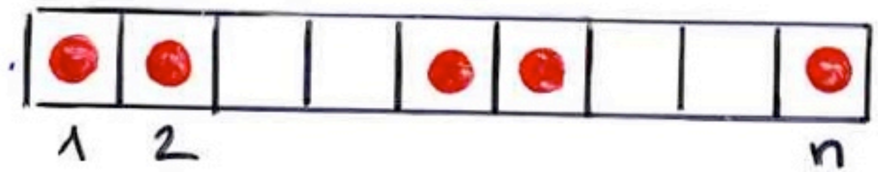


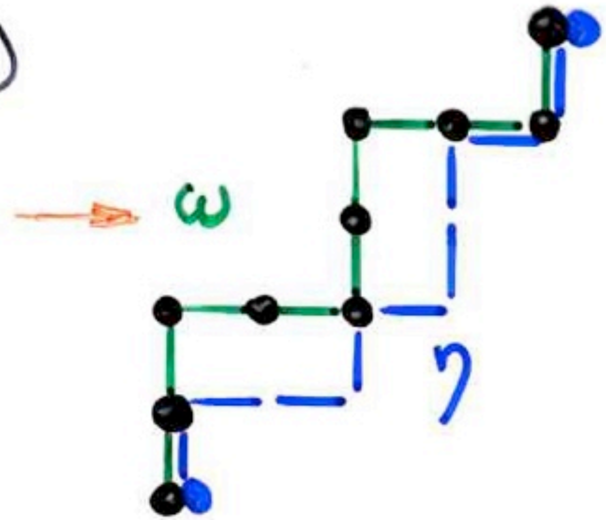


$\mathcal{P}_n(s) =$

Shapiro, Zeilberger, 1982

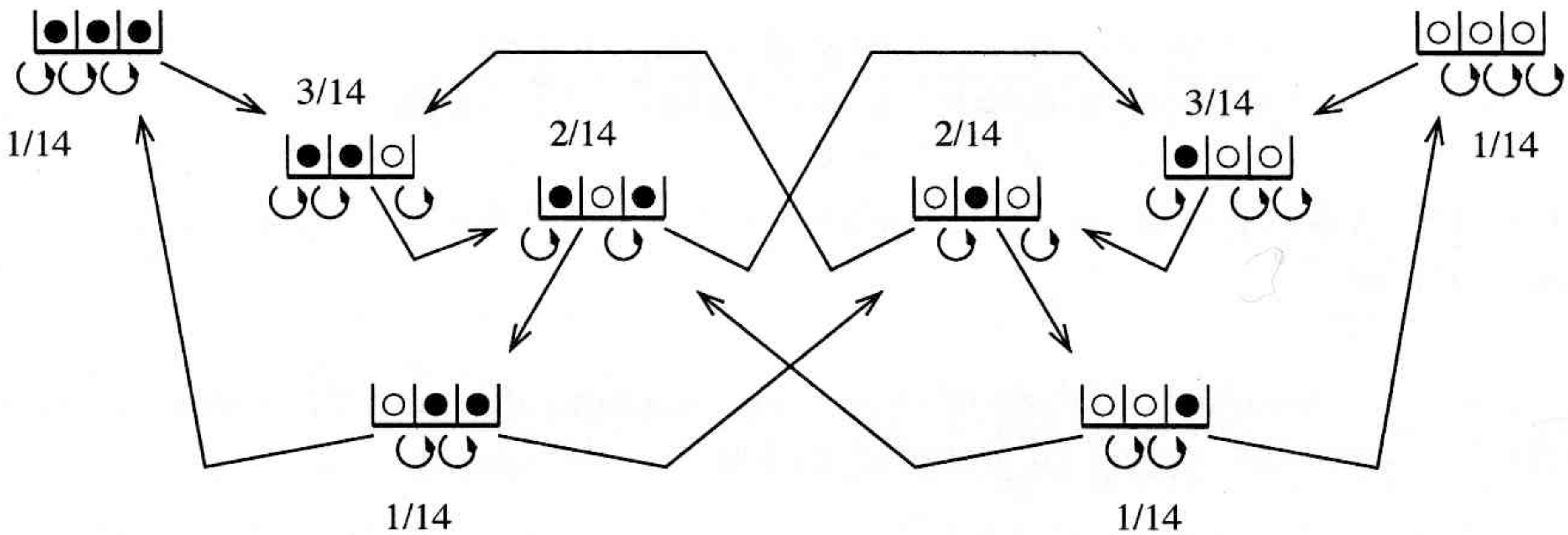
state $s = (\tau_1, \dots, \tau_n)$





$$\mathcal{P}_n(s) = \frac{1}{C_{n+1}} \left(\begin{array}{c} \text{number of paths } \eta \\ \text{below the path } \omega \\ \text{associated to } s \end{array} \right)$$

Shapiro, Zeilberger, 1982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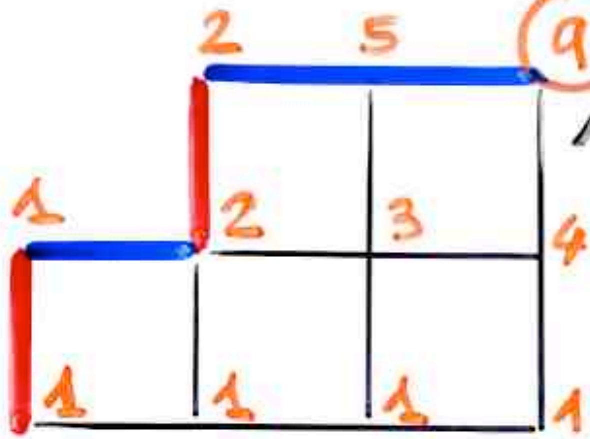
$\delta = (1,0,1,0,0) \quad \lambda = (1,2,2)$



$9 / C_6 = P(1,0,1,0,0)$

Combinatorics of the PASEP

TASEP

Brak, Essam (2003), Duchi, Schaeffer, (2004),
Angel (2005), xgv, (2007)

(P) ASEP

Brak, Corteel, Essam, Parviainen, Rechnitzer (2006)
Corteel, Williams (2006) (2008) (2009) xgv, (2008)
Corteel, Stanton, Stanley, Williams (2010)

Derrida, ...
Mallick, ... Golinelli, Mallick (2006)

- Orthogonal polynomials

→ Sasamoto (1999)

→ Blythe, Evans, Colaiori, Essler (2000)

q-Hermite polynomial

$\alpha, \beta, q \qquad \gamma = \delta = 1$

$$D = \frac{1}{1-q} + \frac{1}{\sqrt{1-q}}\,\hat{a}$$

$$E = \frac{1}{1-q} + \frac{1}{\sqrt{1-q}}\,\hat{a}^{+}$$

$$\hat{a}\,\hat{a}^{+} - q\,\hat{a}^{+}\hat{a} = 1$$

→ Uchiyama, Sasamoto, Wadati (2003)

$\alpha, \beta, \gamma, \delta, \qquad q$

Askey-Wilson polynomials

The Matrix Ansatz

Derrida, Evans, Hakim, Pasquier

$$\mathcal{P}_n(\tau_1, \dots, \tau_n) = f_n(\tau_1, \dots, \tau_n) / Z_n$$

$$Z_n = \sum_{\tau} f_n(\tau_1, \dots, \tau_n)$$

partition function

Derrida, Evans, Hakim, Pasquier (1993)

"matrix ansatz"

D E matrices,

V column vector, W row vector

$$\begin{cases} DE = qED + D + E \\ (\beta D - \delta E)|V\rangle = |V\rangle \\ \langle W|(\alpha E - \gamma D) = \langle W| \end{cases}$$

Then

$f_n(\tau_1, \dots, \tau_n)$

Derrida, Evans, Hakim, Pasquier (1993)

"matrix ansatz"

D E matrices,

V column vector, W row vector

TASEP

q=0

$$\begin{cases} DE = \boxed{} + D + E \\ (\beta D - \boxed{}) |V\rangle = |V\rangle \\ \langle W | (\alpha E - \boxed{}) = \langle W | \end{cases}$$

Then

$$f_n(\tau_1, \dots, \tau_n) = \langle W | \prod_{i=1}^{n} \left(\tau_i D + (1-\tau_i) E \right) | V \rangle$$

examples:

TASEP

$$\begin{cases} DE = D + E \\ D|V\rangle = \bar{\beta}\,|V\rangle \\ \langle W|E = \bar{\alpha}\,\langle W| \end{cases}$$

examples:

TASEP

$$D = \begin{bmatrix} 0 & \bar{\beta} & 0 & \cdots \\ 0 & 0 & \bar{\beta} & \cdots \\ \vdots & & \ddots & \ddots \\ 0 & \cdots & & 0 \end{bmatrix} \qquad E = \begin{bmatrix} \bar{\alpha} & 0 & 0 & 0 & \cdots \\ \bar{\alpha}\beta & 1 & 0 & 0 & \cdots \\ \bar{\alpha}\beta^2 & \beta & 1 & 0 & \cdots \\ \bar{\alpha}\beta^3 & \beta^2 & \beta & 1 & \cdots \\ \vdots & \vdots & \vdots & & \ddots \end{bmatrix}$$

(infinite matrices)

$$\bar{\beta} = \frac{1}{\beta}, \quad \bar{\alpha} = \frac{1}{\alpha}$$

$$\langle W| = (1, 0, \cdots 0, \cdots) \qquad |V\rangle = (1, 1, \cdots 1, \cdots)^T$$

$$D = \begin{bmatrix} 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & & \ddots & 1 \\ 0 & \cdots & & 0 \end{bmatrix} \qquad E = \begin{bmatrix} \bar{\beta} & & & \\ \bar{\beta} & 1 & & \\ \vdots & \vdots & \ddots & \\ \bar{\beta} & 1 & \cdots & 1 \end{bmatrix}$$

(infinite matrices)

$$\bar{\beta} = \frac{1}{\beta}, \quad \bar{\alpha} = \frac{1}{\alpha}$$

$$\langle W| = (1, 0, \cdots) \qquad |V\rangle = (1, \bar{\alpha}, \bar{\alpha}^2, \cdots)^T$$

examples:

TASEP

$$D = \begin{bmatrix} \bar{\beta} & \kappa & & & 0 \\ & 1 & 1 & & \\ & & 1 & \ddots & \\ & 0 & & \ddots & 1 \\ 0 & & & & 1 \end{bmatrix} \qquad E = \begin{bmatrix} \bar{\alpha} & 0 & & & 0 \\ \kappa & 1 & & & \\ & 1 & 1 & & 0 \\ 0 & & \ddots & \ddots & \\ & & & 1 & 1 \end{bmatrix}$$

(infinite matrices)

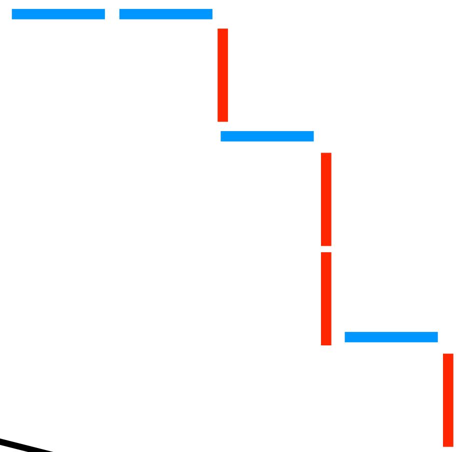
$$\langle W| = (1, 0, \cdots, 0) \qquad |V\rangle = (1, 0, \cdots, 0)$$

$$\bar{\alpha} = \frac{1}{\alpha} \qquad \bar{\beta} = \frac{1}{\beta} \qquad \kappa^2 = \bar{\alpha} + \bar{\beta} - \bar{\alpha}\bar{\beta}$$

The PASEP algebra

$$DE = qED + E + D$$

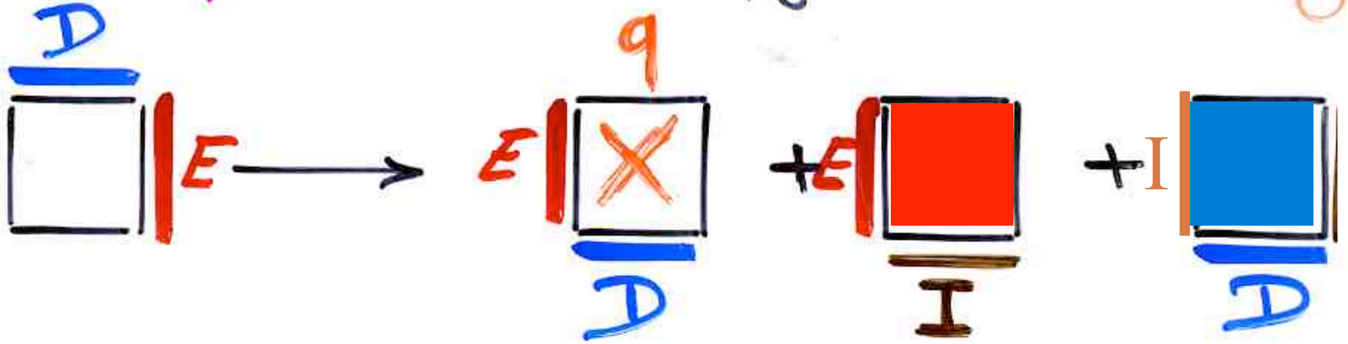
D D E D E E D E



D DE (D E) E D E

DDE(E)EDE + DDE(ED)EDE + DDE(D)EDE

Proof: "planarization" of the rewriting rules



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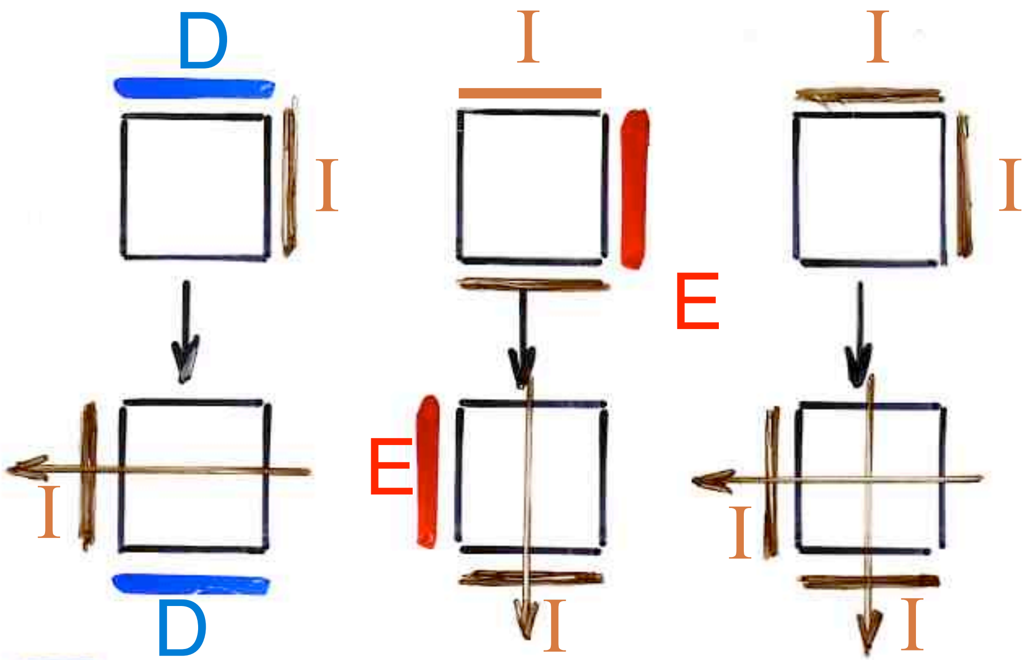

