Chapter 4b

Alternative tableaux and the PASEP

13, 18 january 2011

Talca

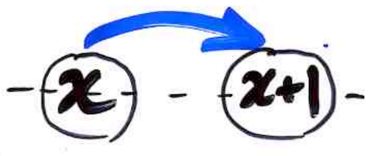
The “exchange-fusion” algorithm

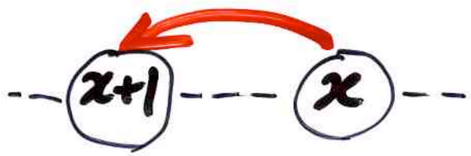
Def- Permutation $\sigma = \sigma(1) \cdots \sigma(n)$

$x = \sigma(i)$, $1 \leq x < n$

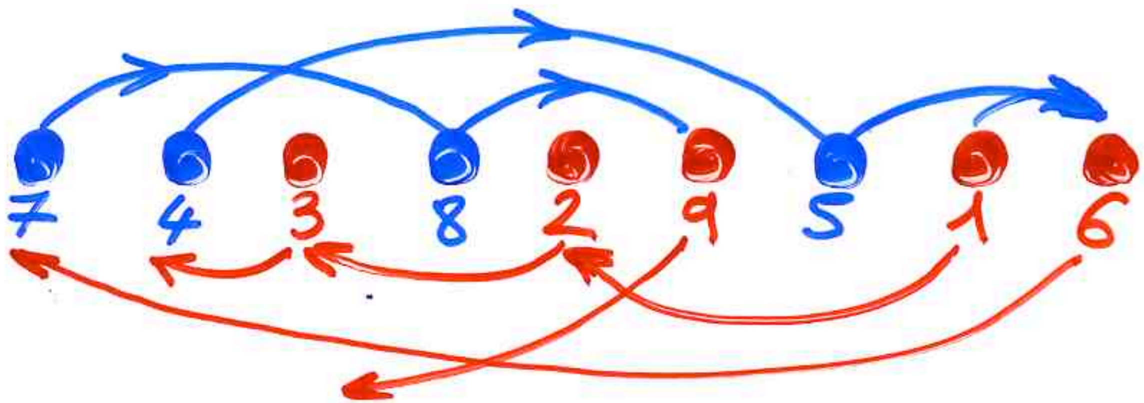
(valeur) x $\begin{cases} \text{avance} \\ \text{recul} \end{cases}$ $x+1 = \sigma(j)$, $\begin{cases} i<j \\ j<i \end{cases}$