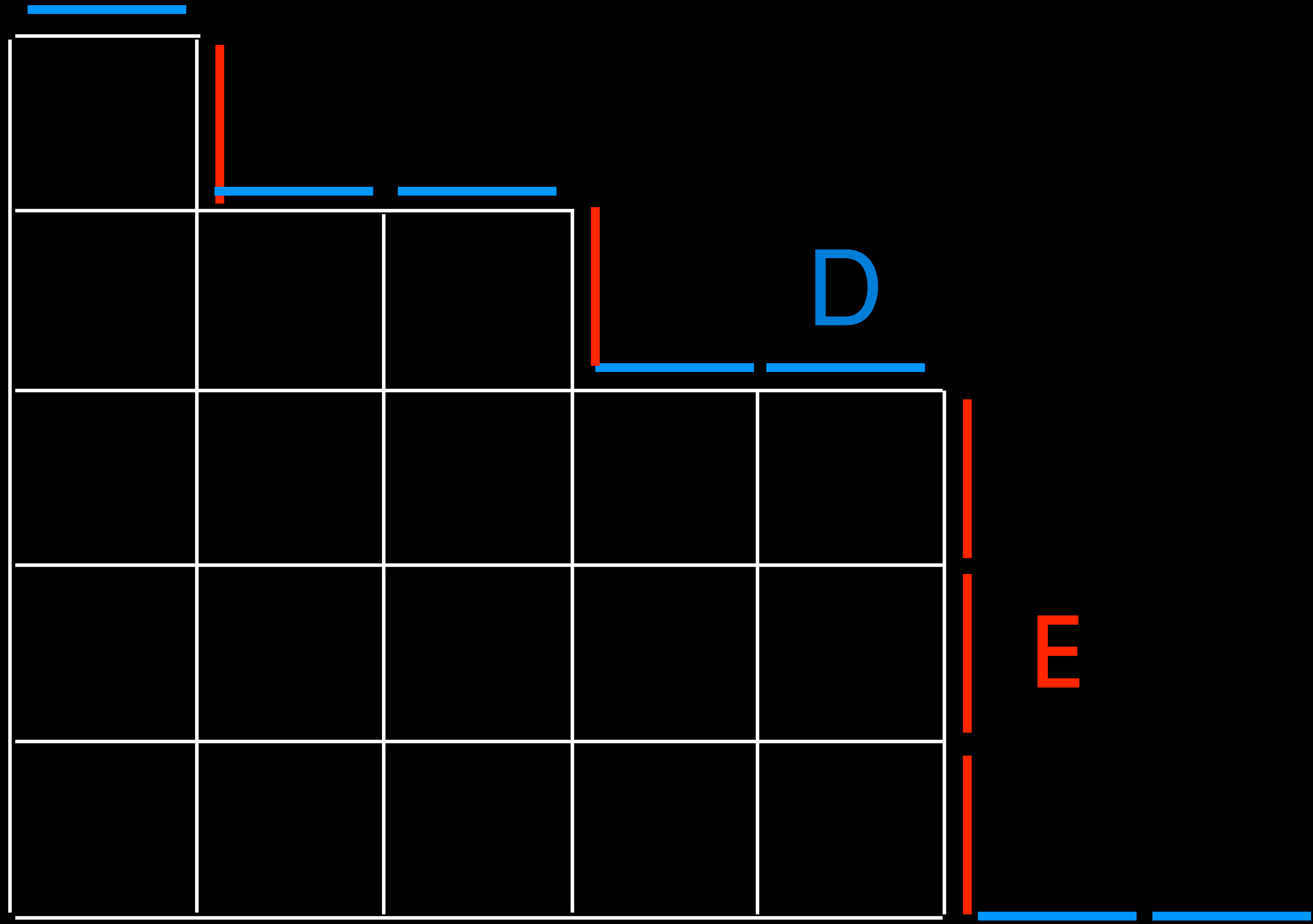
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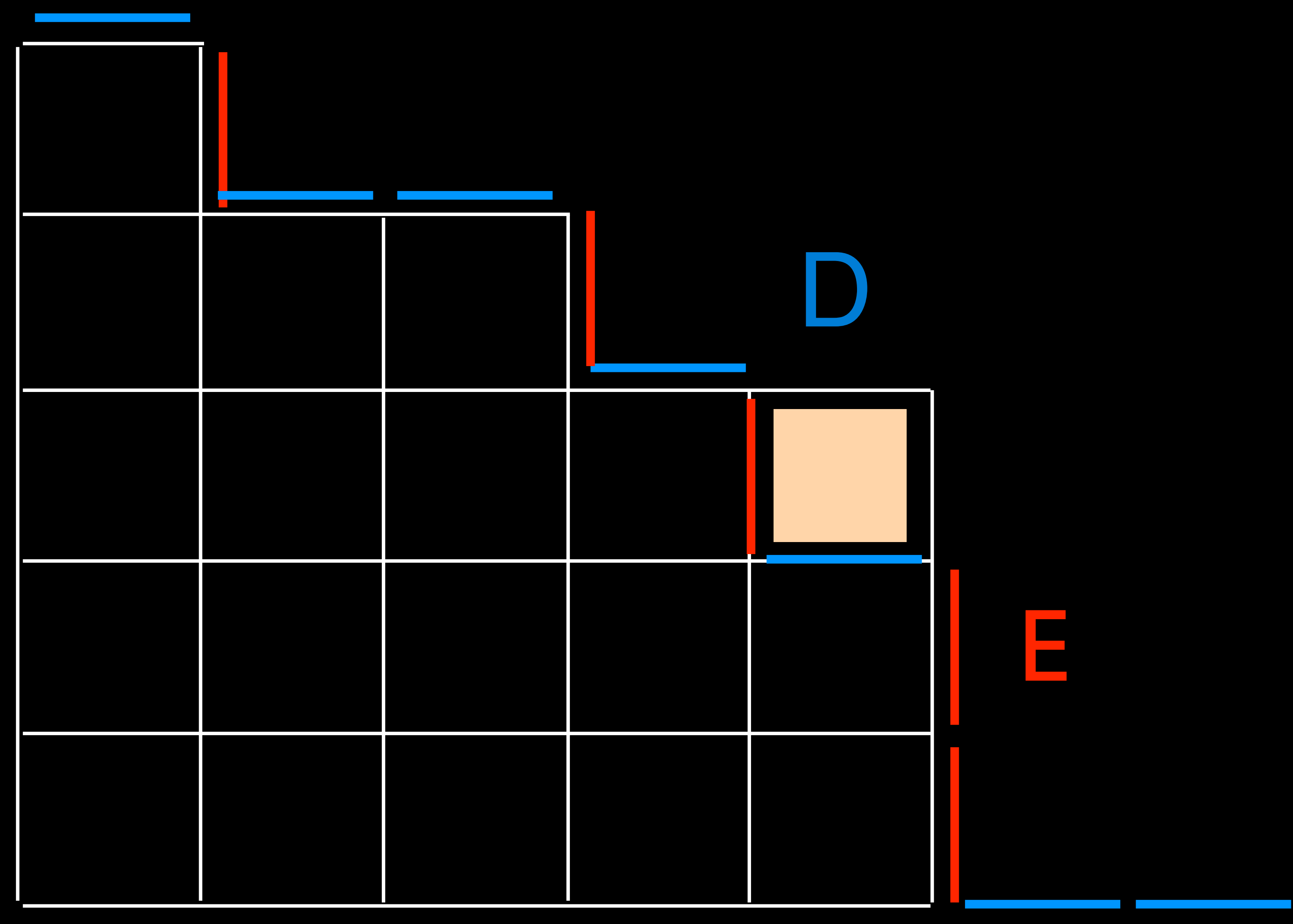
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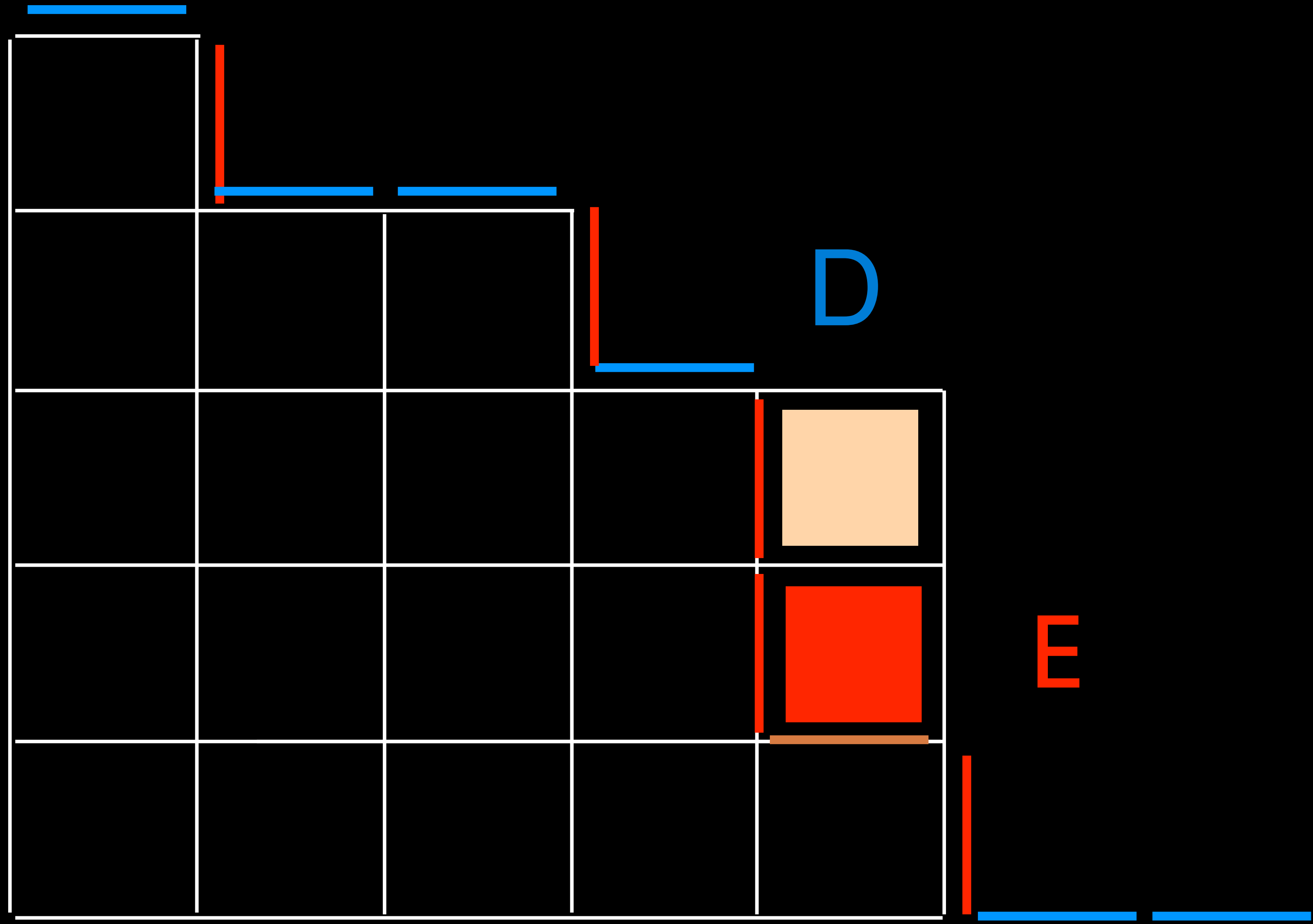
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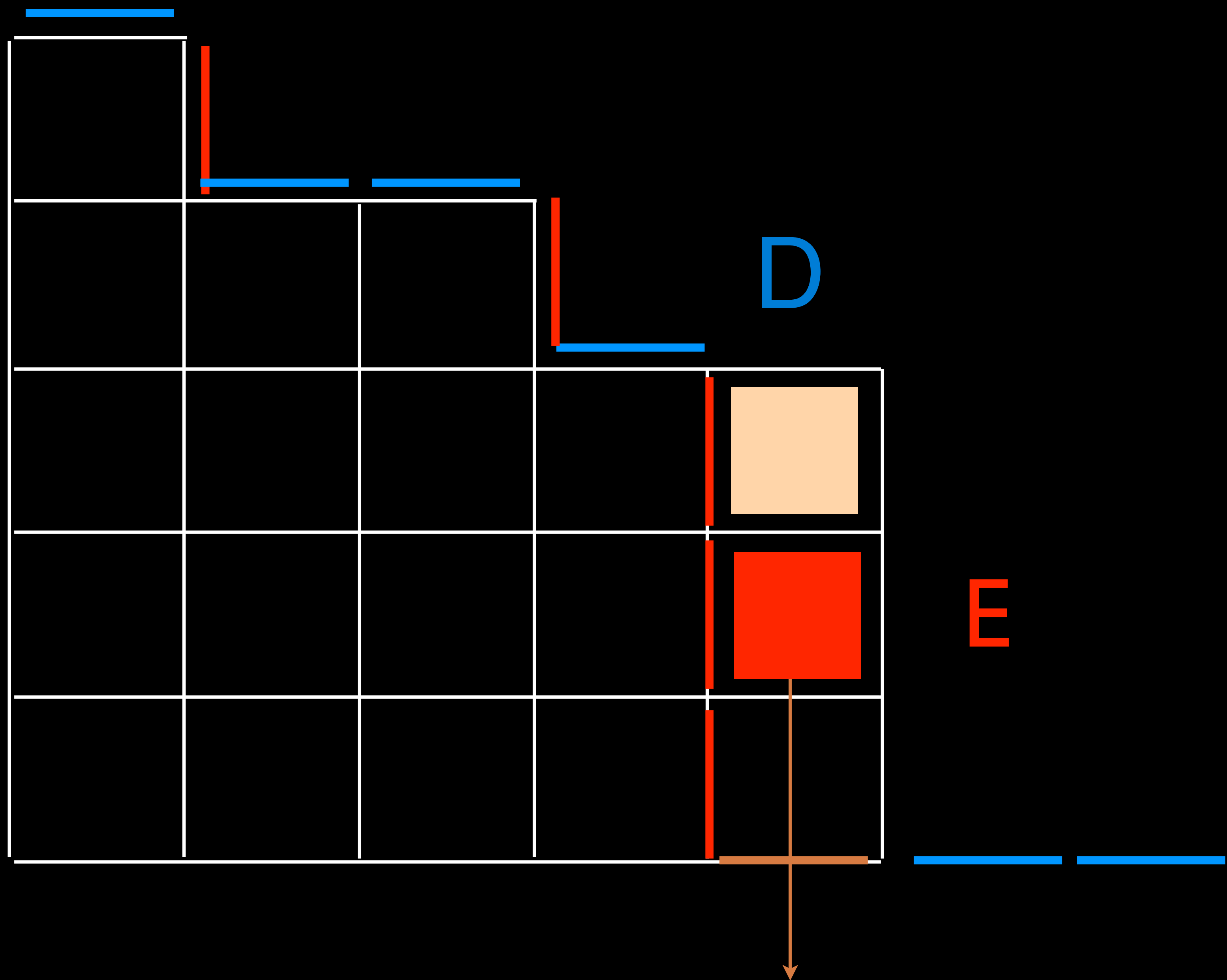
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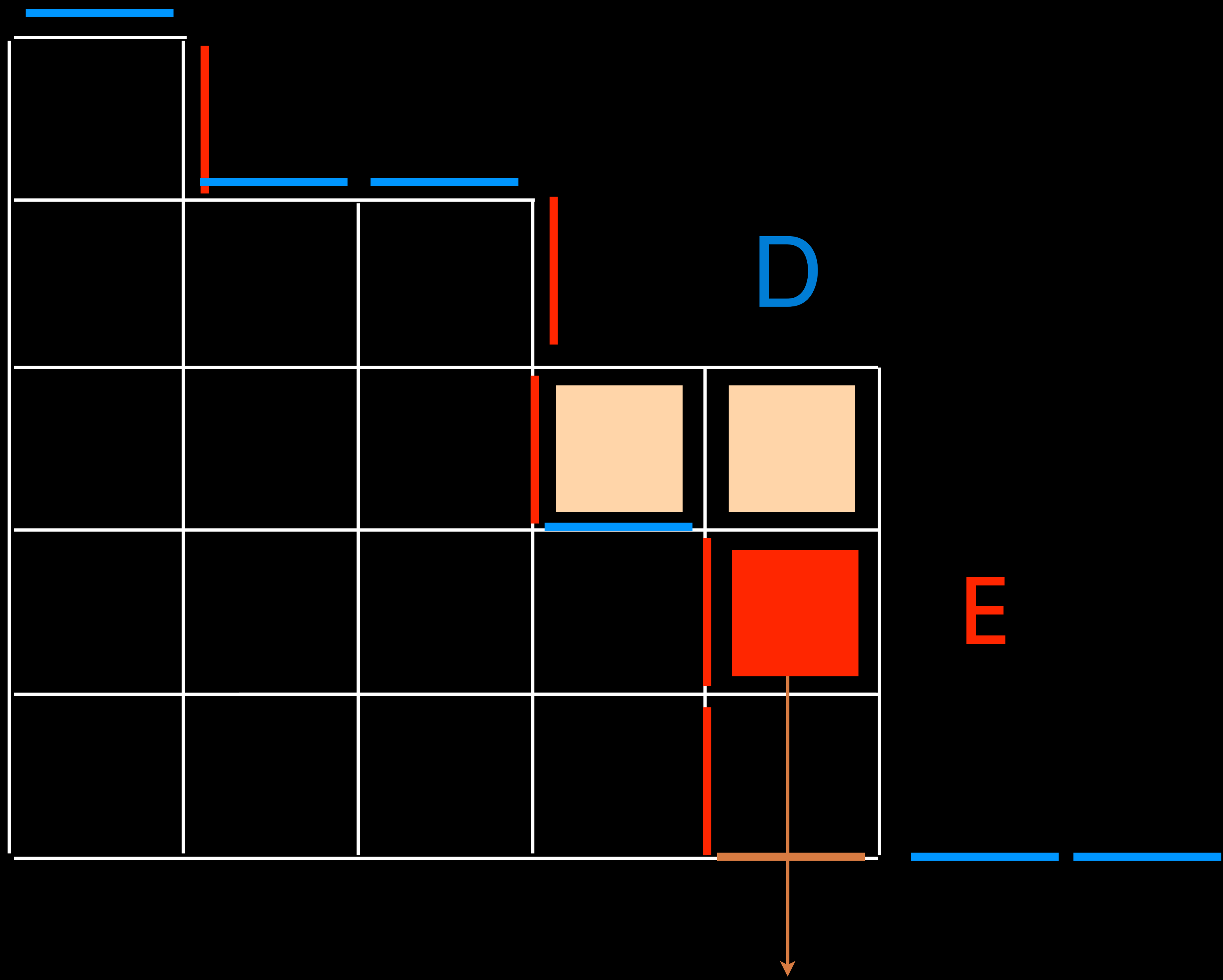
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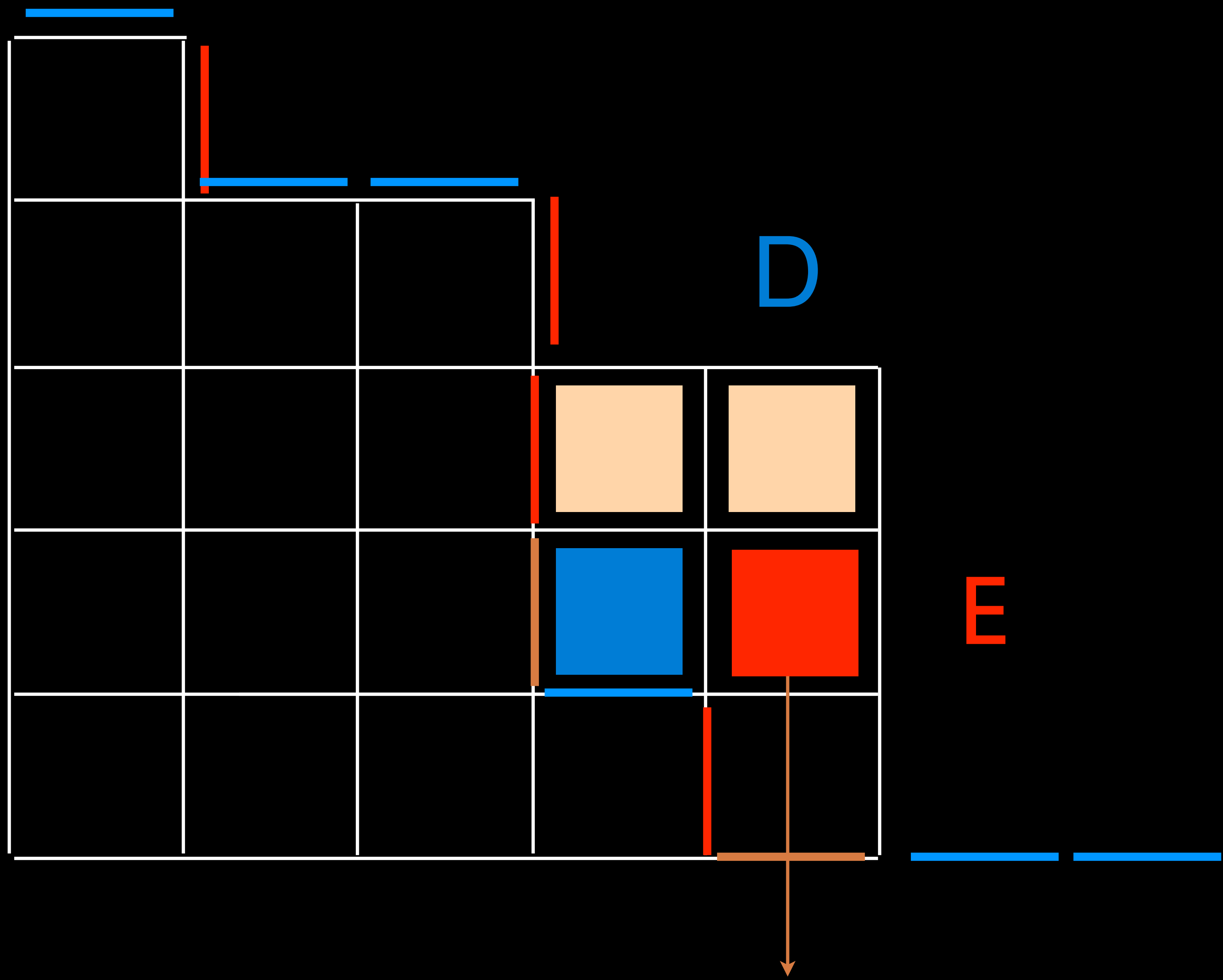
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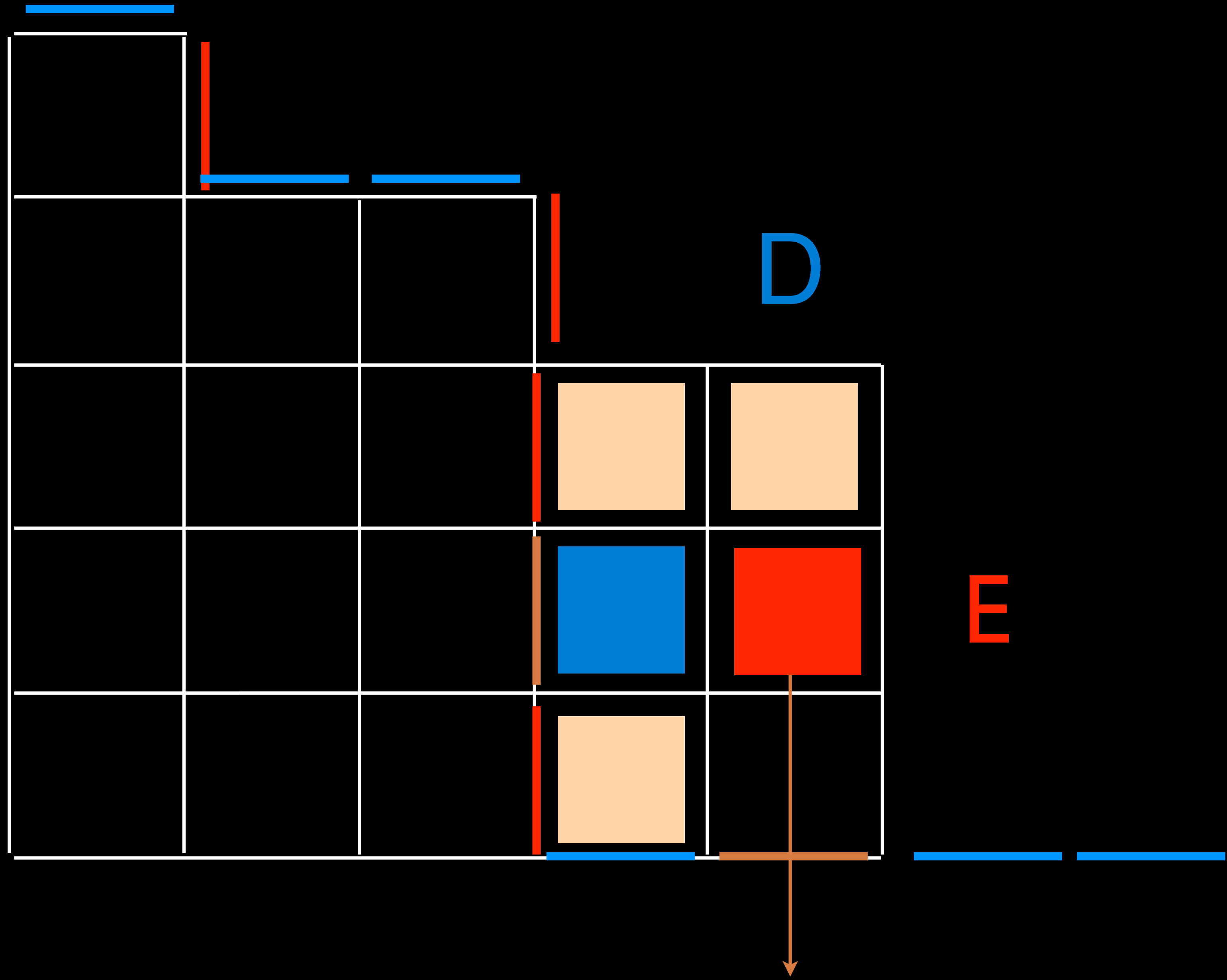
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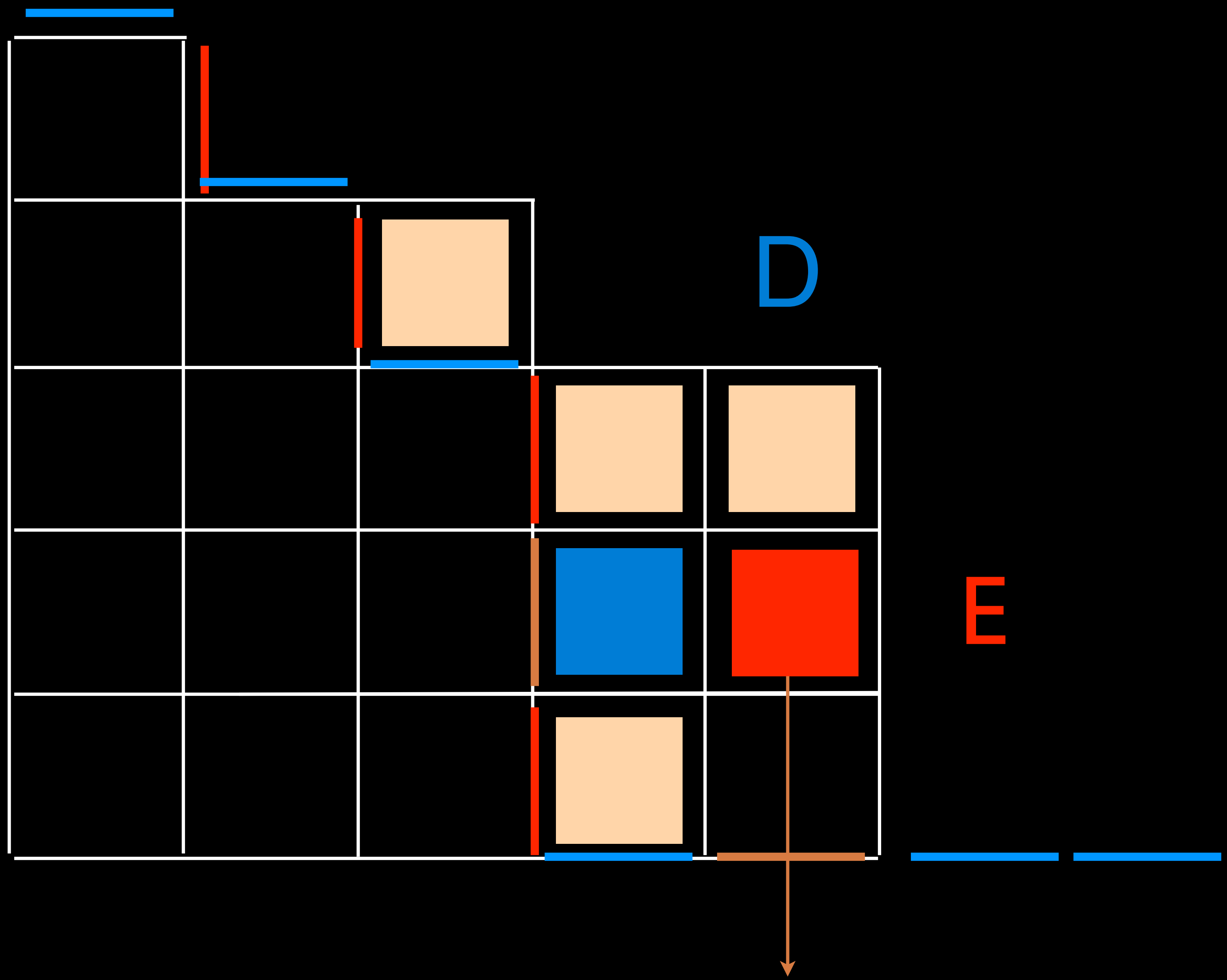
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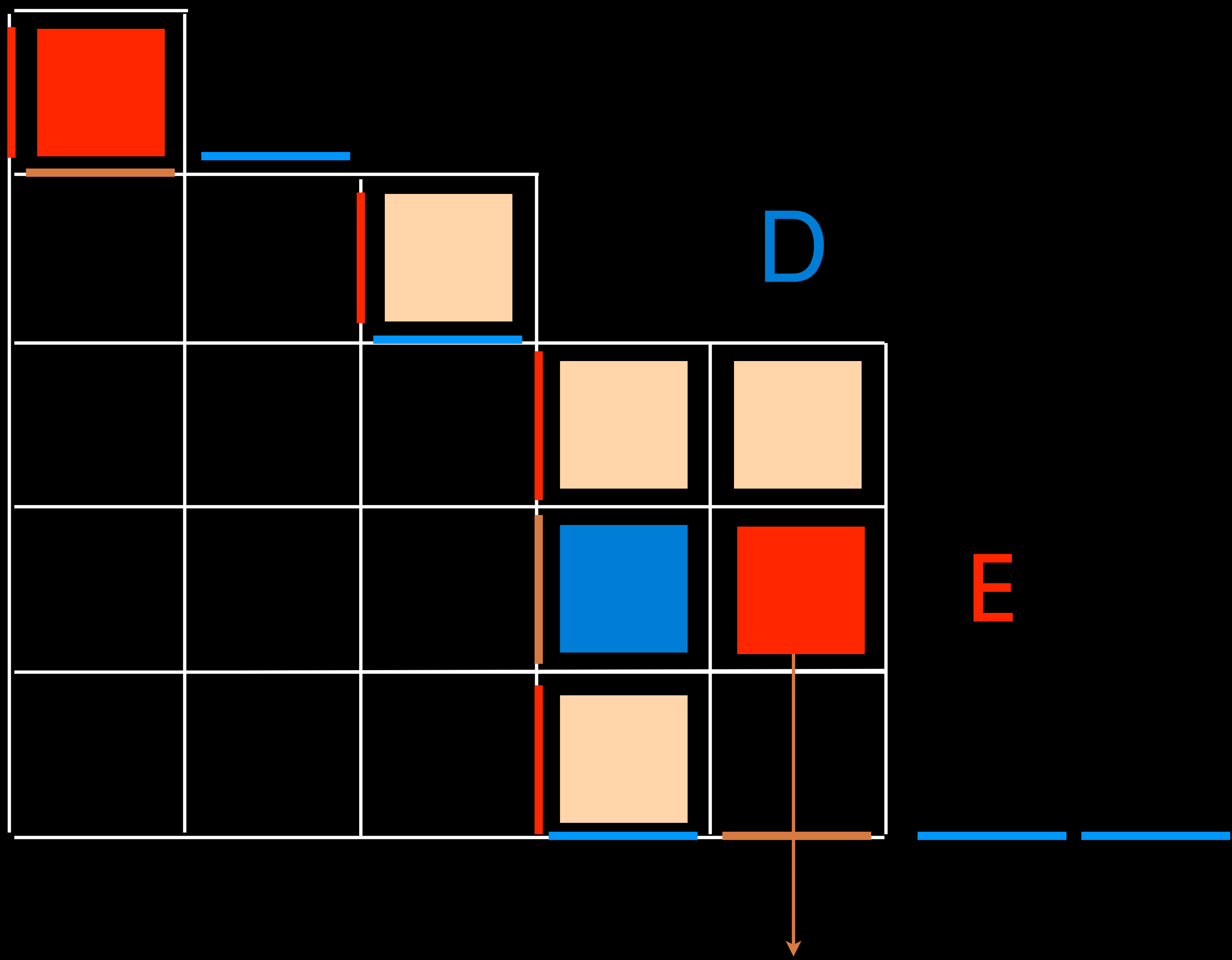
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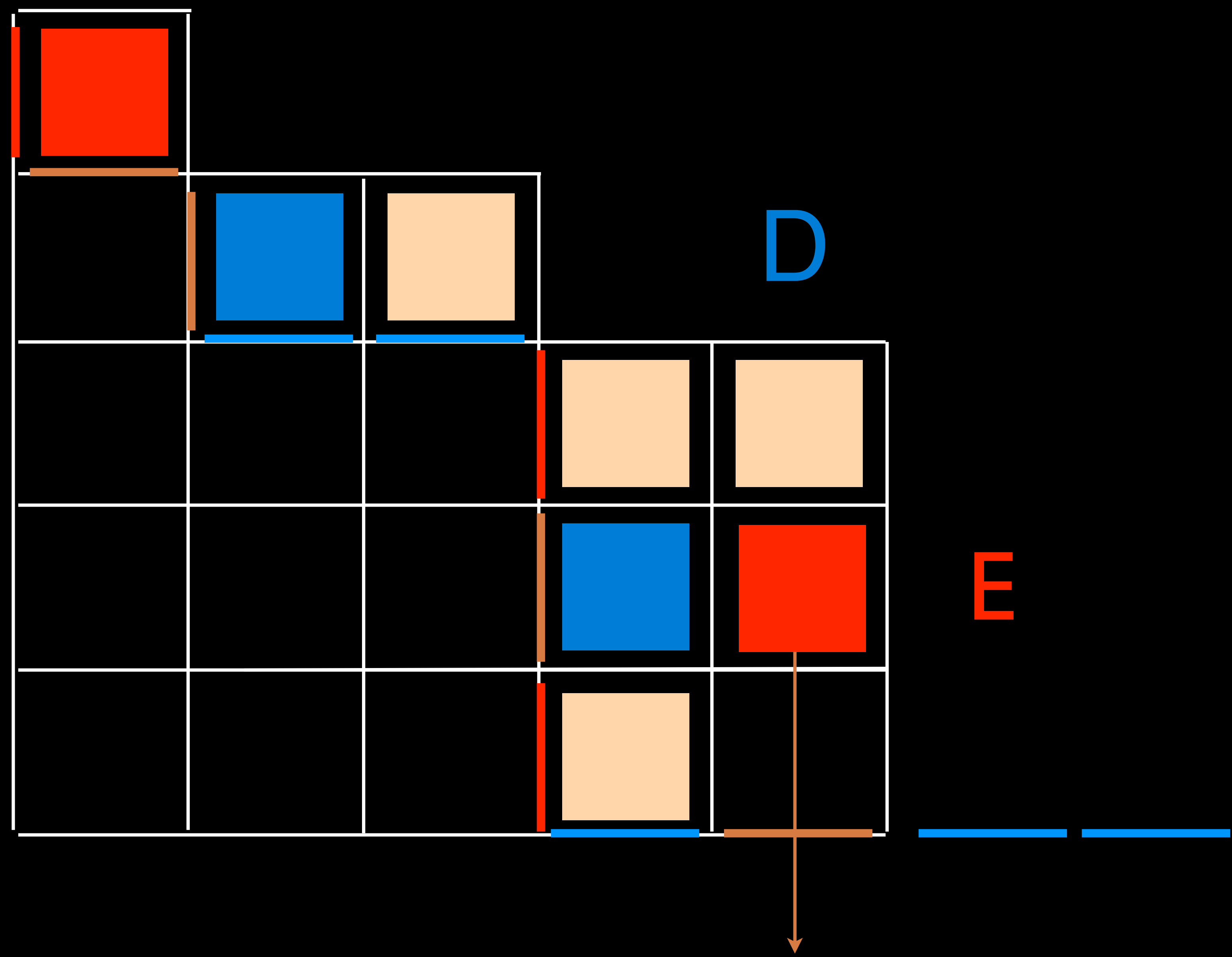
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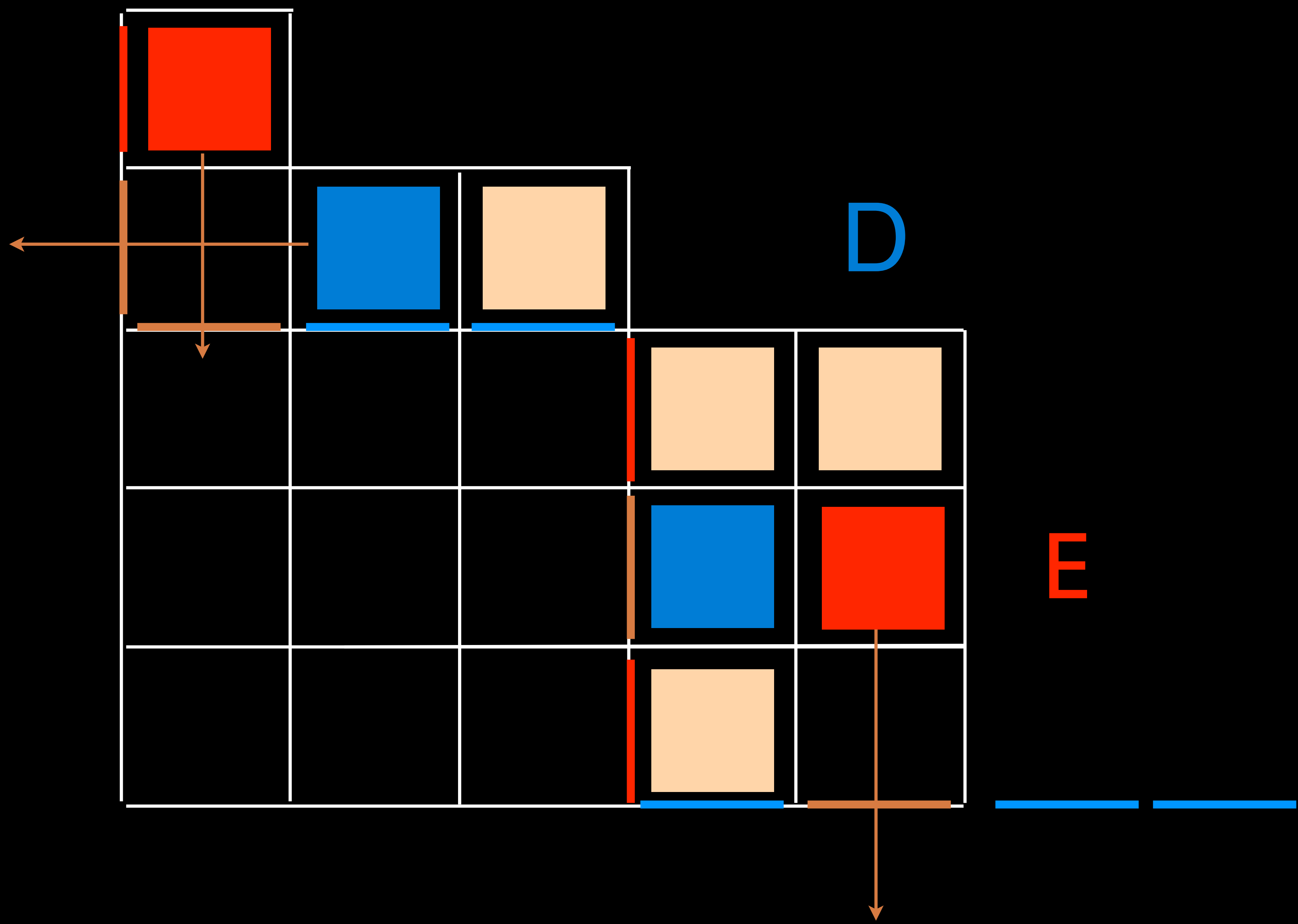
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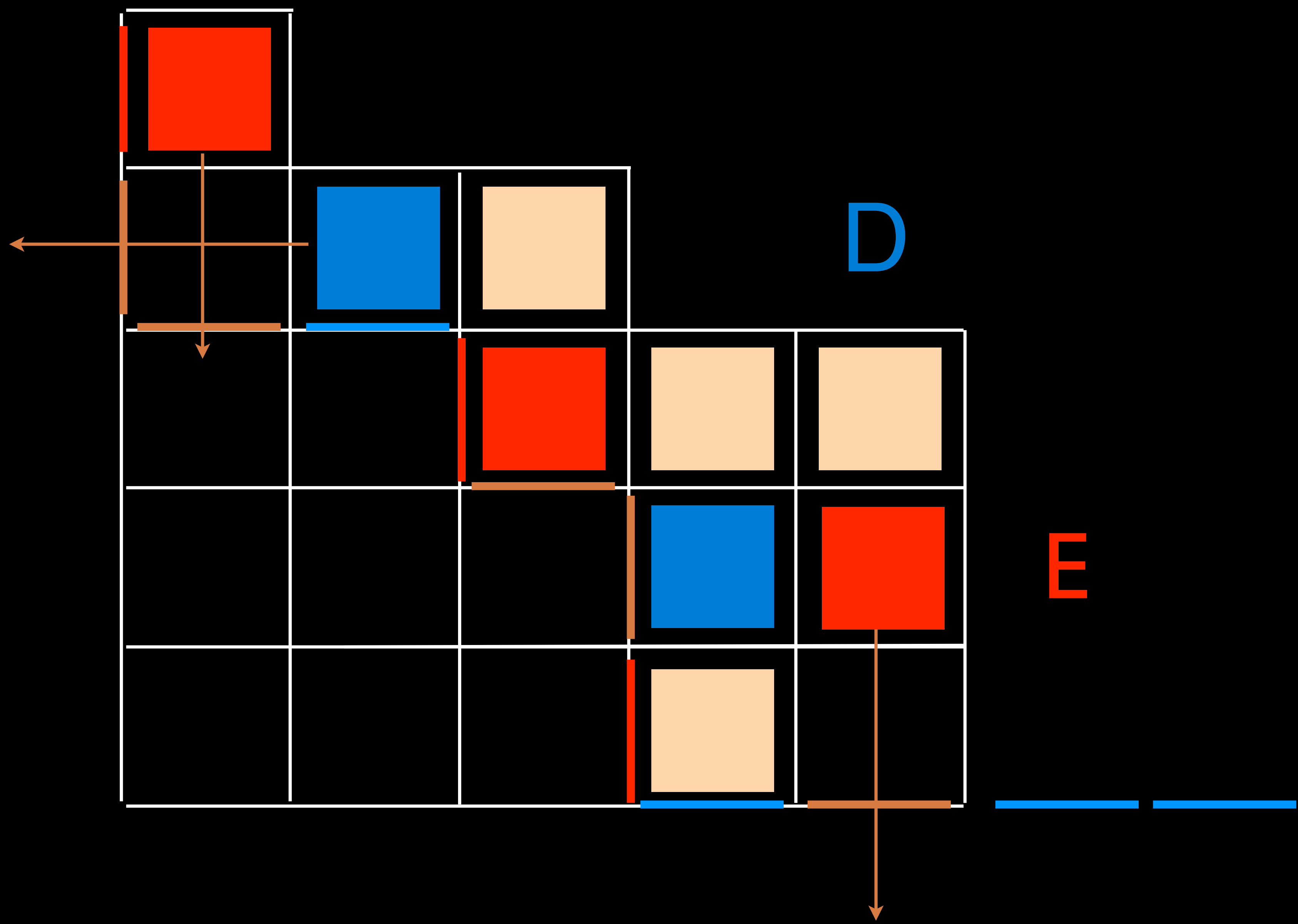
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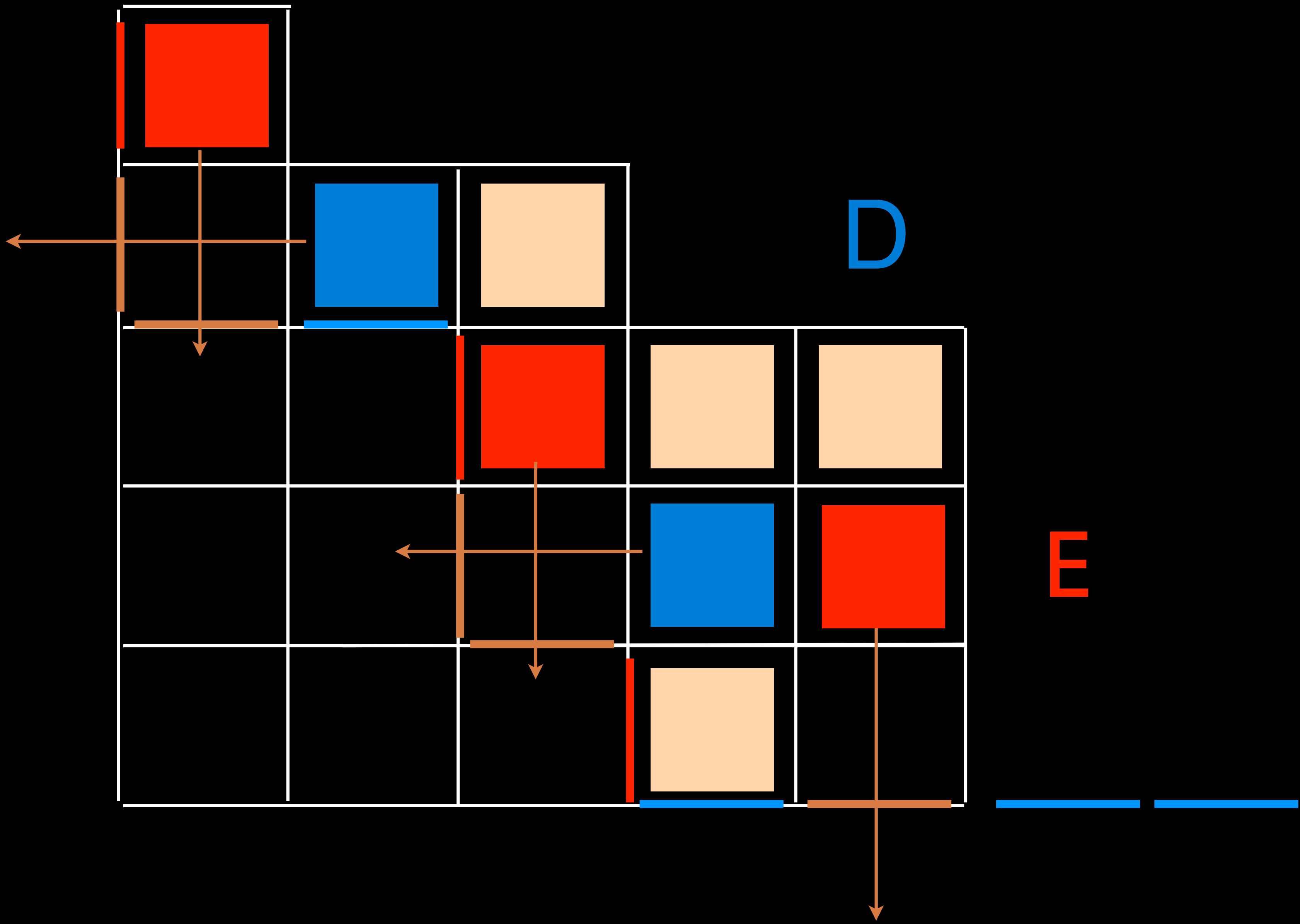
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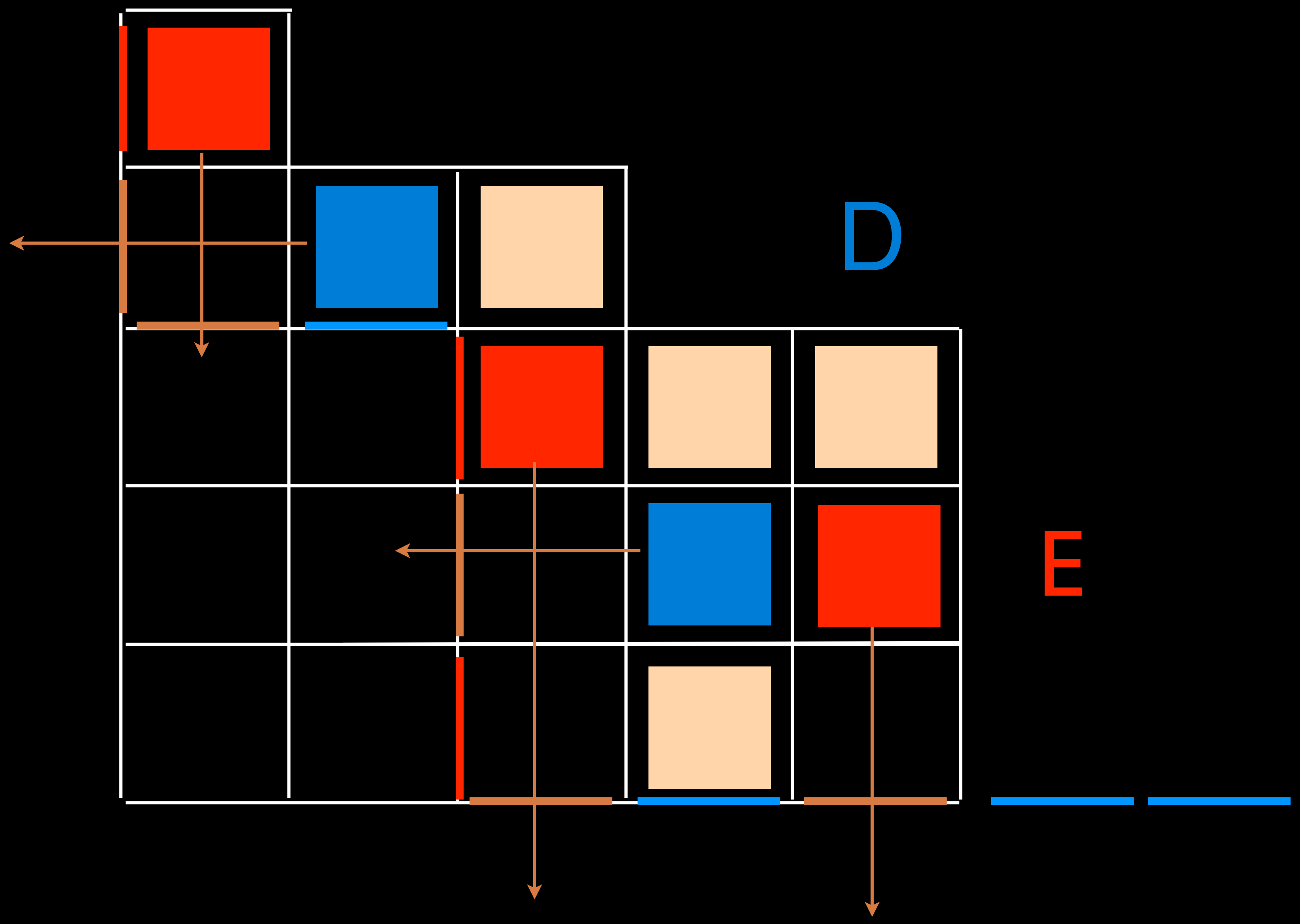
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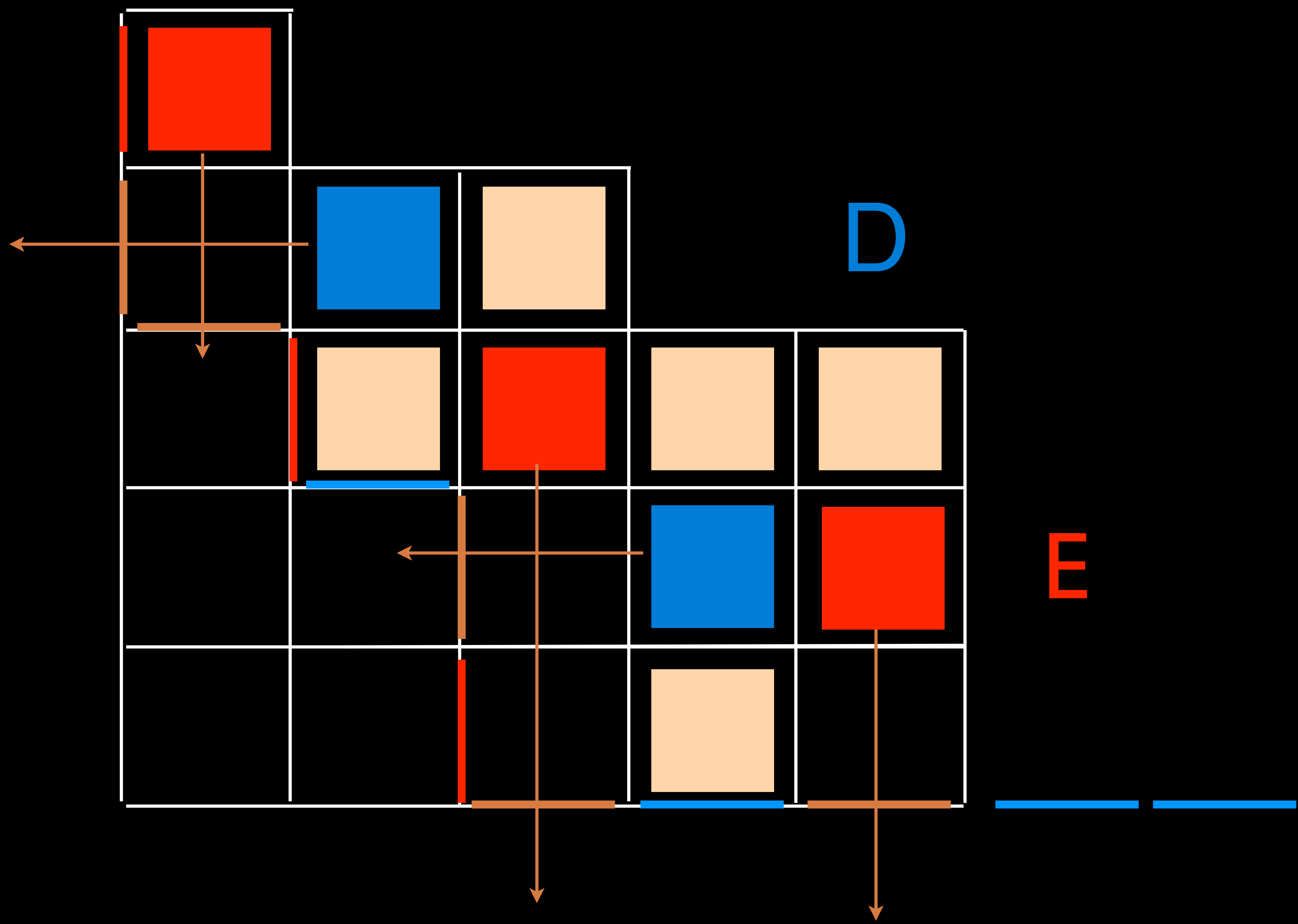

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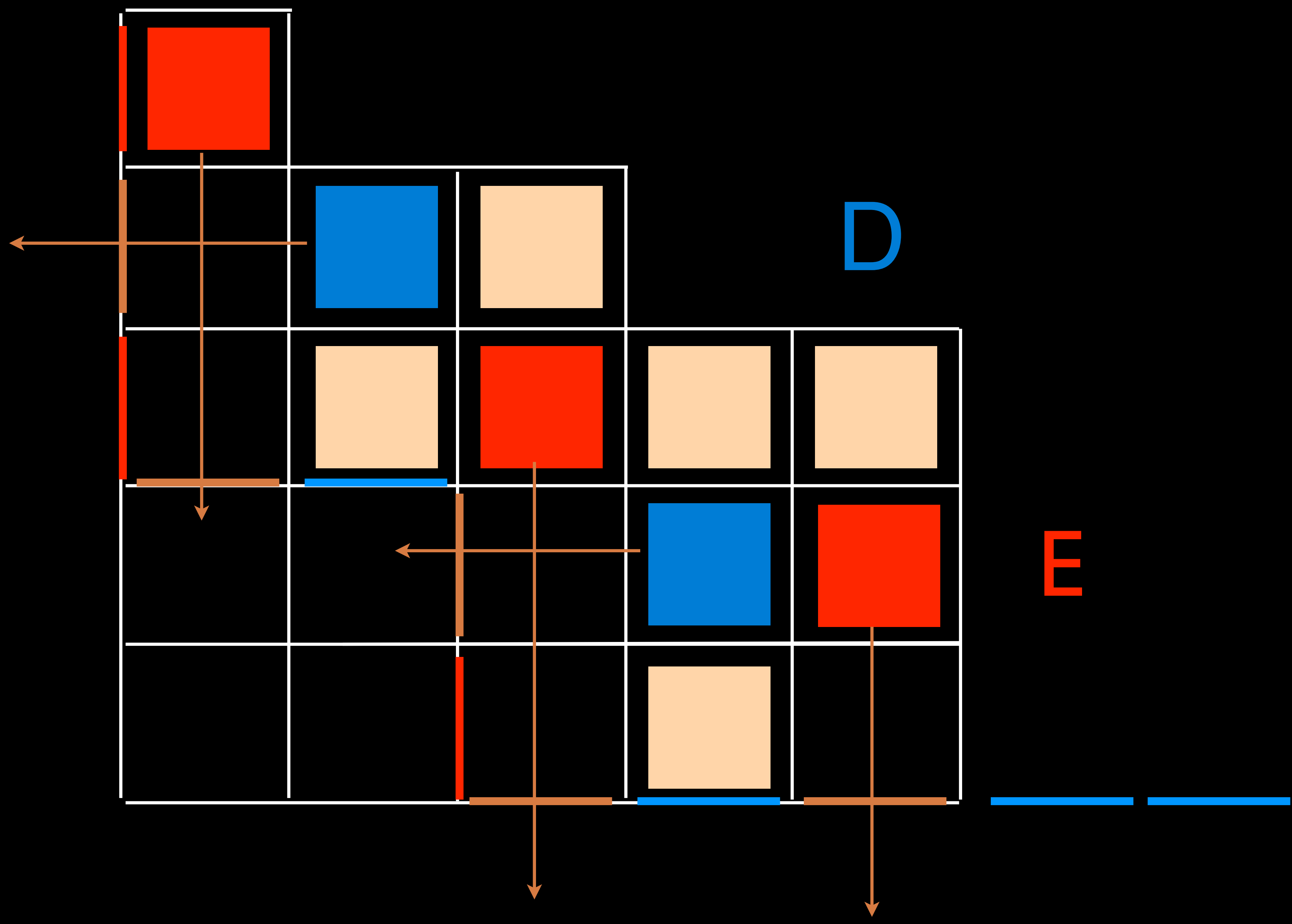
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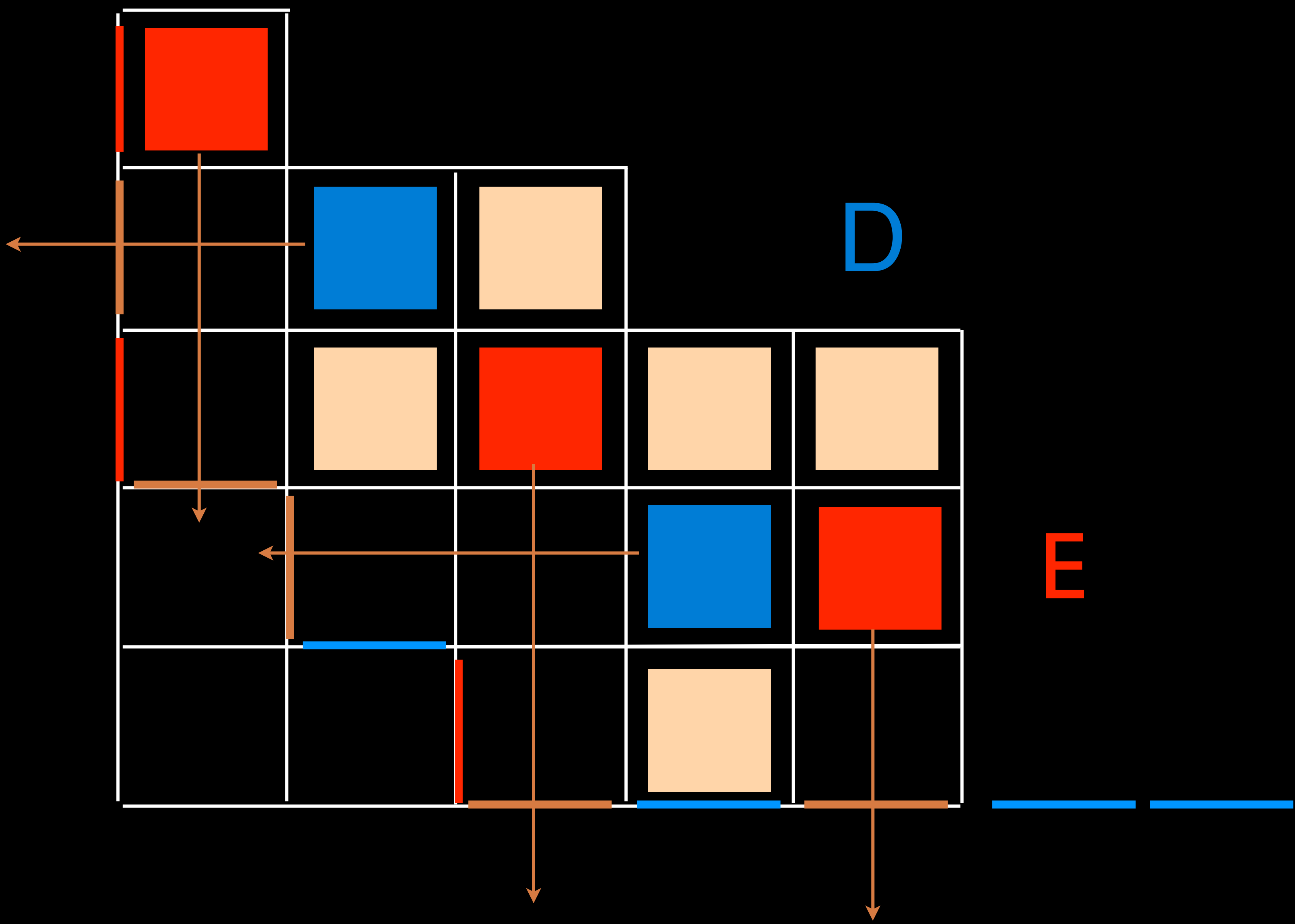
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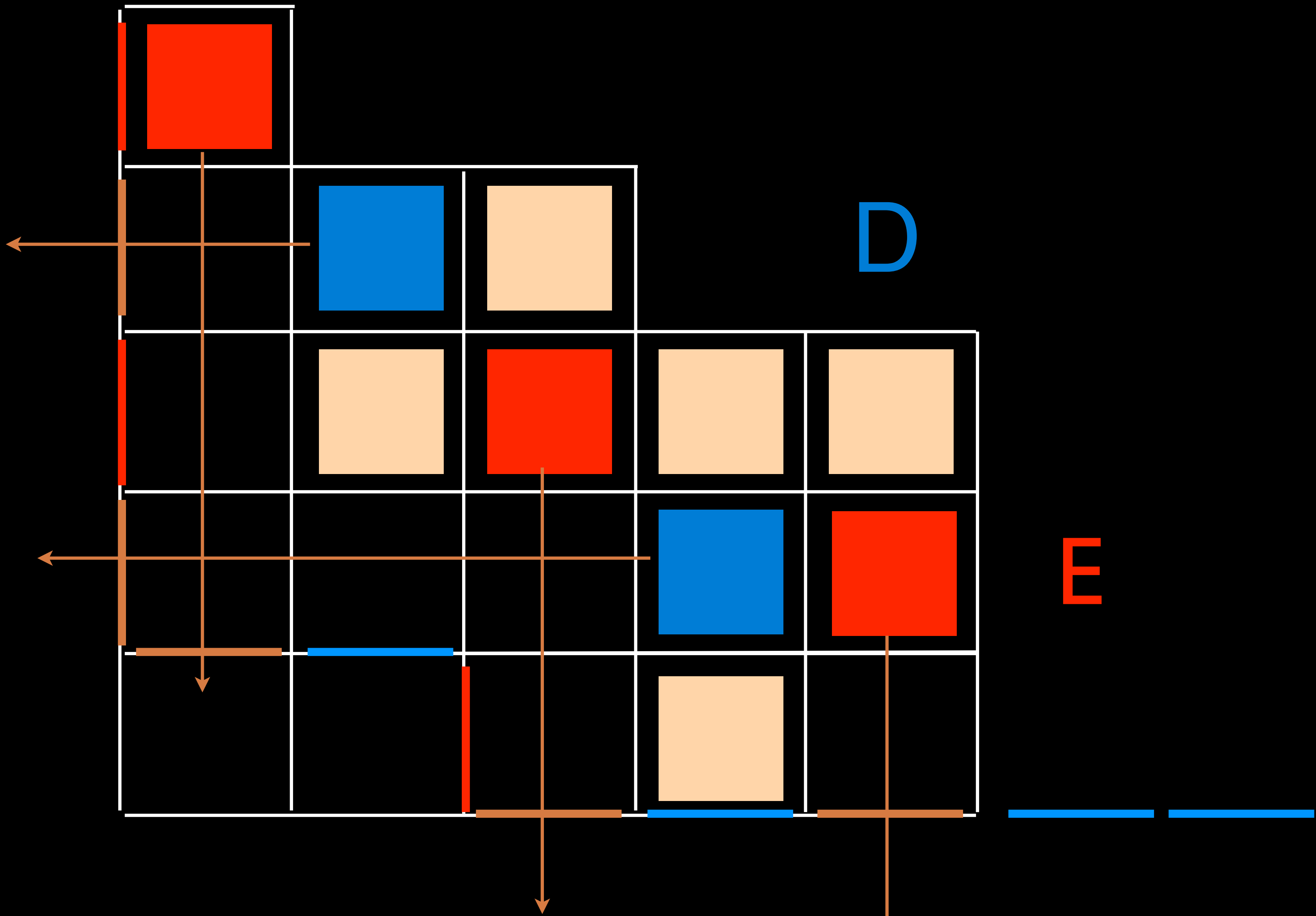
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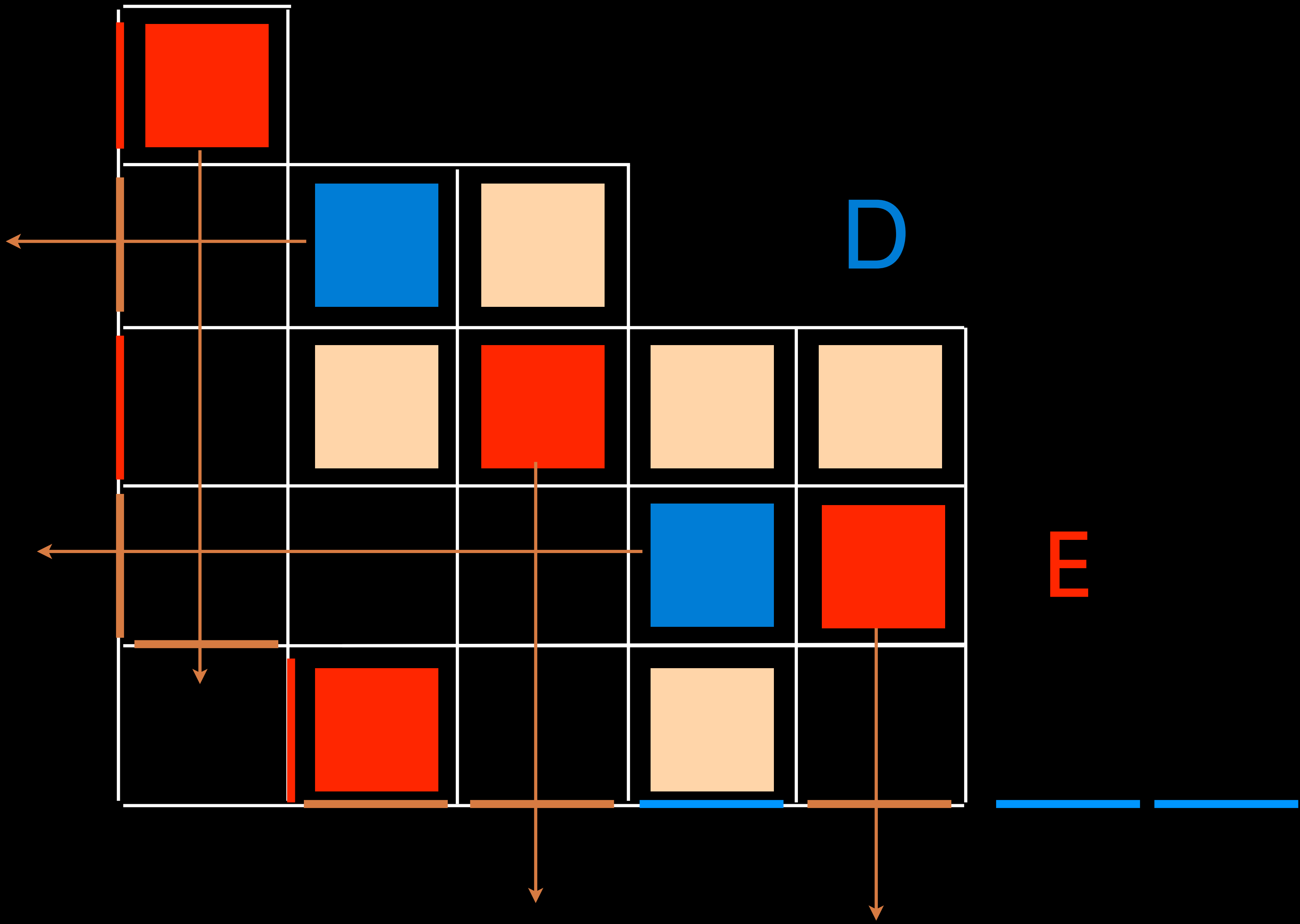
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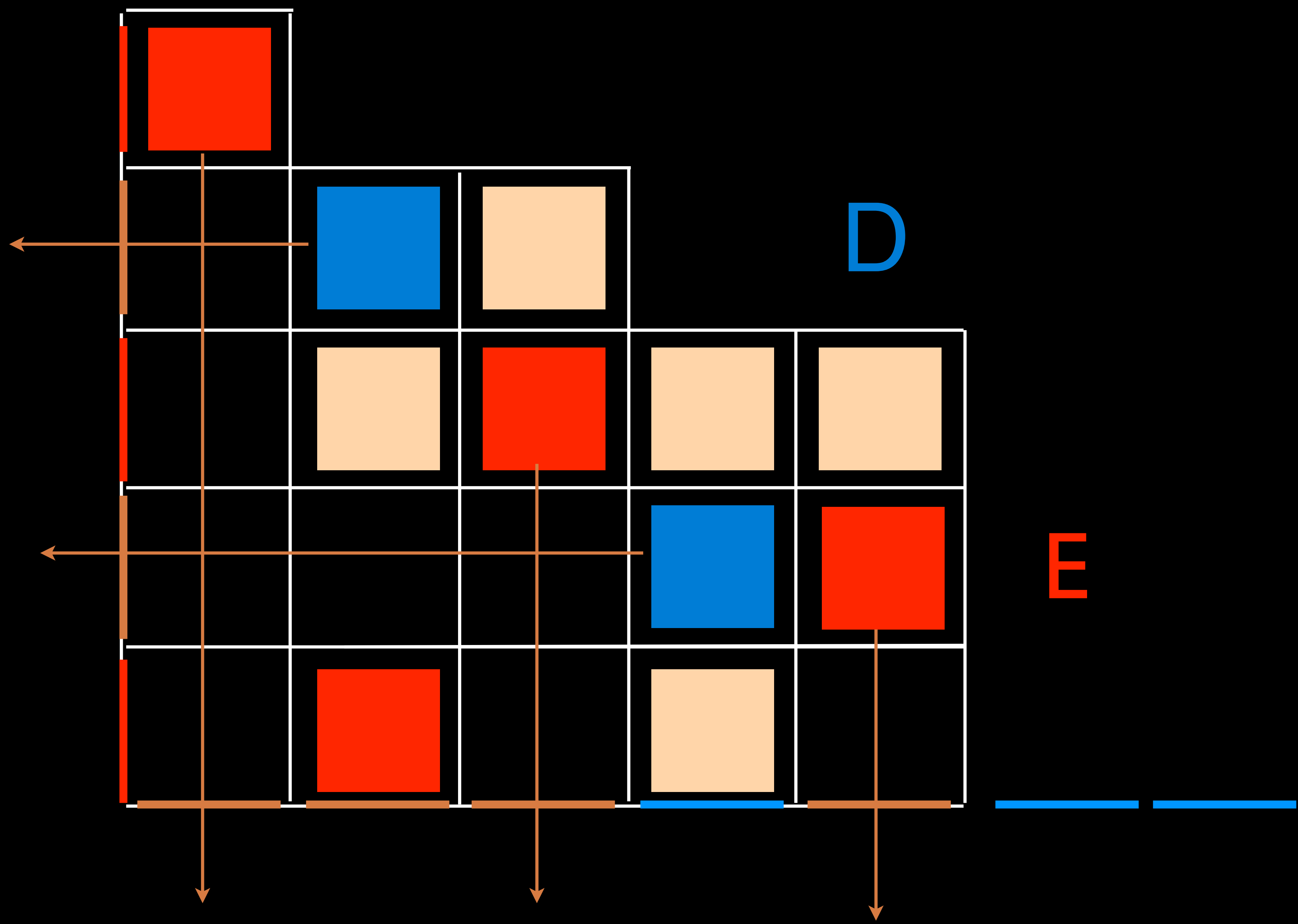
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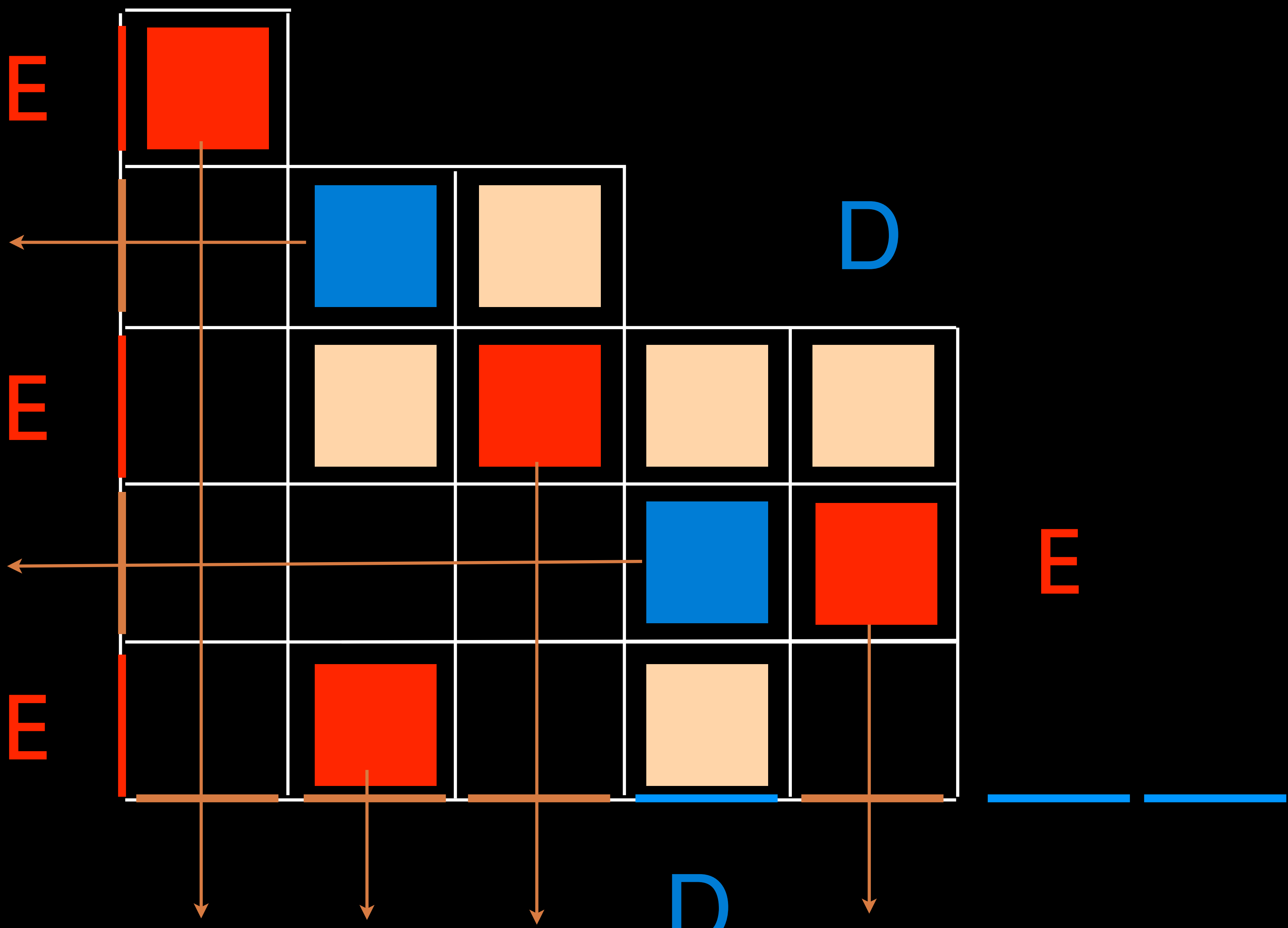