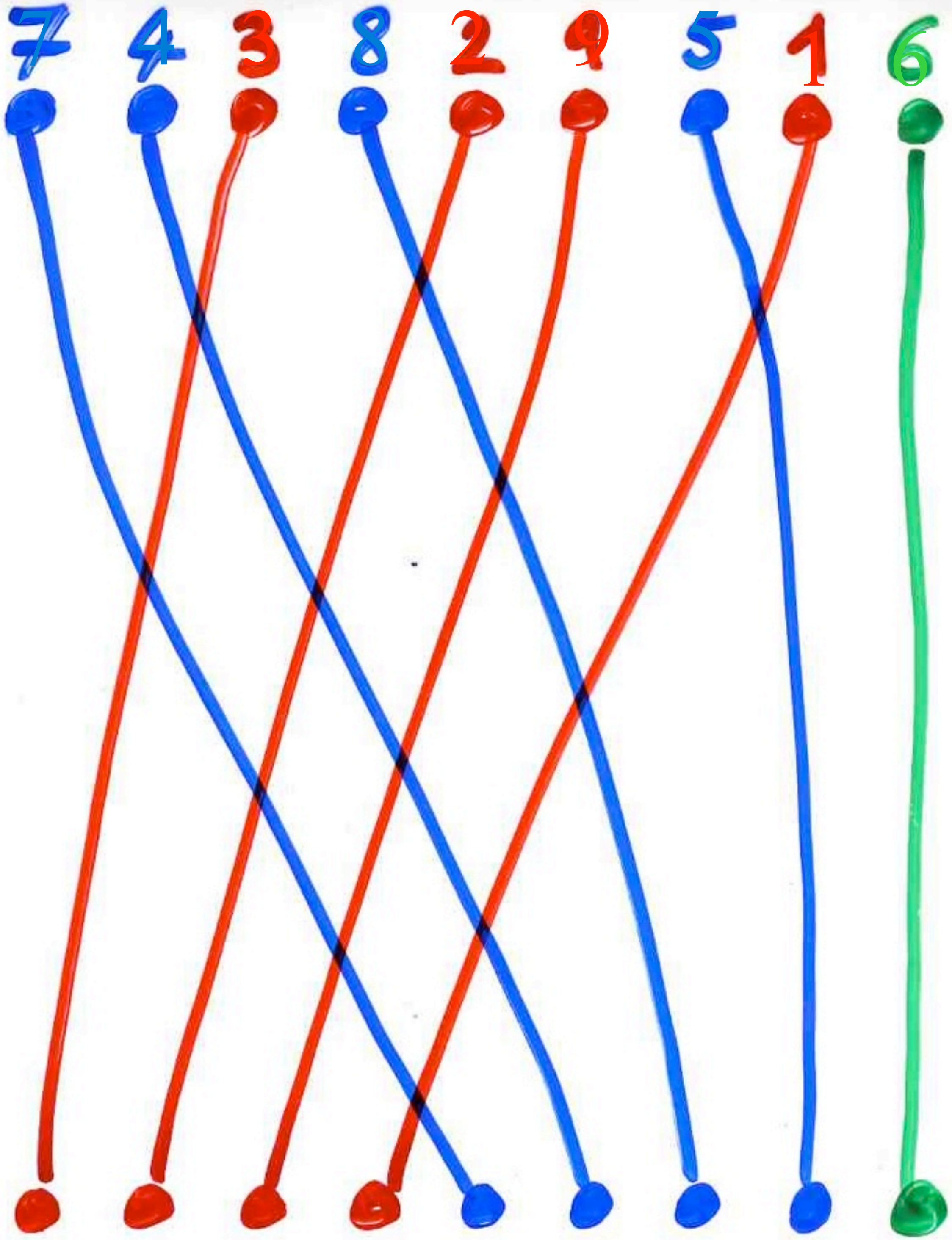
- convention $x = n$ est un recul





$\sigma = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