E
D
E
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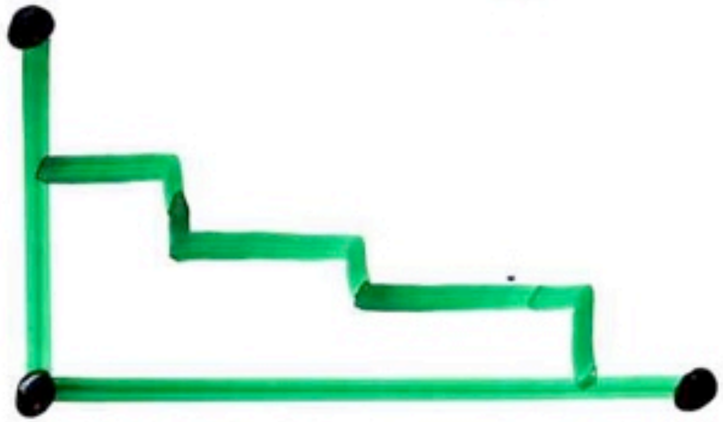
E
D

alternative tableaux

alternative tableau

- Ferrers diagram F



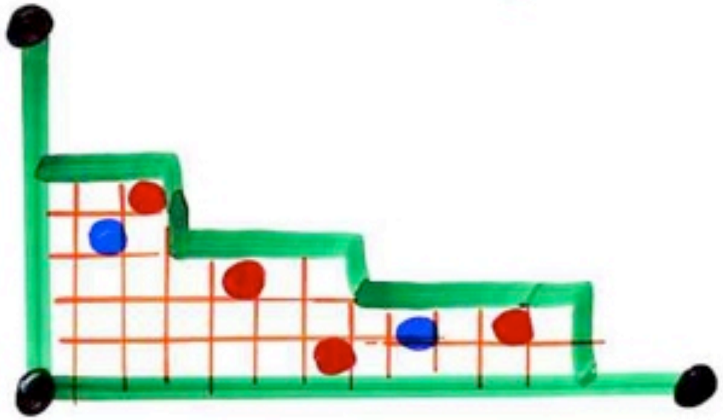
(possibly empty rows or column)

(nb of rows) + (nb of columns) $= n$

alternative tableau

- Ferrers diagram F

(possibly empty rows or column)

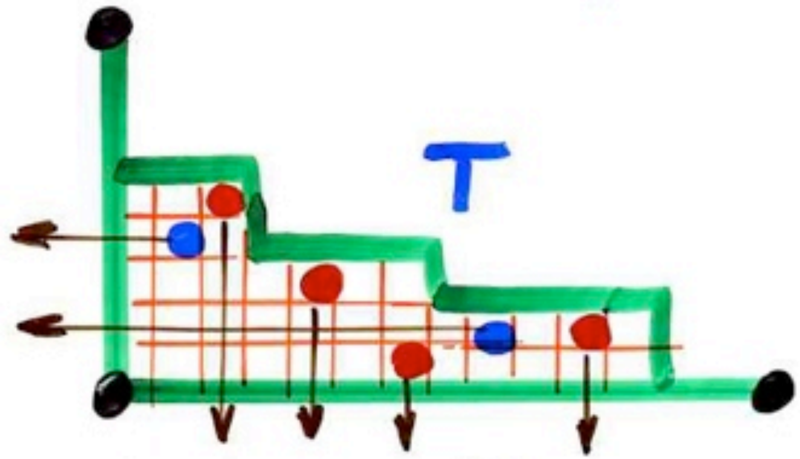


(nb of rows) + (nb of columns) $= n$

- some cells are coloured **red** or **blue**

alternative tableau T

- Ferrers diagram F (possibly empty rows or column)



(nb of rows) + (nb of columns) $= n$

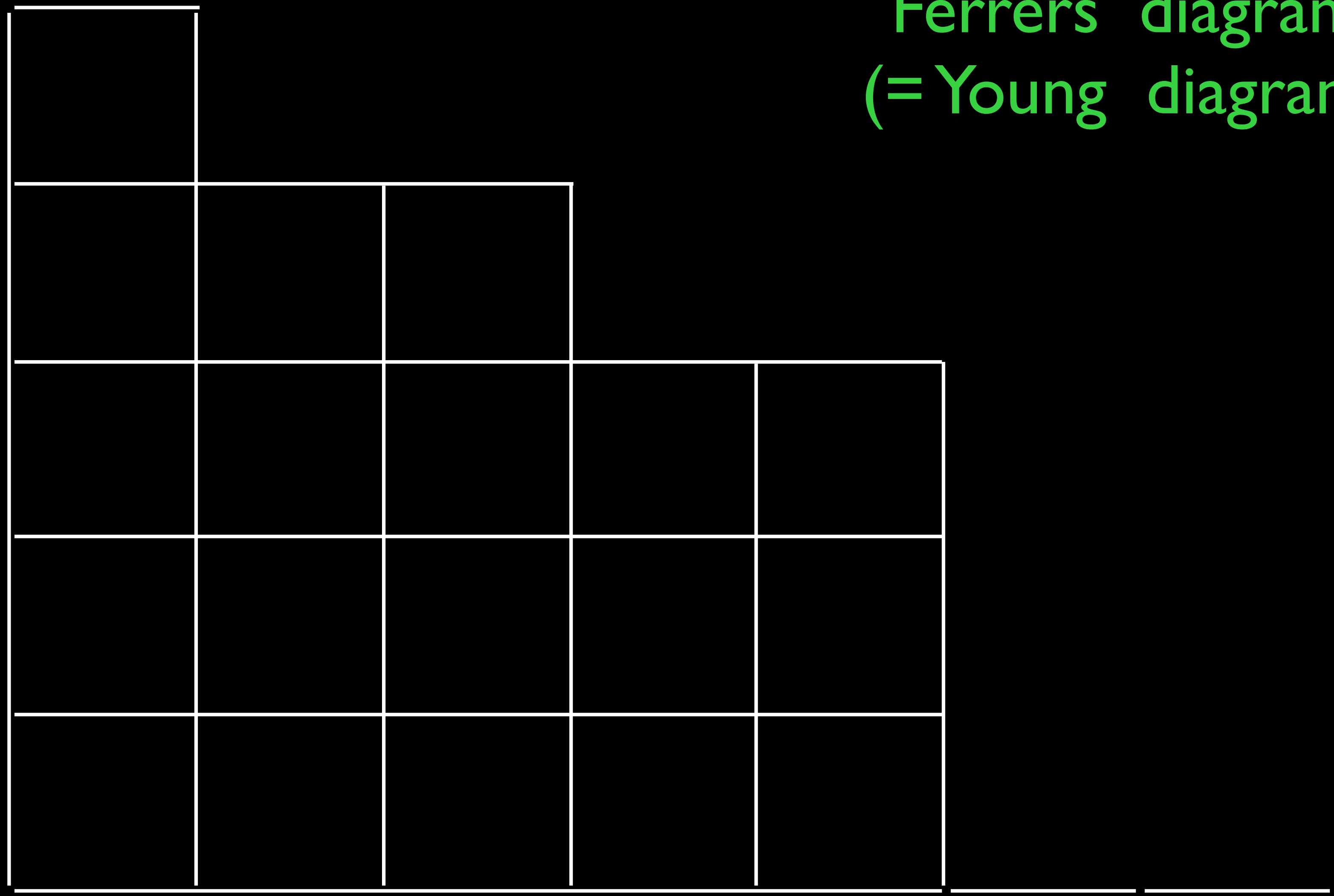
- some cells are coloured **red** or **blue**

- { no coloured cell at the left of
  { no coloured cell below

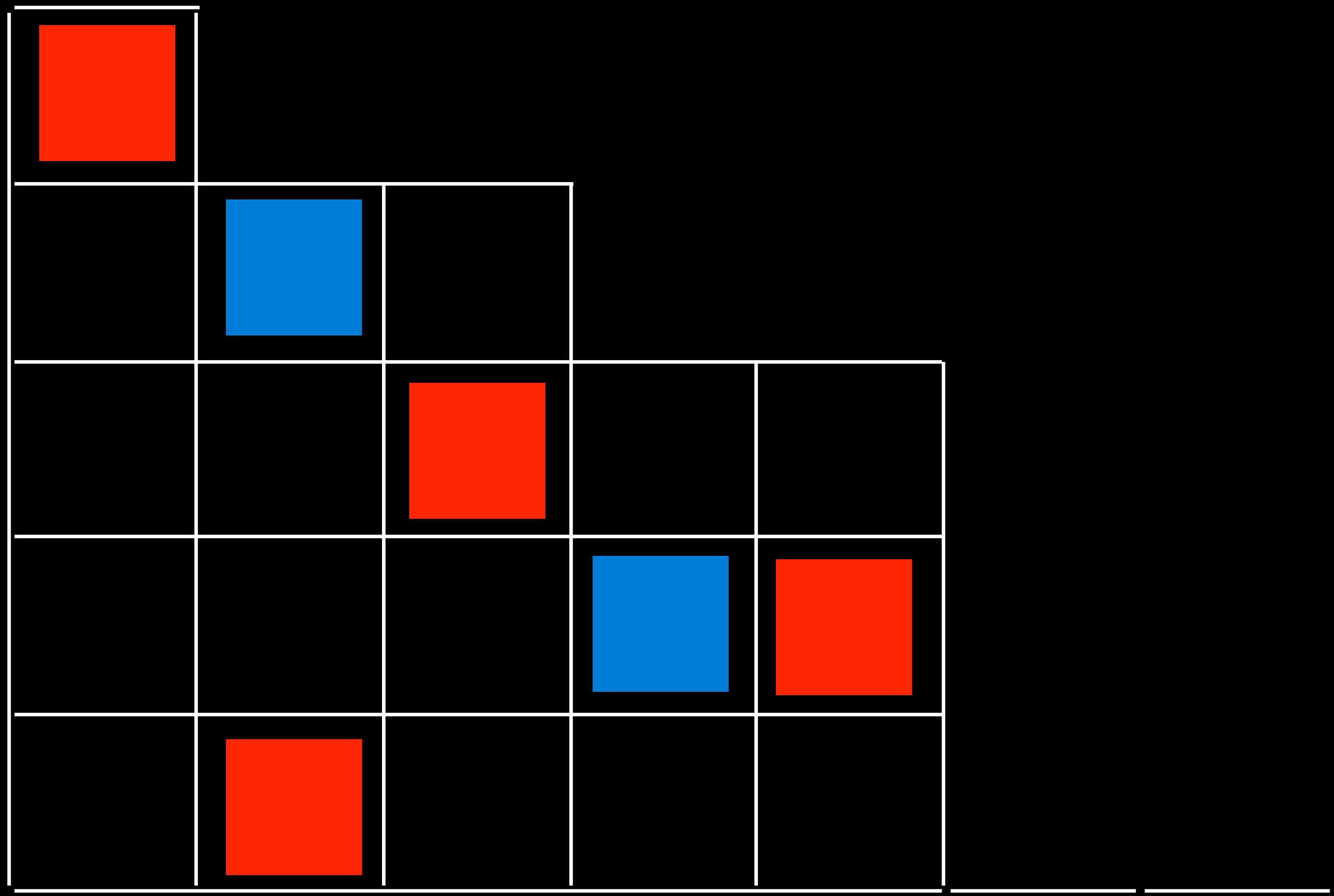
n size of T

alternative tableau

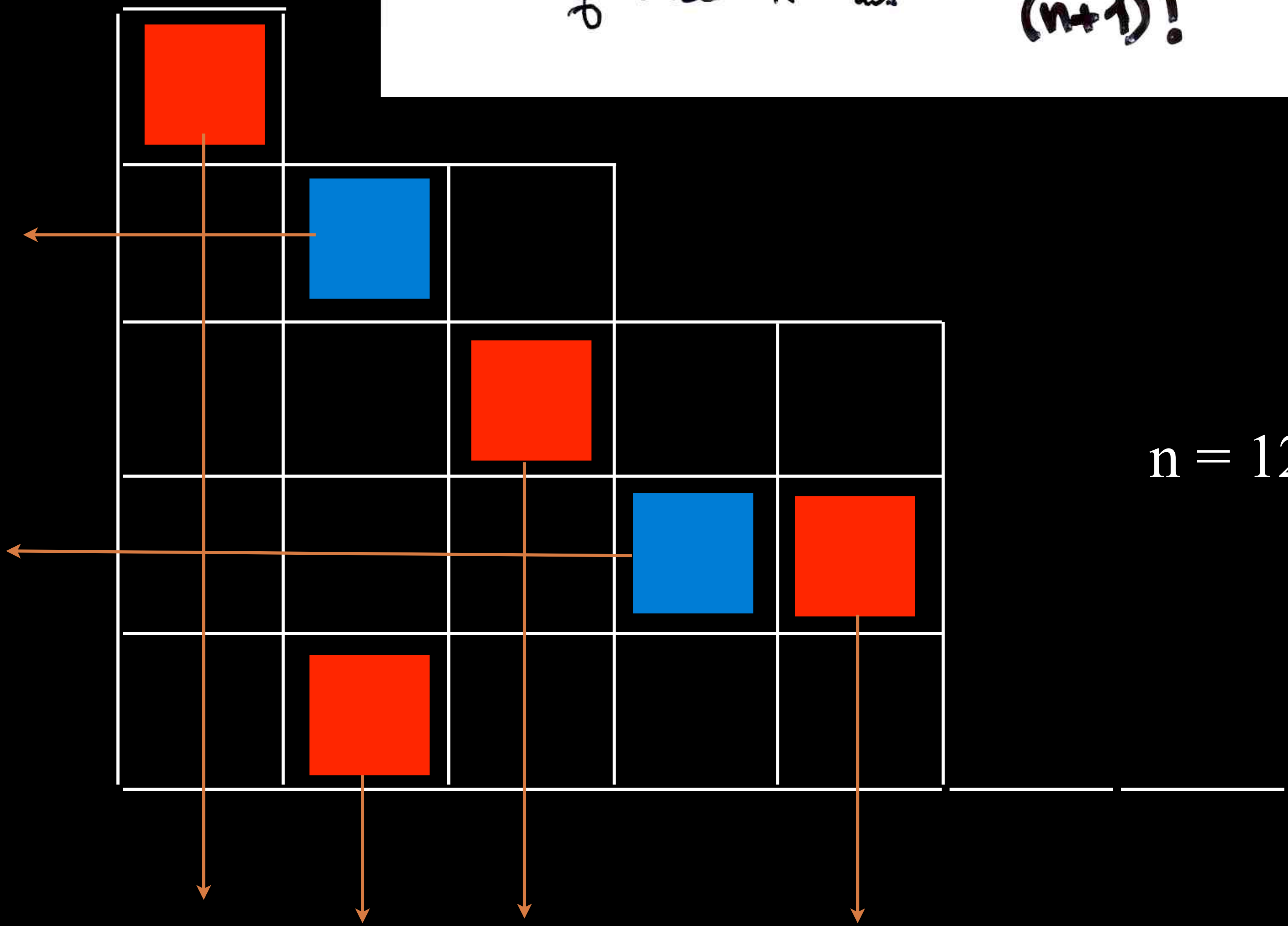
Ferrers diagram
(= Young diagram)



alternative tableau



Prop. The number of alternative tableaux of size n is $(n+1)!$



$n = 12$

Def - profile of an alternative tableau

word $w \in \{E, D\}^*$

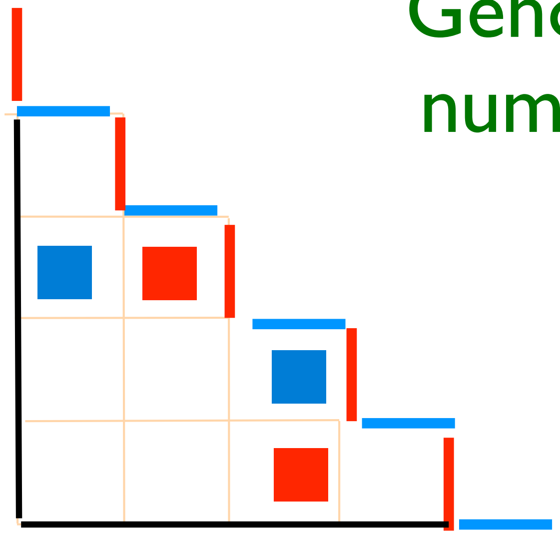
nombres de Genocchi

$$G_{2n} = 2(2^{2n}-1)\, B_{2n}$$

Bernoulli

$$2^{2n}\, G_{2n+2} = (n+1)\, T_{2n+1}$$

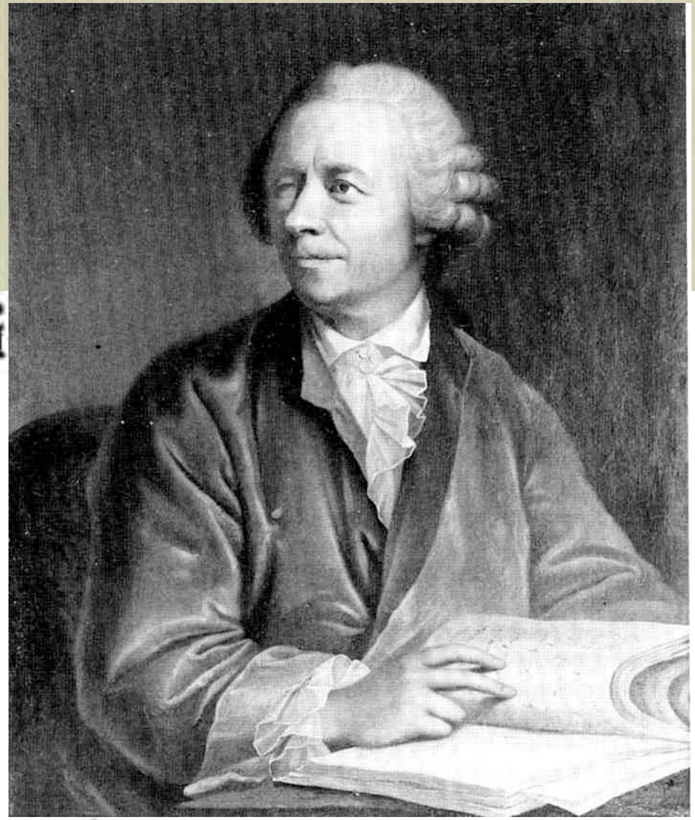
Genocchi numbers



alternating profile

Angelo Genocchi
1817 - 1889

Hinc igitur calculo instituto reperi



$$A = 1$$
$$B = 1$$
$$C = 3$$
$$D = 17$$
$$E = 155 = 5.31$$
$$F = 2073 = 691.3$$
$$G = 38227 = 7.5461 = 7.\frac{127.129}{3}.$$
$$H = 929569 = 3617.257$$
$$I = 28820619 = 43867.9.73 \quad \&c.$$

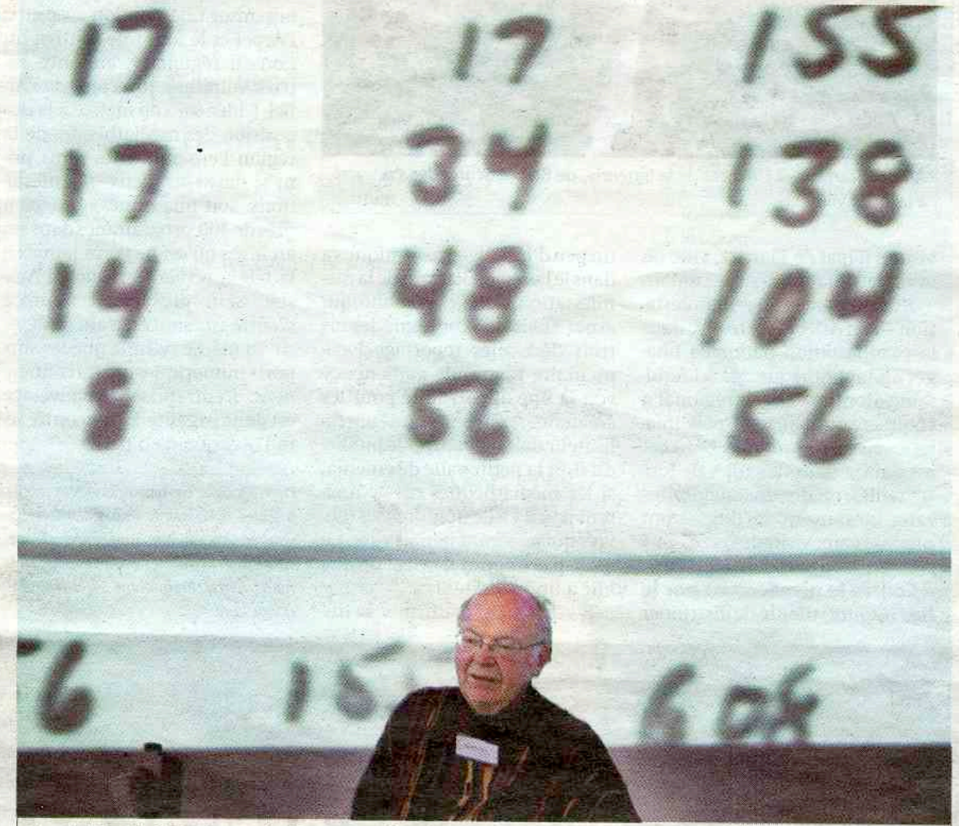
BORDEAUX 1. Le professeur Donald Knuth consacre sa vie à la programmation informatique, considérée comme un art. Il vient d'être sacré docteur honoris causa à Bordeaux

L'ermite de l'informatique

: Bernard Broustet

Une sommité de l'informatique mondiale a séjourné en Gironde ces derniers jours. Donald Knuth, 69 ans, a été sacré mardi docteur honoris causa de l'université Bordeaux 1, après avoir été lundi au centre d'une journée d'échanges qui réunissait une bonne partie du gratin français et européen de la recherche en informatique (1).

Depuis son premier contact, il y a un demi-siècle, avec un monumental et dinosaurien IBM 650, Donald Knuth n'a cessé d'être habité par la passion de l'informatique. Physicien, puis mathématicien de formation, ce géant affable et modeste a voué sa vie à ce qu'il appelle « l'art de la programmation informatique ». Car, à ses yeux, plus qu'une technique, c'est une forme d'activité qui requiert à la fois rigueur, intuition et sens esthétique. Les programmes informatiques réussis ont une sorte de beauté à laquelle même les non-spécialistes peuvent être sensibles.



Donald Knuth, à Bordeaux, le 29 octobre. À 69 ans, il animait une journée d'échanges avec le gratin européen de la recherche en informatique PHOTO LAURENT THEILLET

Une encyclopédie. Au long de sa carrière académique (pour l'essentiel à l'université californienne de Stanford), Donald Knuth a fait preuve d'une grande fécondité, en jouant notamment un rôle essentiel dans le développement de langages toujours utilisés par la communauté des mathématiciens. Mais, à 55 ans, le professeur Knuth a décidé de prendre sa retraite de Stanford. Il trouve que les fonctions administratives sont trop absorbantes pour lui permettre de mener à bien l'œuvre entamée à la fin des années 60 sous le titre de « Art of computer programming », sorte d'encyclopédie de l'algorithmique et de la programmation informatique. Donald Knuth a publié, il y a quelque temps déjà, les trois premiers volumes de cette gigantesque somme, traduite en russe, en japonais, en polonais, etc. mais pas en français. Le quatrième tome est pour bientôt. Et Donald Knuth se dit décidé à poursuivre sa tâche tant qu'il en aura la force. Ses ouvrages, dont les ventes cumulées au fil des ans approchent le million d'exemplaires, visent essentiellement les informaticiens et créateurs de programmes. Une communauté certes minoritaire à travers le monde, mais qui se trouve investie d'une mission considérable. En quelques décennies, l'écriture informatique a aidé à résoudre d'innombrables problèmes. « Mais il y en a tant d'autres qui attendent des solutions, notamment dans le domaine médical », affirme le professeur émérite de Stanford.

Un chèque de 2,56 dollars. Pour mener à bien sa tâche, Donald Knuth s'est imposé une vie d'ermite. D'ordinaire, sa journée débute par la bibliothèque ou la piscine. Après quoi, il passe tout le reste de son temps à sa table de travail, dimanche compris. Il n'a plus d'e-mail depuis le début des années 90, considérant que le courrier électronique représente une perte de temps, dès lors qu'on veut aller au fond des choses et non pas rester à leur surface. Une secrétaire lui fait passer les messages considérés comme les plus urgents. Pour le reste, Donald Knuth demande qu'on lui écrive par courrier ordinaire ou par fax, dont il prend parfois connaissance avec des mois de retard. Il s'oblige, en revanche, à tenir aussi scrupuleusement que possible sa promesse d'envoyer un chèque de 2,56 dollars à tout lecteur ayant détecté une erreur dans un de ses livres. Par ailleurs, pour se détendre, il pratique l'orgue, appris dans sa prime jeunesse auprès de son père qui partagea sa vie entre la musique et l'enseignement.

L'orgue de Sainte-Croix. Donald Knuth n'est pas fermé aux choses de ce monde. Sur son site Internet, à la rubrique « Questions qui ne me sont pas fréquemment posées », il demande entre autres : « Pourquoi mon pays a-t-il le droit d'occuper l'Irak ? », « Pourquoi mon pays ne soutient-il pas une Cour internationale de justice ? » Mais cet homme de conscience ne se veut pas militant, pas plus qu'il n'aspire au vedettariat et à la richesse. « Beaucoup de gens, dit-il, ont tendance à considérer que l'informatique, c'est surtout des histoires de business, d'entreprise. Ce n'est pas mon cas. » Sortant de sa semi-réclusion, Donald Knuth s'est donc laissé convaincre d'accepter les hommages de l'université de Bordeaux, après celles de Harvard, d'Oxford, de Tübingen. Il a eu le coup de foudre pour la beauté et l'agrément de la ville. Et il n'oubliera sans doute pas de sitôt l'orgue illustre de l'église Sainte-Croix (2), sur lequel il a eu le bonheur d'exercer son talent.