$

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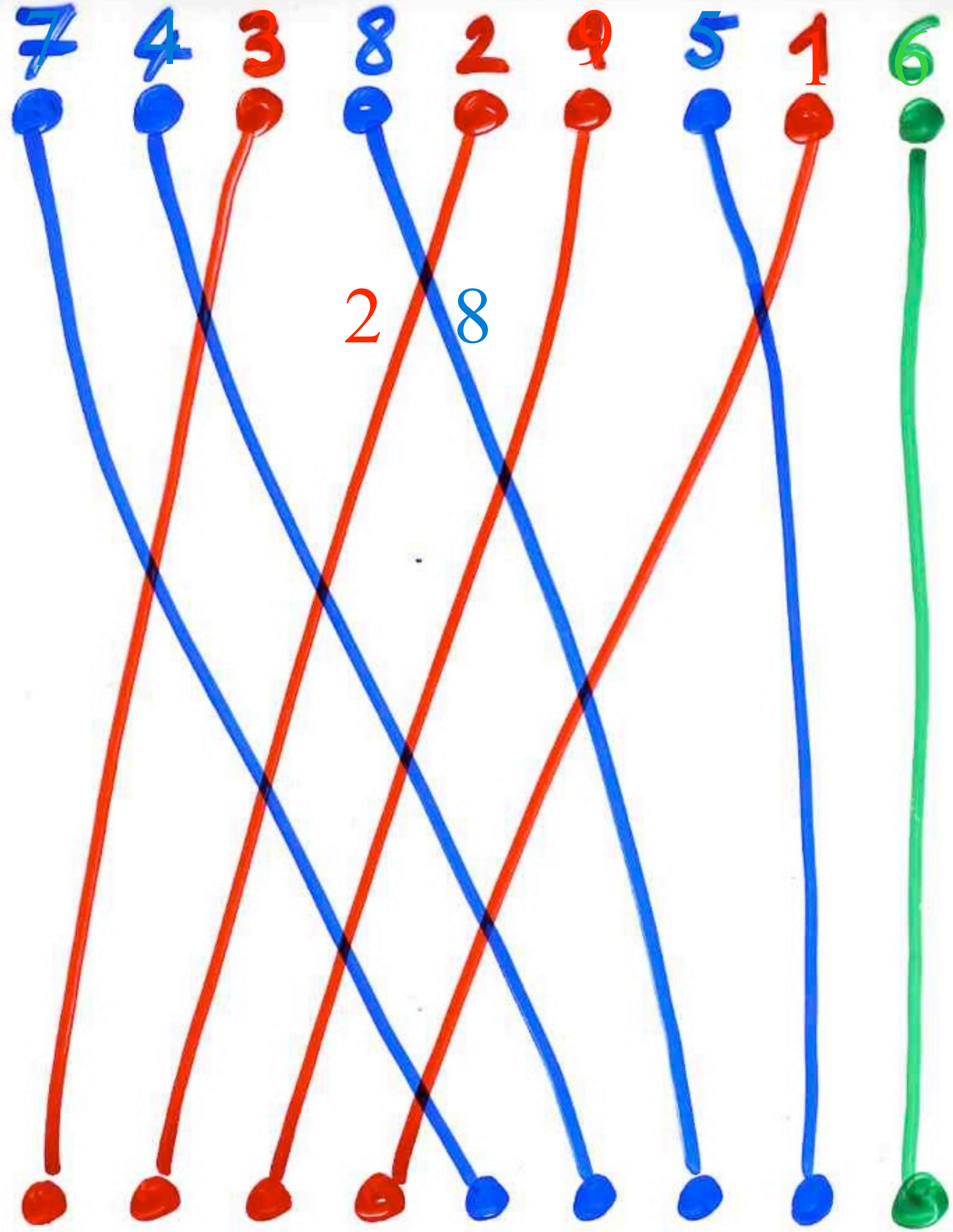