(1) Ces journées étaient organisées par le Laboratoire bordelais de recherche en informatique (Labri).

(2) Thierry Semenoux, professeur d'orgue au conservatoire de Bordeaux, a joué dans ce domaine un rôle de cicérone auprès de Donald Knuth.

stationary probabilities for the PASEP

$$DE = qED + E + D$$

$$w(E,D) = \sum_{T} q^{k(T)} E^{i(T)} D^{j(T)}$$

T alternative tableau with profile w

$k(T)$ = nb of $\square$

$i(T)$ = nb of rows without blue cell

$j(T)$ = nb of columns without red cell

stationary probabilities

permutation tableau

S. Corteel, L. Williams

(2007) (2008) (2009)

$$\begin{cases} DE = qED + D + E \\ DV = \bar{\beta} V & \bar{\beta} = 1/\beta \\ WE = \bar{\alpha} W & \bar{\alpha} = 1/\alpha \end{cases}$$

$$W E^i D^j V = \bar{\alpha}^i \bar{\beta}^j \underbrace{WV}_{1}$$

Cor. The stationary probability associated to the state $\tau = (\tau_1, \dots, \tau_n)$ (PASEP) is

$$\mathrm{proba}_\tau(q;\alpha,\beta) = \frac{1}{Z_n} \sum_{T} q^{L(T)} \alpha^{-f(T)} \beta^{-u(T)}$$

alternative tableaux
profile τ

$$\begin{cases} f(T) \\ u(T) \end{cases} \text{nb of} \begin{pmatrix} \text{rows} \\ \text{columns} \end{pmatrix} \text{without} \quad \text{cell}$$

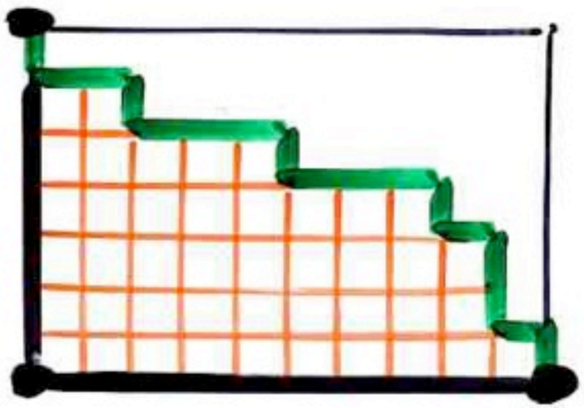
$L(T)$ nb of cells $\boxed{\times}$

permutation tableaux

Permutation Tableau

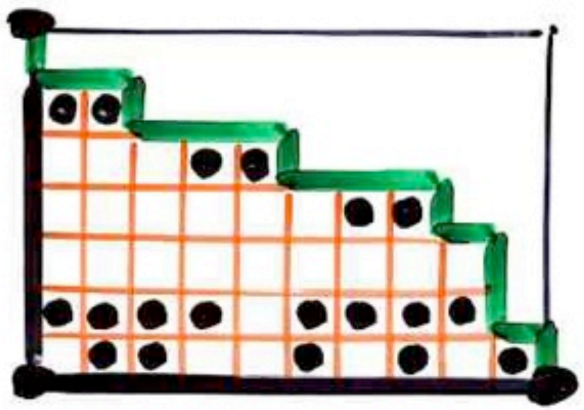
Ferrers diagram $F \subseteq k \times (n-k)$

rectangle



Permutation Tableau

Ferrers diagram $F \subseteq k \times (n-k)$ rectangle



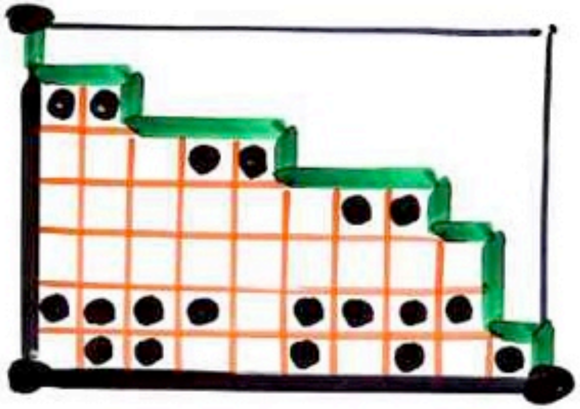
filling of the cells with 0 and 1

(i)

(ii)

Permutation Tableau

Ferrers diagram $F \subseteq k \times (n-k)$
rectangle



filling of the cells with 0 and 1

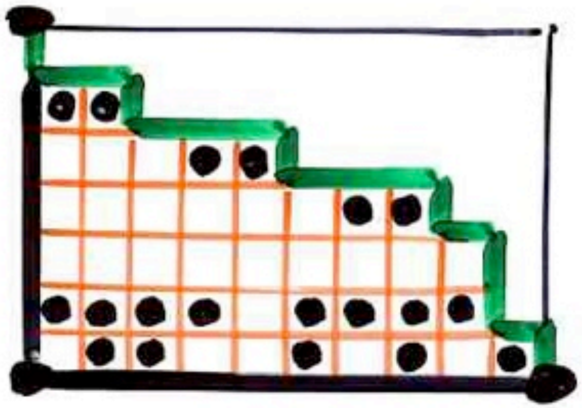
(i) in each column : at least one 1

(ii)

Permutation Tableau

Ferrers diagram $F \subseteq k \times (n-k)$

rectangle



□ = 0 ⊡ = 1

filling of the cells with 0 and 1

(i) in each column: at least one 1

(ii) $\begin{array}{ccc} 1 & \cdots & 0 \\ & & \vdots \\ & & 1 \end{array}$ forbidden

permutation tableau

A. Postnikov (2001, ...)

totally nonnegative part of the Grassmannian

E. Steingrímsson, L. Williams (2005)

Corteel, Williams (2006) PASEP

Partially Asymmetric Exclusion Process

The total number of permutation tableaux (n fixed, $1 \leqslant k \leqslant n$) is

$$n!$$

bijection

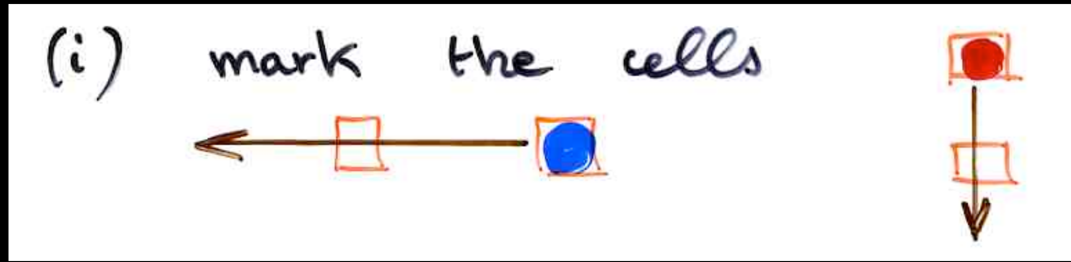
permutations $\longleftrightarrow$ permutation tableaux

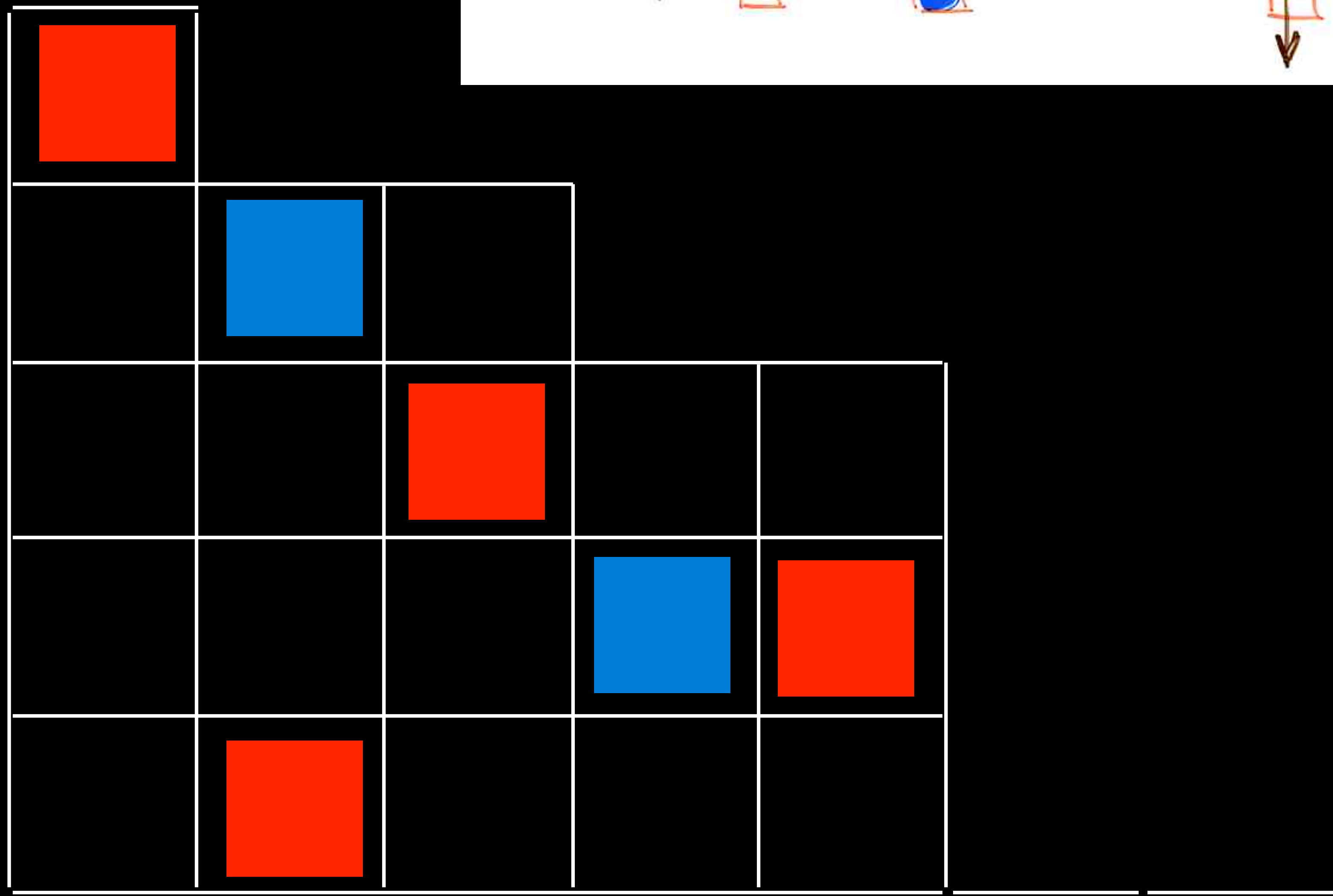
- Postnikov, Steingrímsson, Williams (2005)
- Corteel (2006)
- Corteel, Nadeau (2007)

bijection

$\updownarrow$ alternative tableaux size n

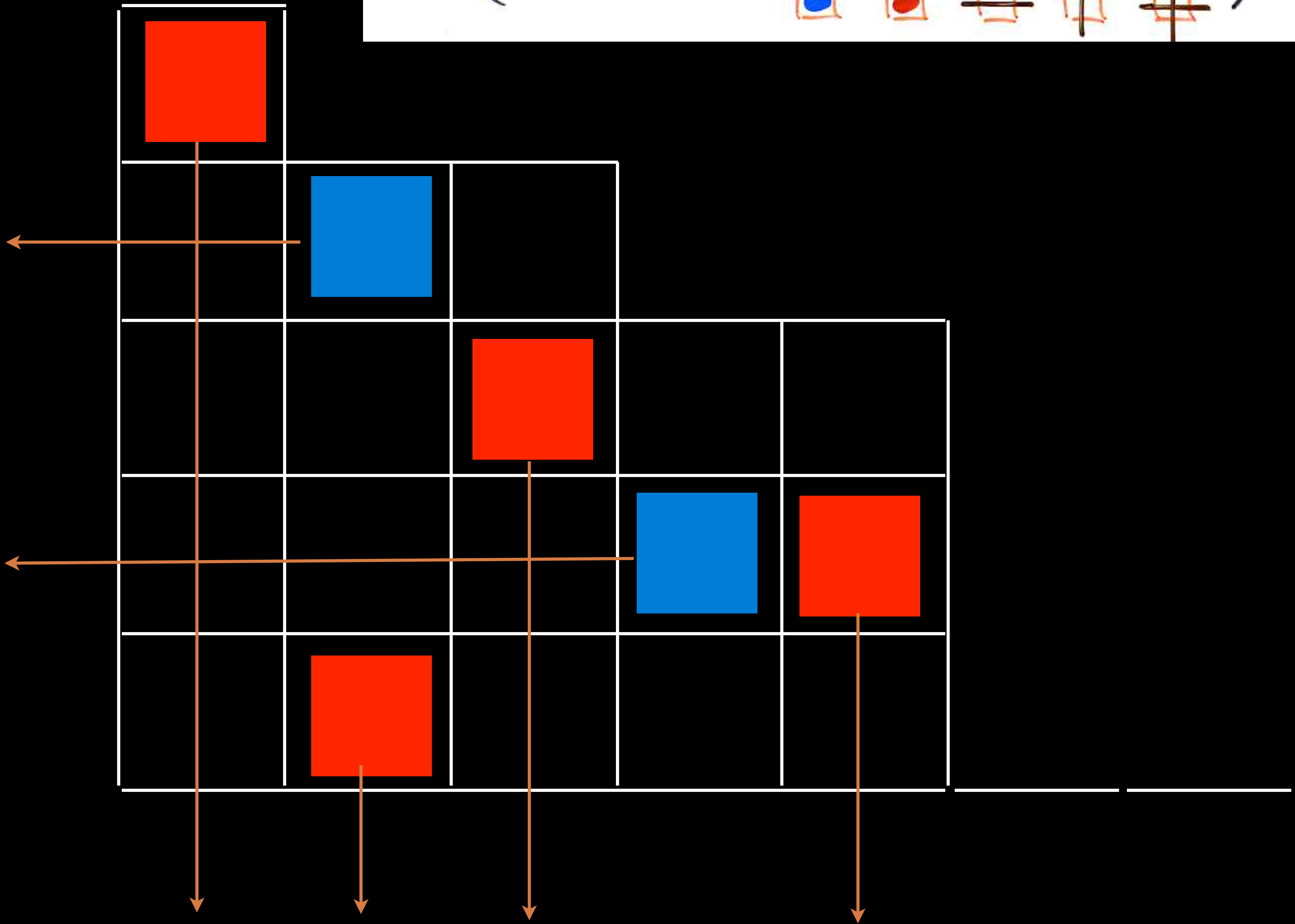
permutation tableaux size $(n+1)$

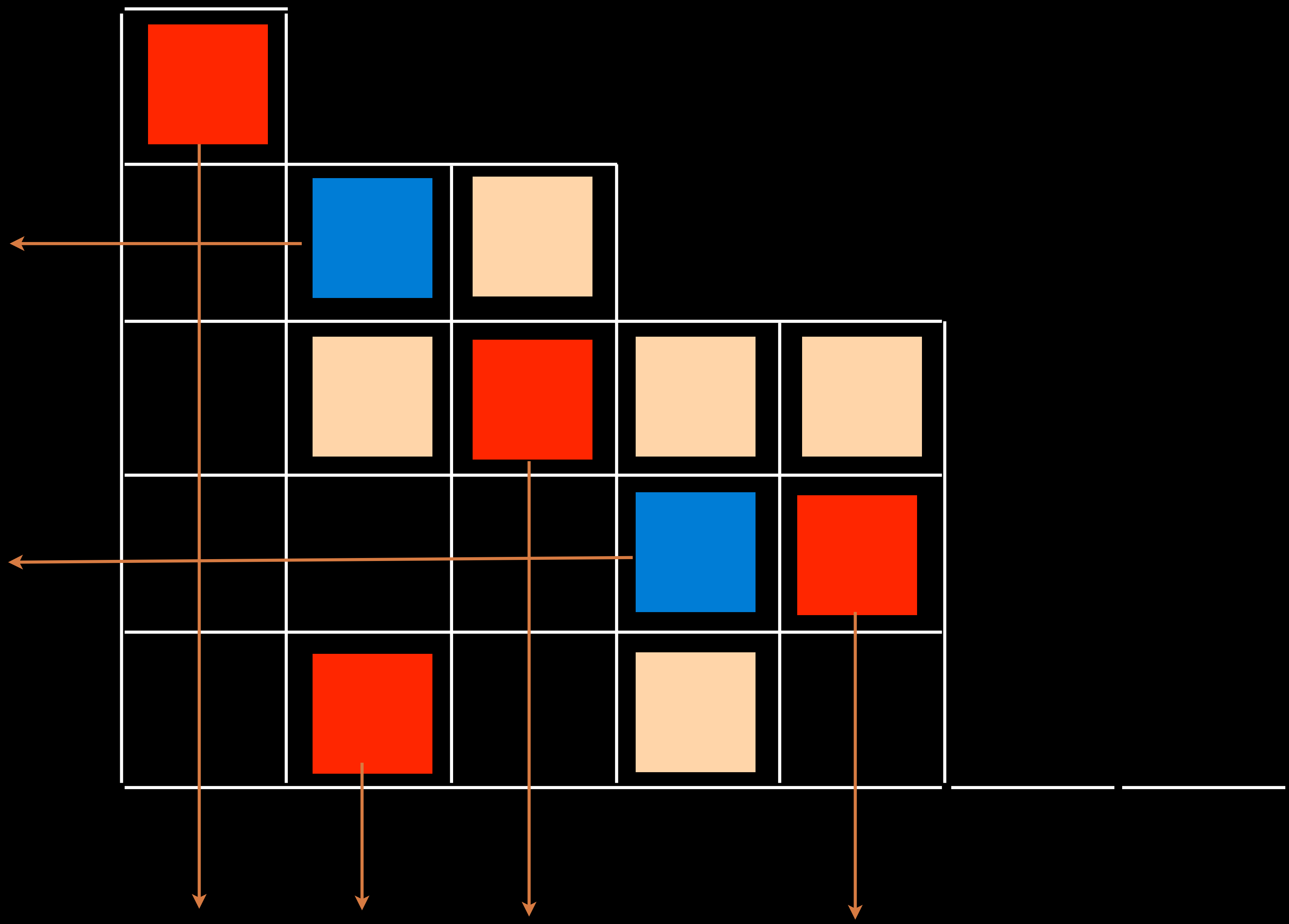




alternative tableau

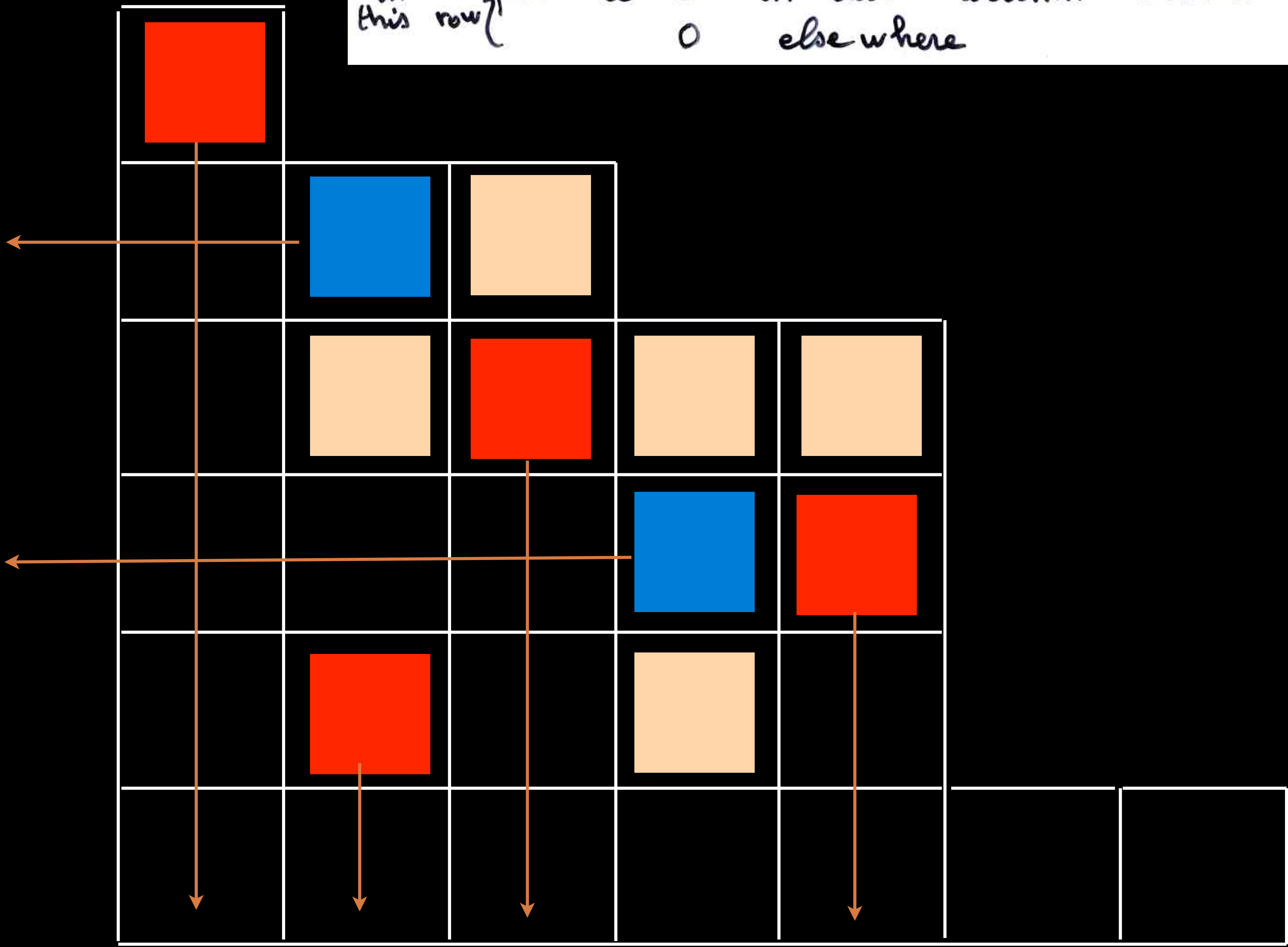
(ii) mark the empty cells by
(other than)



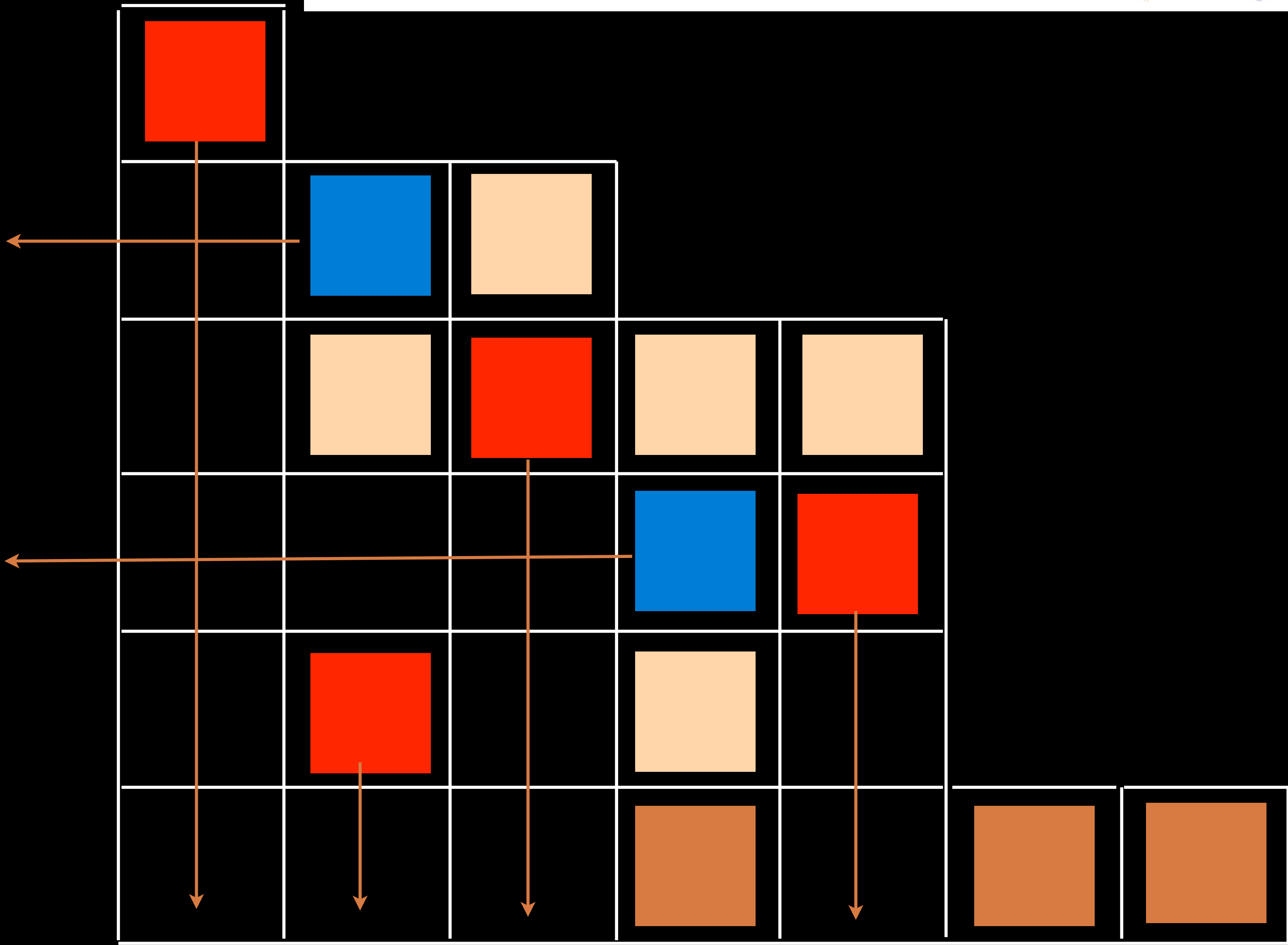


(iv) add a new row below F

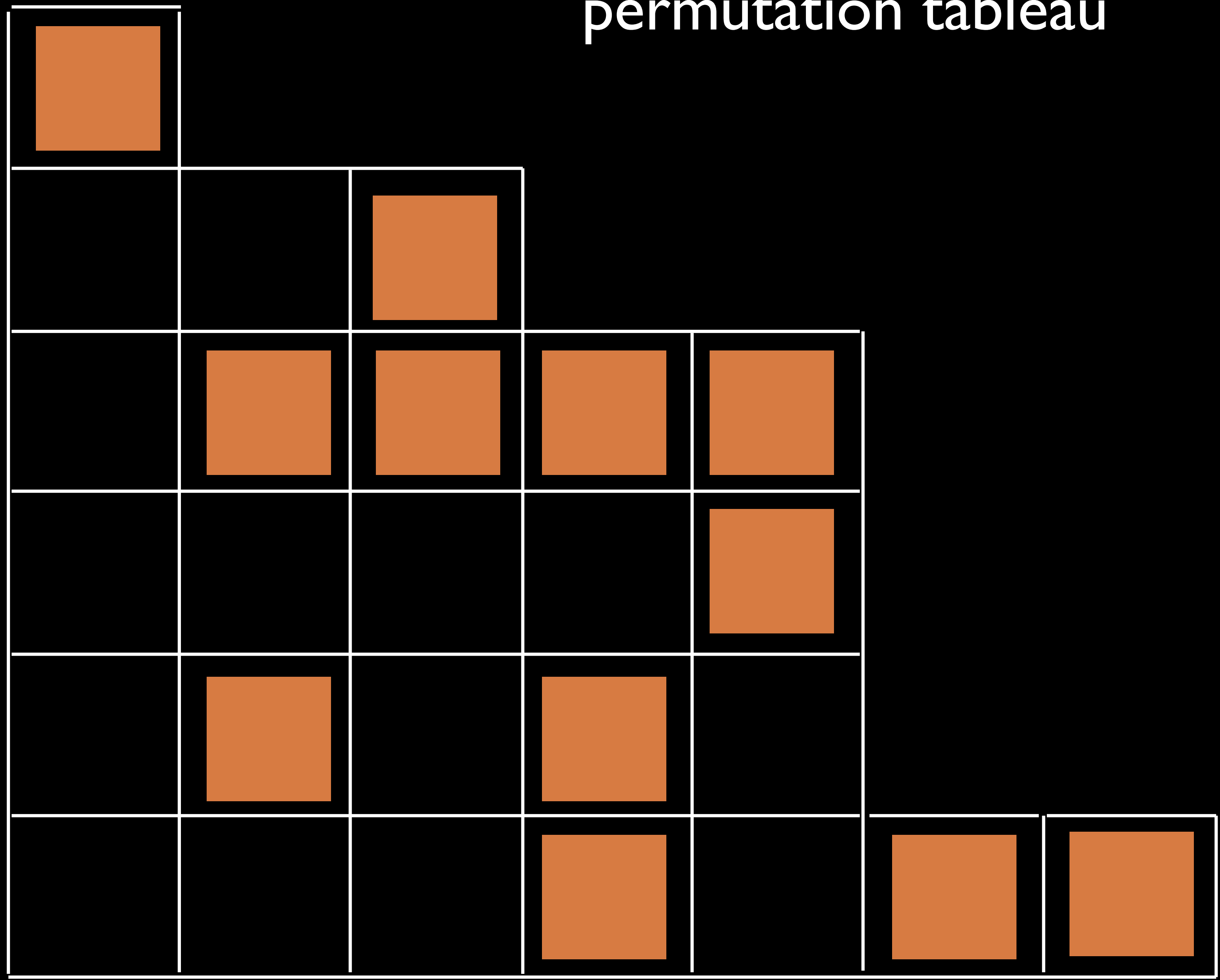
in this row $\begin{cases} \text{put a } 1 \text{ in each column without} \\ 0 \text{ elsewhere} \end{cases}$



(iii) • replace the cells or by 1
• replace the cells by 0

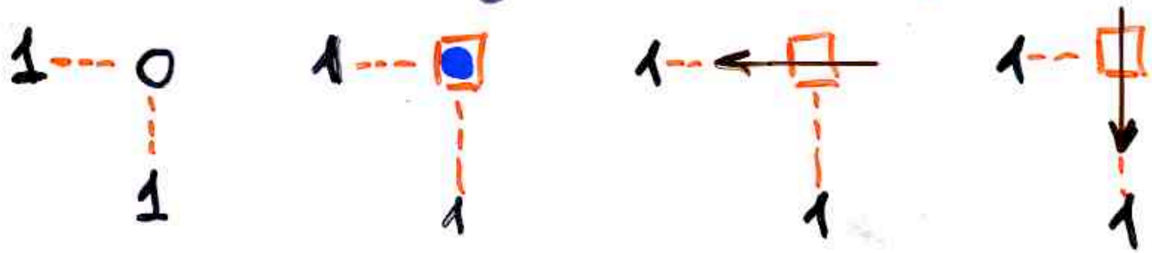


permutation tableau



check: $AT \xrightarrow{\varphi} PT$ size $(n+1)$

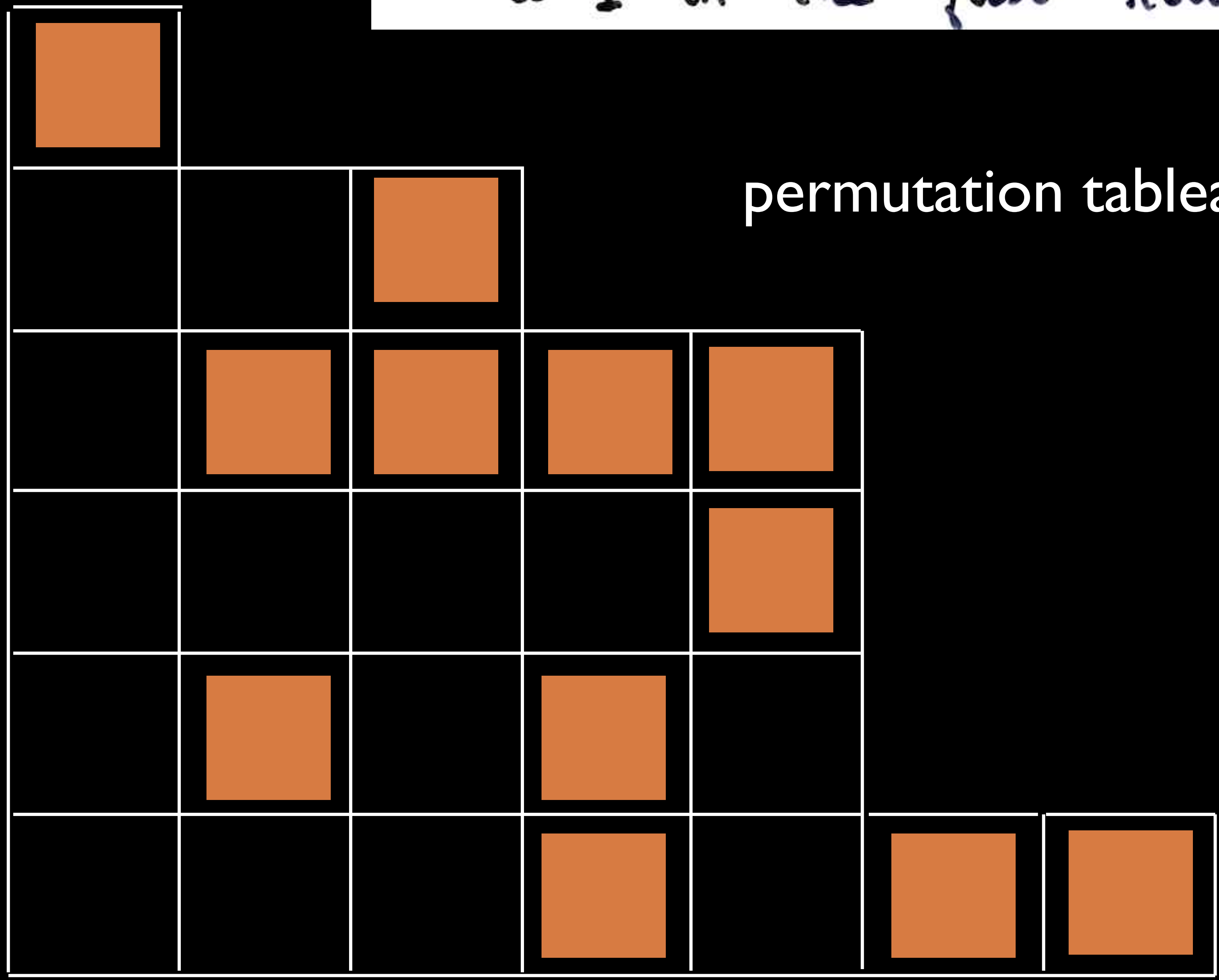
- the exist at least a 1 in each column of $PT = \varphi(AT)$



impossible

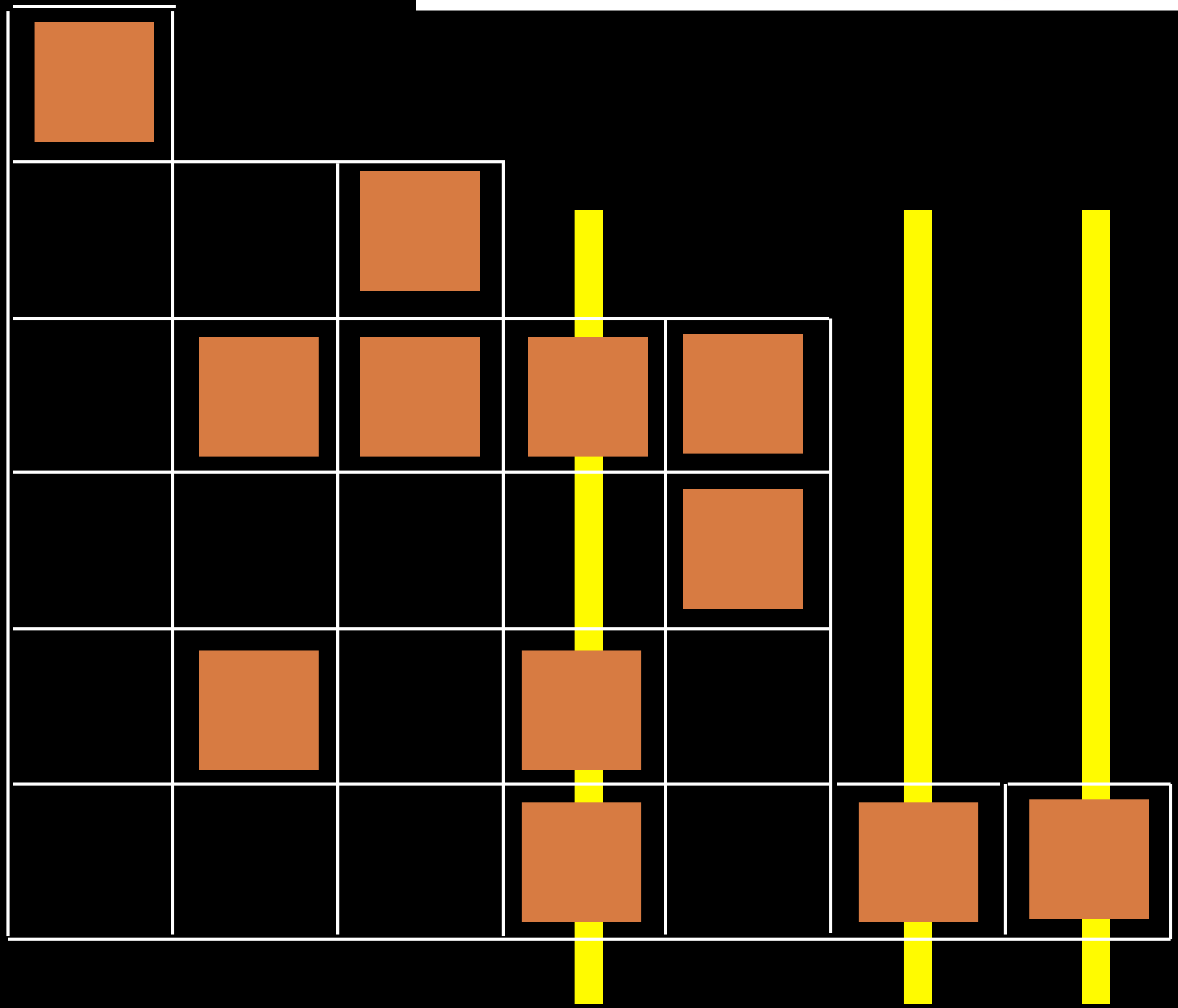
inverse bijection $\psi = \varphi^{-1}$

(i) mark the columns with a 1 in the first row



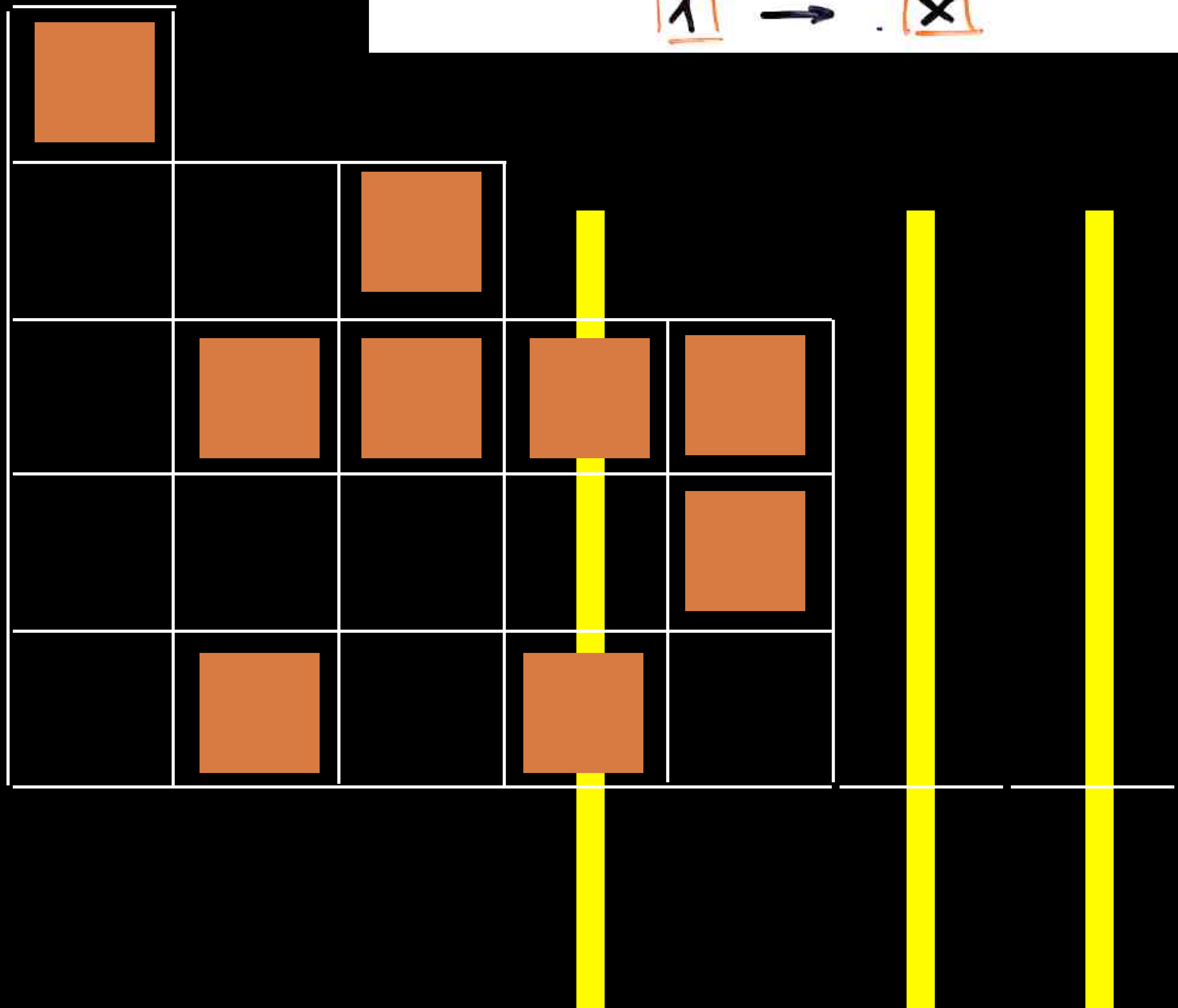
permutation tableau

(ii) delete the first row

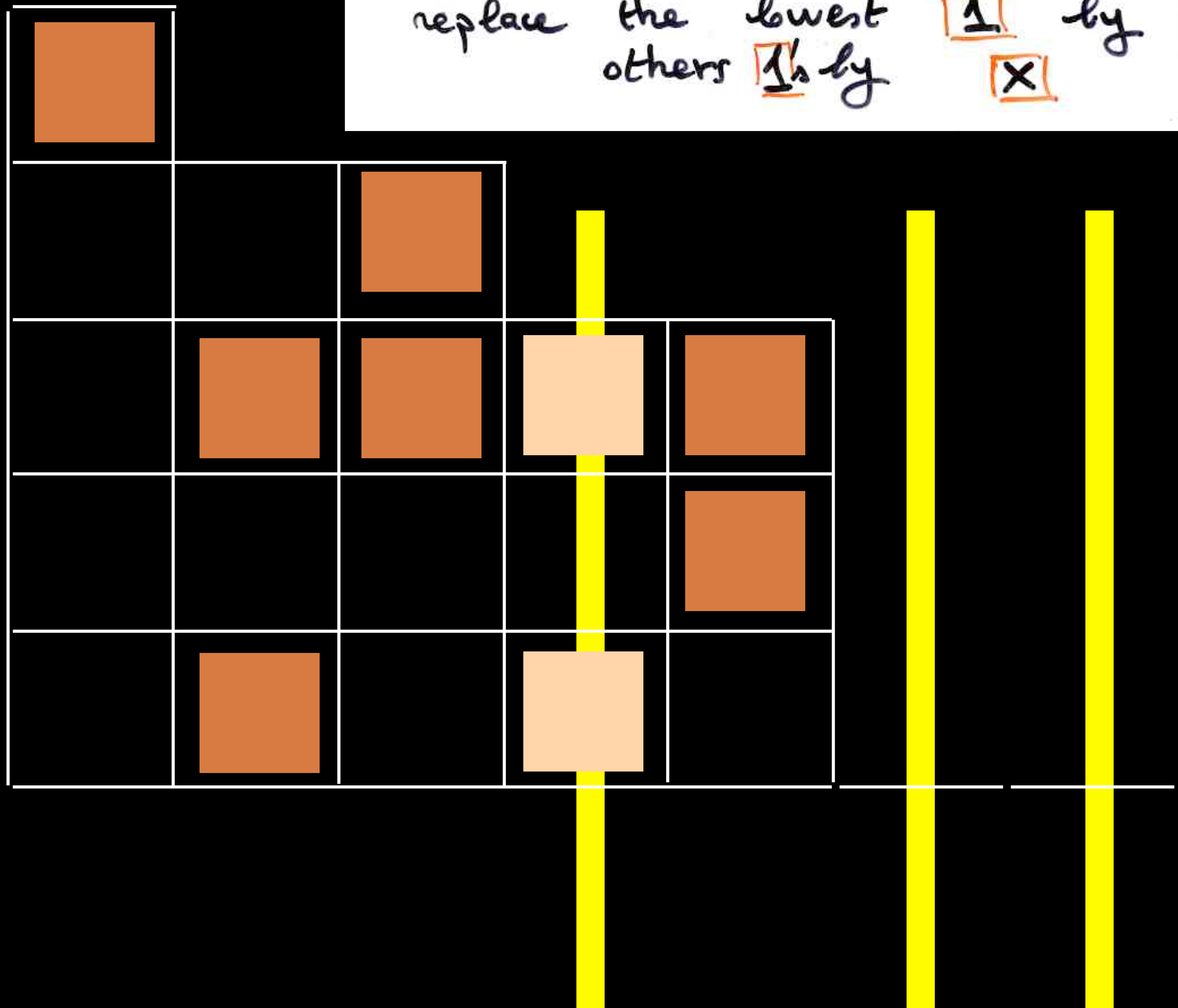


(iii) in each marked column

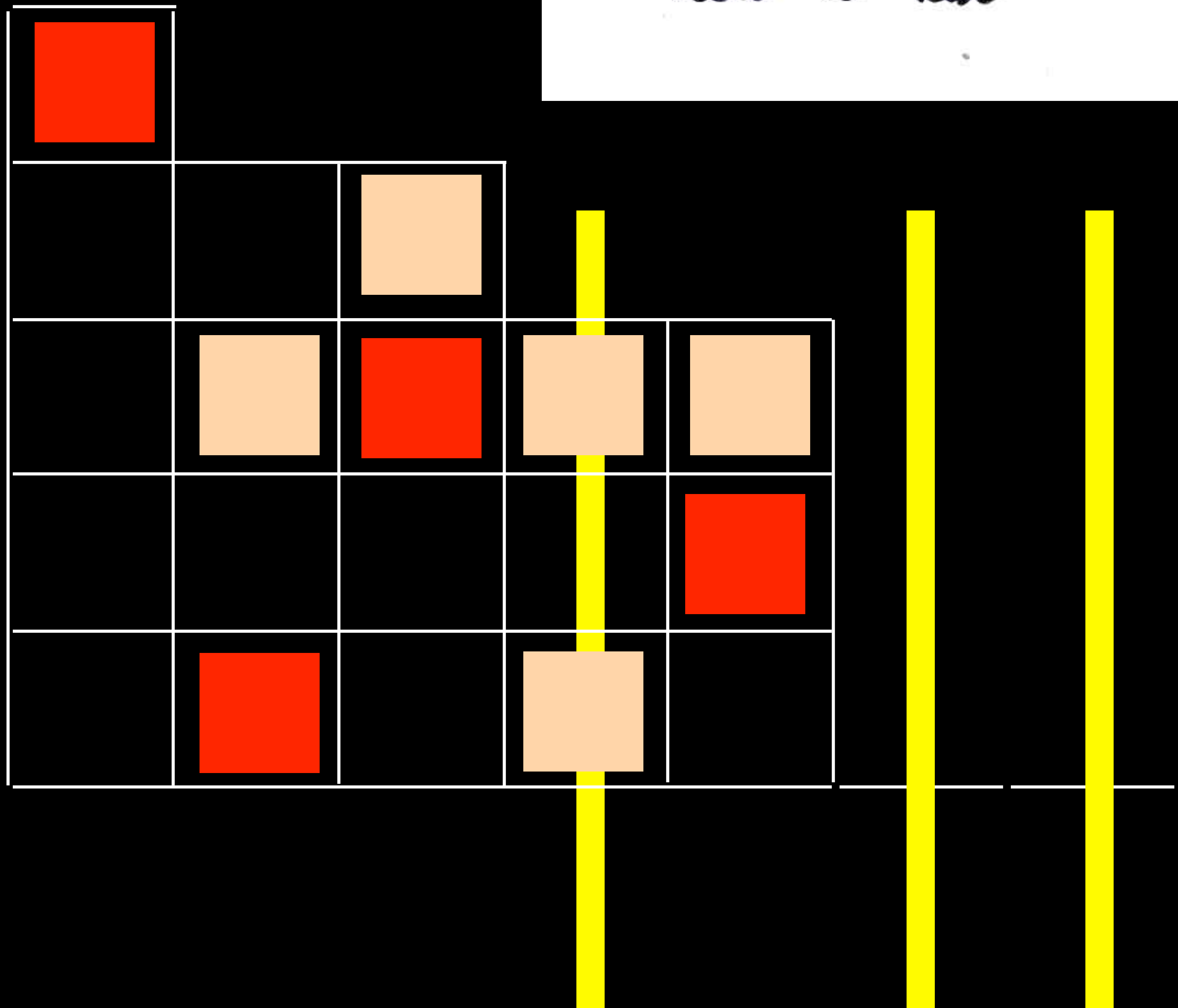
$\boxed{1} \longrightarrow \boxed{\times}$



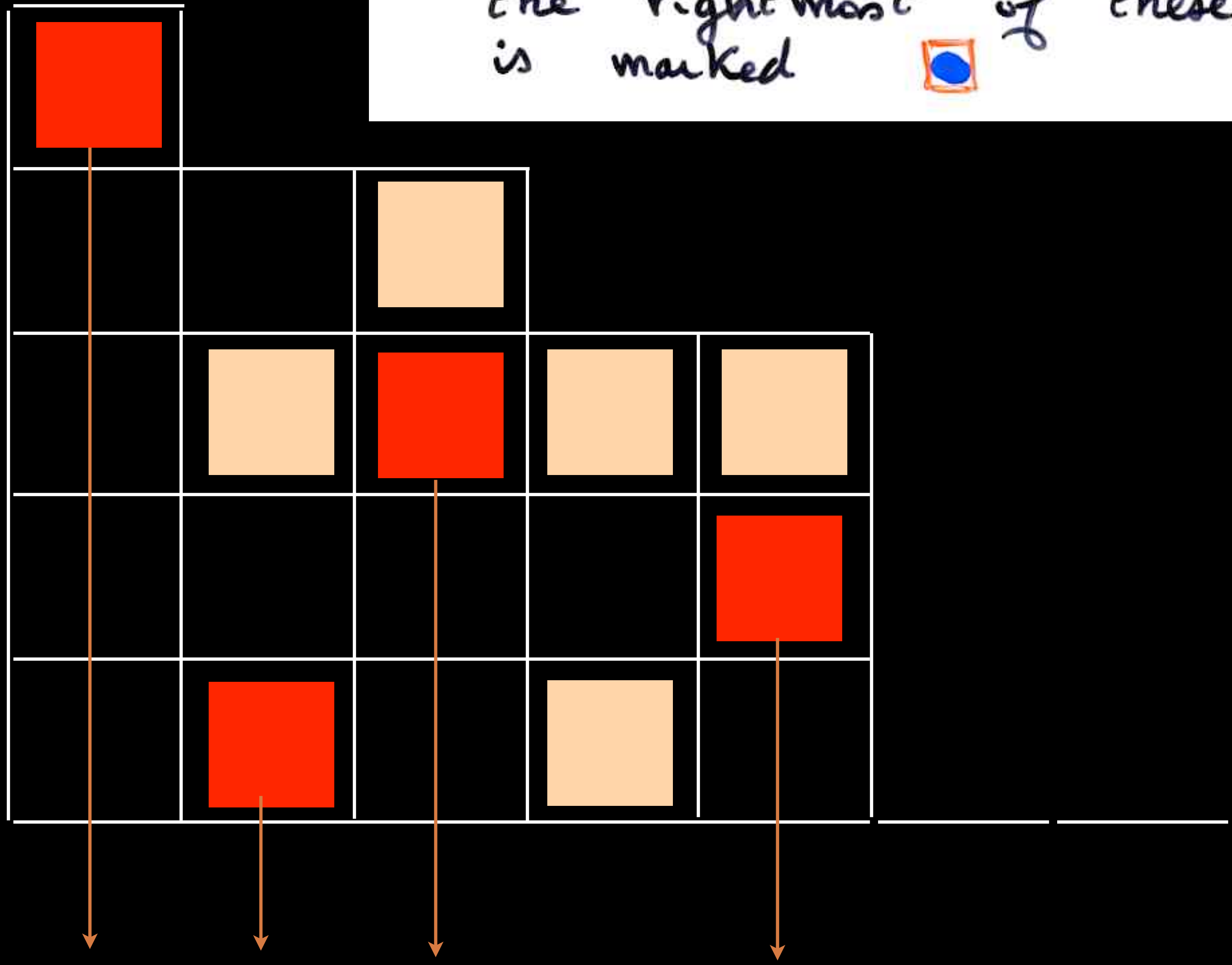
(iv) in each non marked column
(∃ some cells with 1)
replace the lowest $\boxed{1}$ by $\boxed{\bullet}$
others $\boxed{1's}$ by $\boxed{\times}$



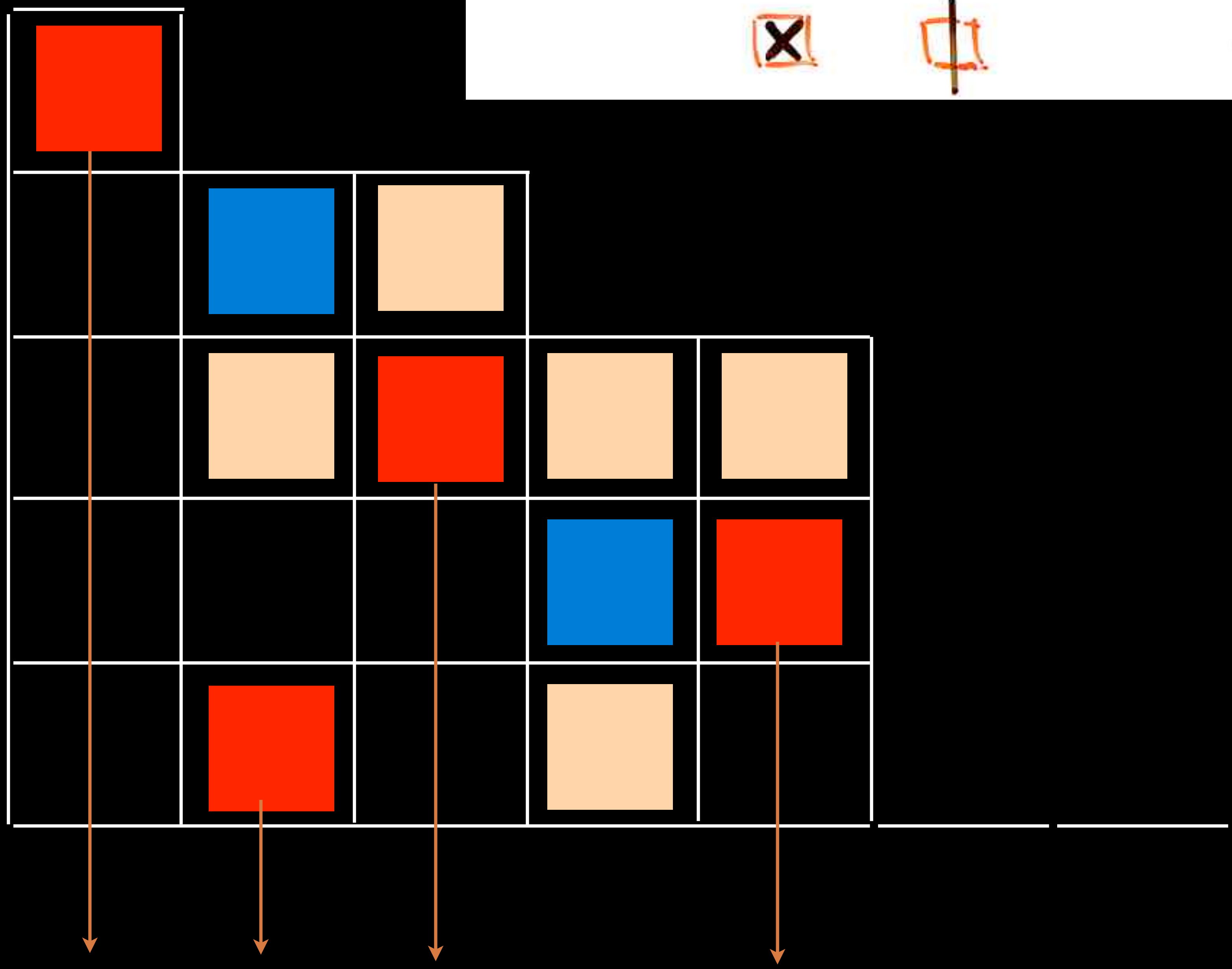
(v) mark the cells below a red



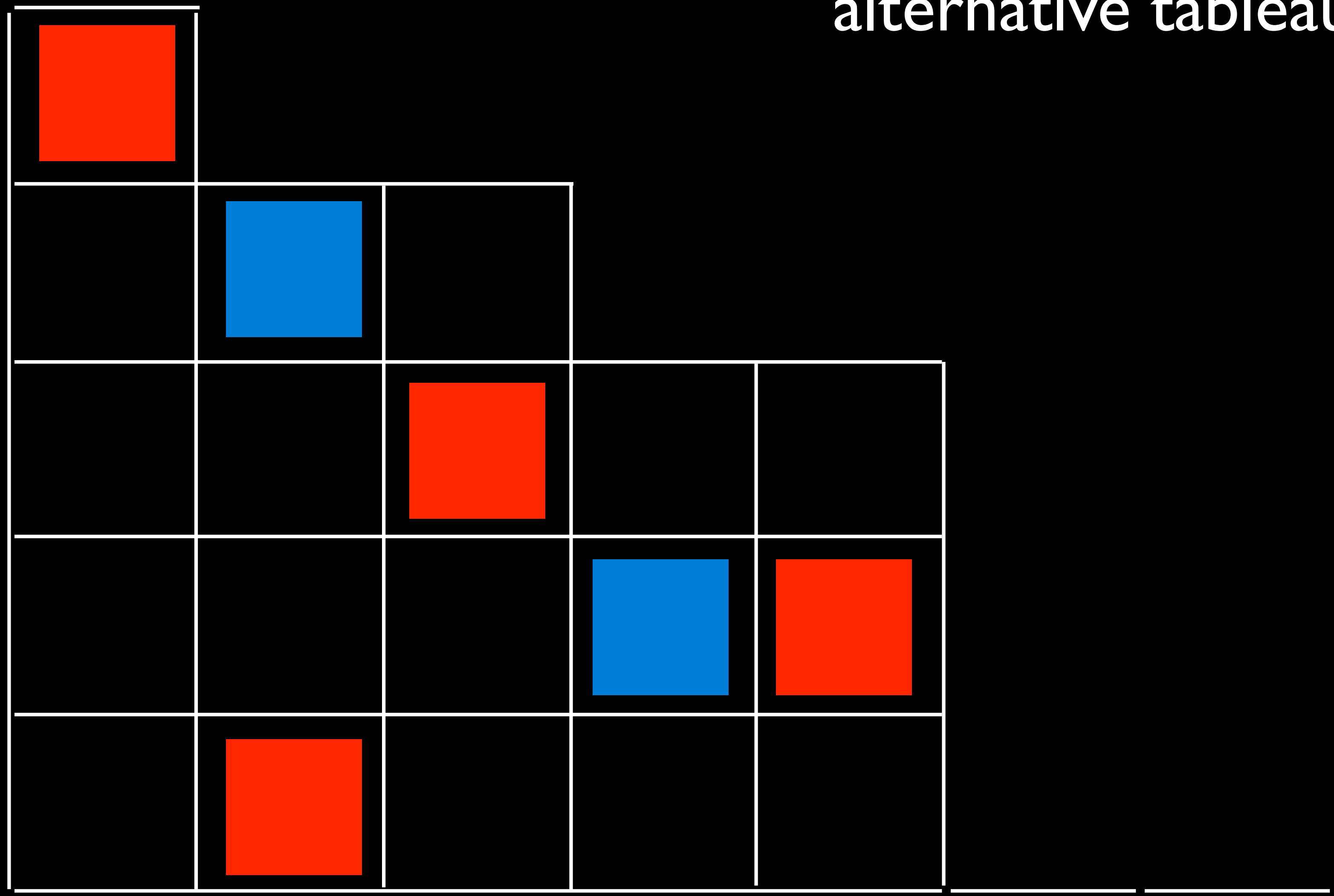
(vi) in each rows where there exist empty cells, the rightmost of these cells is marked



(vii) delete the marks



alternative tableau



check

- $\psi(PT)$ is an alternative tableau
- $\psi = \varphi^{-1}$.

notations. T tableau de permutations

- $\mathrm{wt}(T)$ = (nb total de 1) − (nb de colonnes)
- $f(T)$ = (nb de 1 sur la 1ère ligne)
- $u(T)$ = (nb de lignes non restreintes)

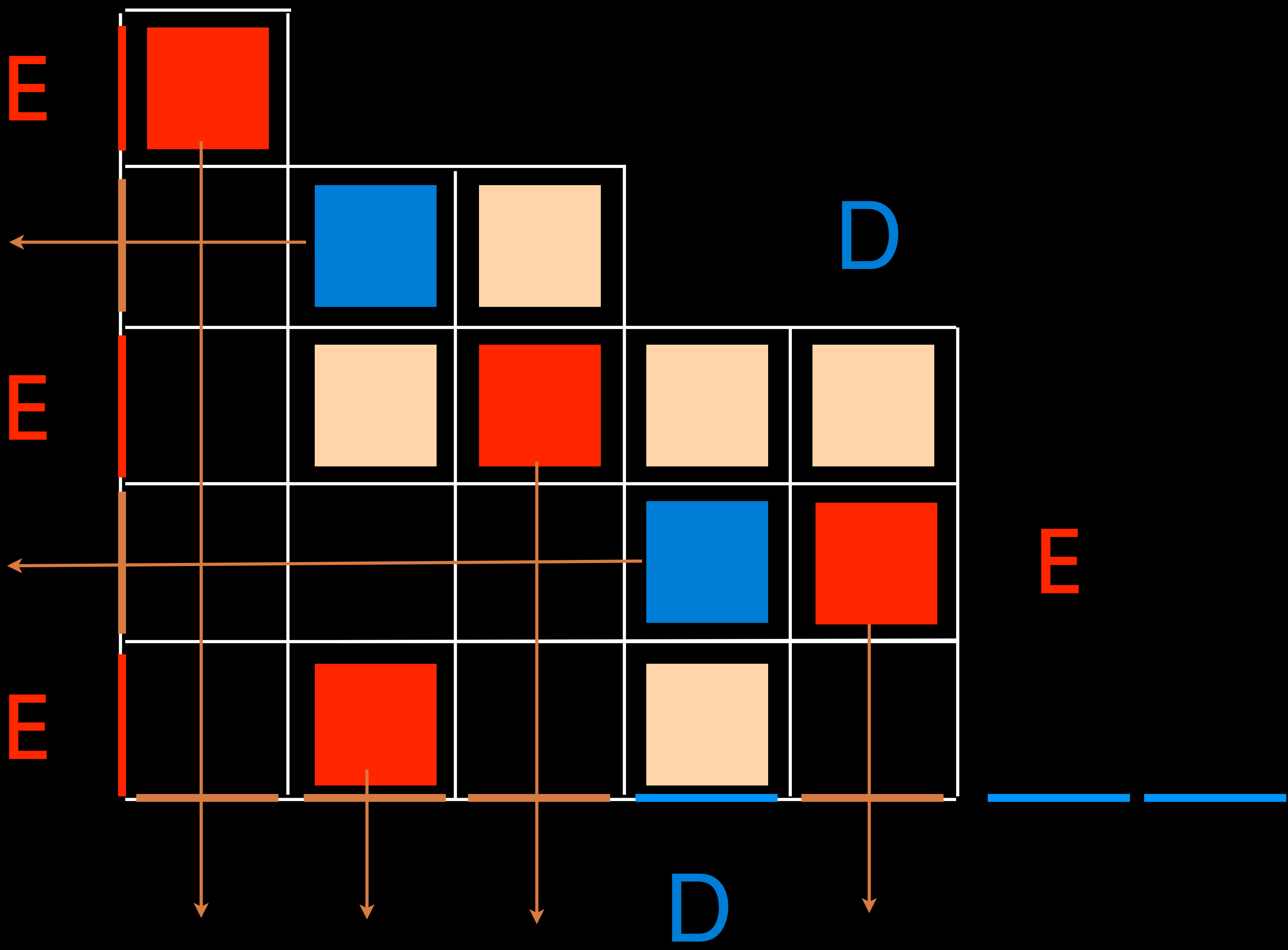
Déf – ligne restreinte : ssi elle a une case restreinte, c.à.d une case contenant un 0 et située au dessus d'un 1.

Cor. La probabilité stationnaire associée à l'état $\tau = (\tau_1, \dots, \tau_n)$ (PASEP) est

$$P_{\tau}(q) = \frac{1}{Z_n} \sum_{T} q^{\mathrm{wt}(T)} \alpha^{-f(T)} \beta^{-u(T)}$$

T tableau de permutation forme F associé à τ

S. Corteel, L. Williams (2007) (2008) (2009)


E
D
E
E
E
D

orthogonal polynomials

- Orthogonal polynomials

→ Sasamoto (1999)

→ Blythe, Evans, Colaiori, Essler (2000)

q- Hermite polynomial

$\alpha, \beta, q \qquad \gamma = \delta = 1$

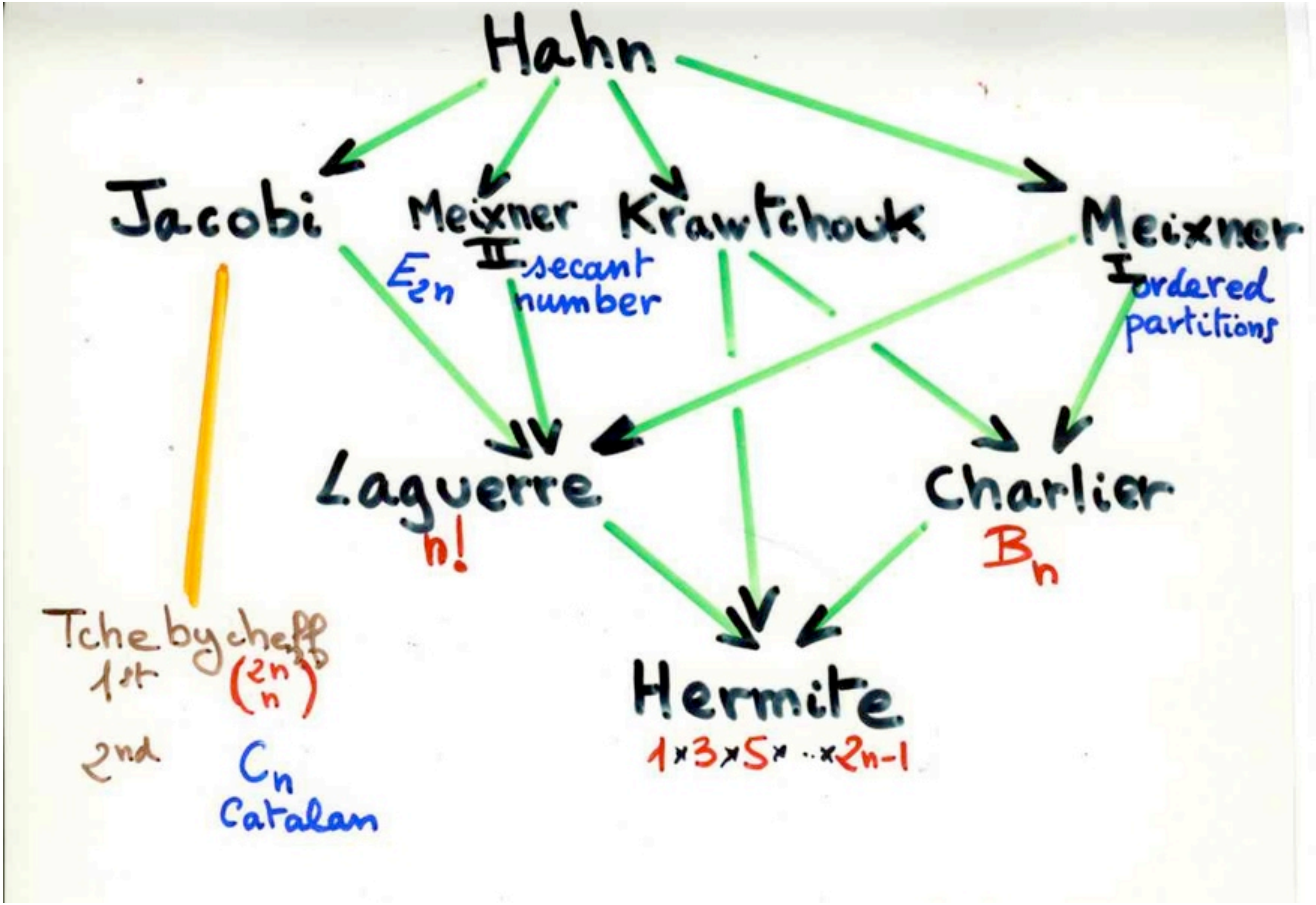
$$D = \frac{1}{1-q} + \frac{1}{\sqrt{1-q}}\,\hat{a}$$
$$E = \frac{1}{1-q} + \frac{1}{\sqrt{1-q}}\,\hat{a}^+$$
$$\hat{a}\,\hat{a}^+ - q\,\hat{a}^+\hat{a} = 1$$

→ Uchiyama, Sasamoto, Wadati (2003)

$\alpha, \beta, \gamma, \delta, \quad q$

Askey-Wilson polynomials

Askey-Wilson



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http://www.labri.fr/perso/viennot/

Recherche, cv, publications, exposés, diaporamas, livres, petite école, photos: voir ma page personnelle ici

Vulgarisation scientifique voir la page de l'association Cont'Science

downloadable papers, slides and lecture notes, etc ... here

(the summary on page “recherches” and most slides are in english)

page “video”

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conference 23 April 2008,

Isaac Newton Institute for Mathematical science

or http://www.newton.cam.ac.uk/ (page “web seminar”)

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The PASEP

The Matrix Ansatz

The PASEP algebra

alternative tableaux

stationary probabilities for the PASEP

permutation tableaux

orthogonal polynomials

references