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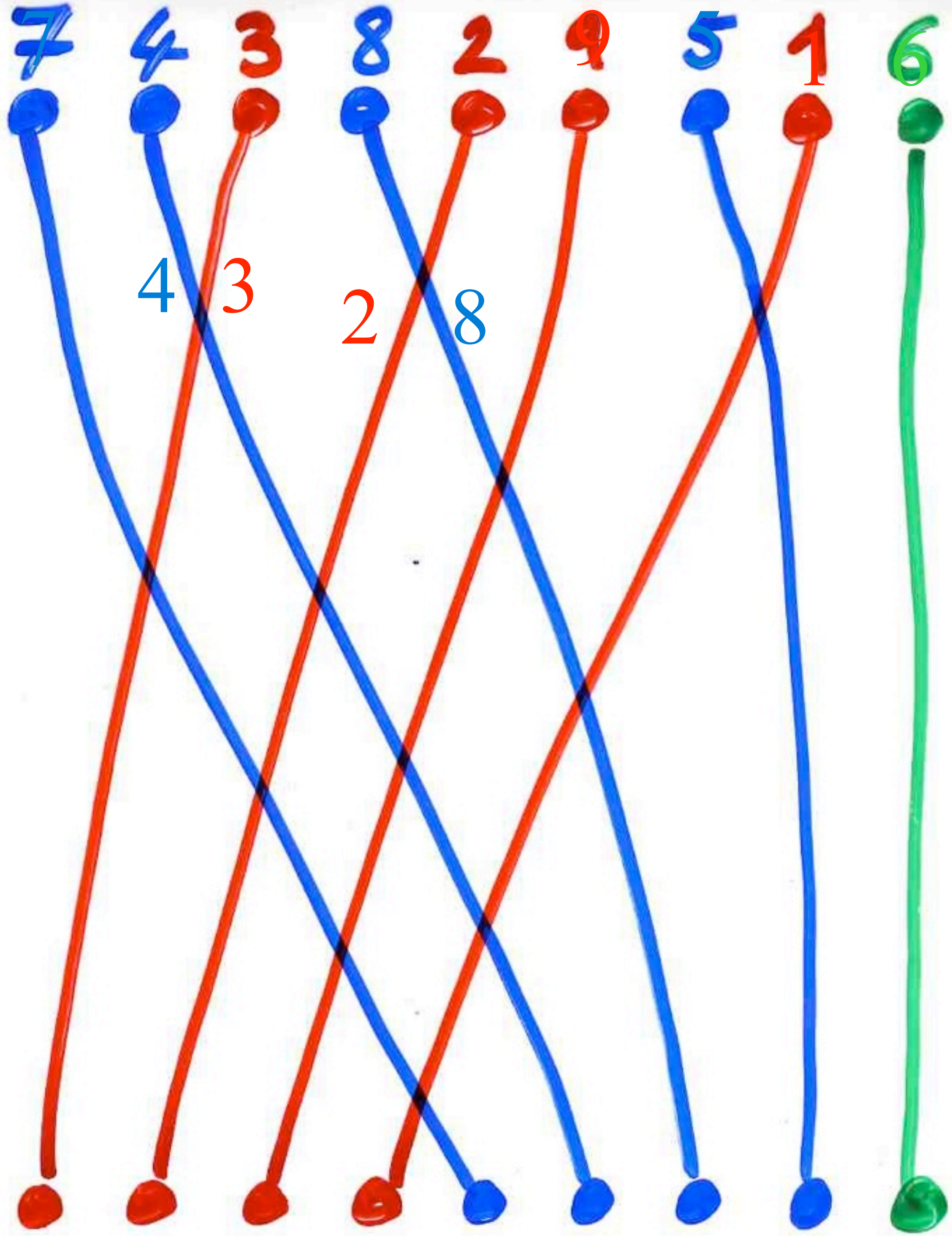
2 8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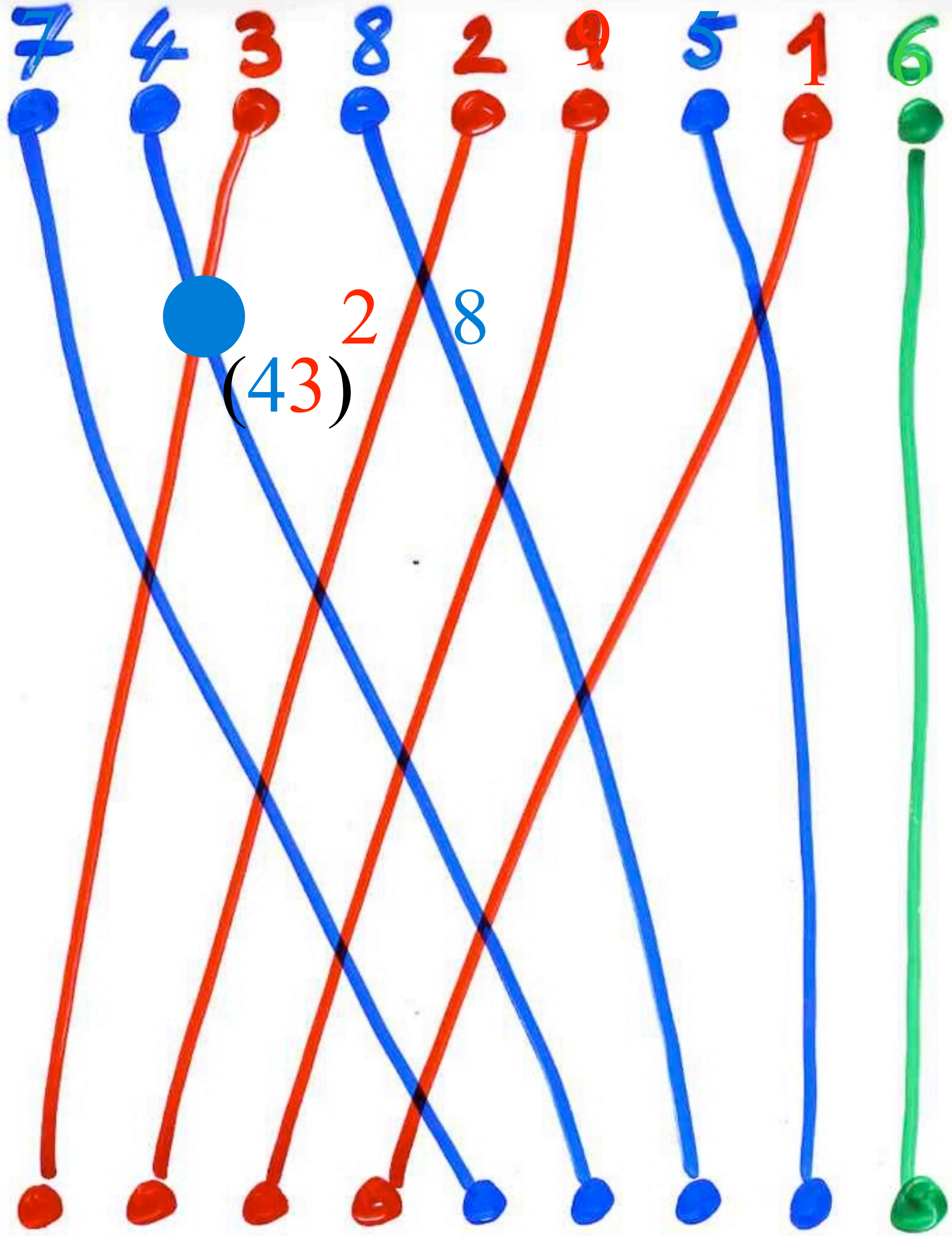
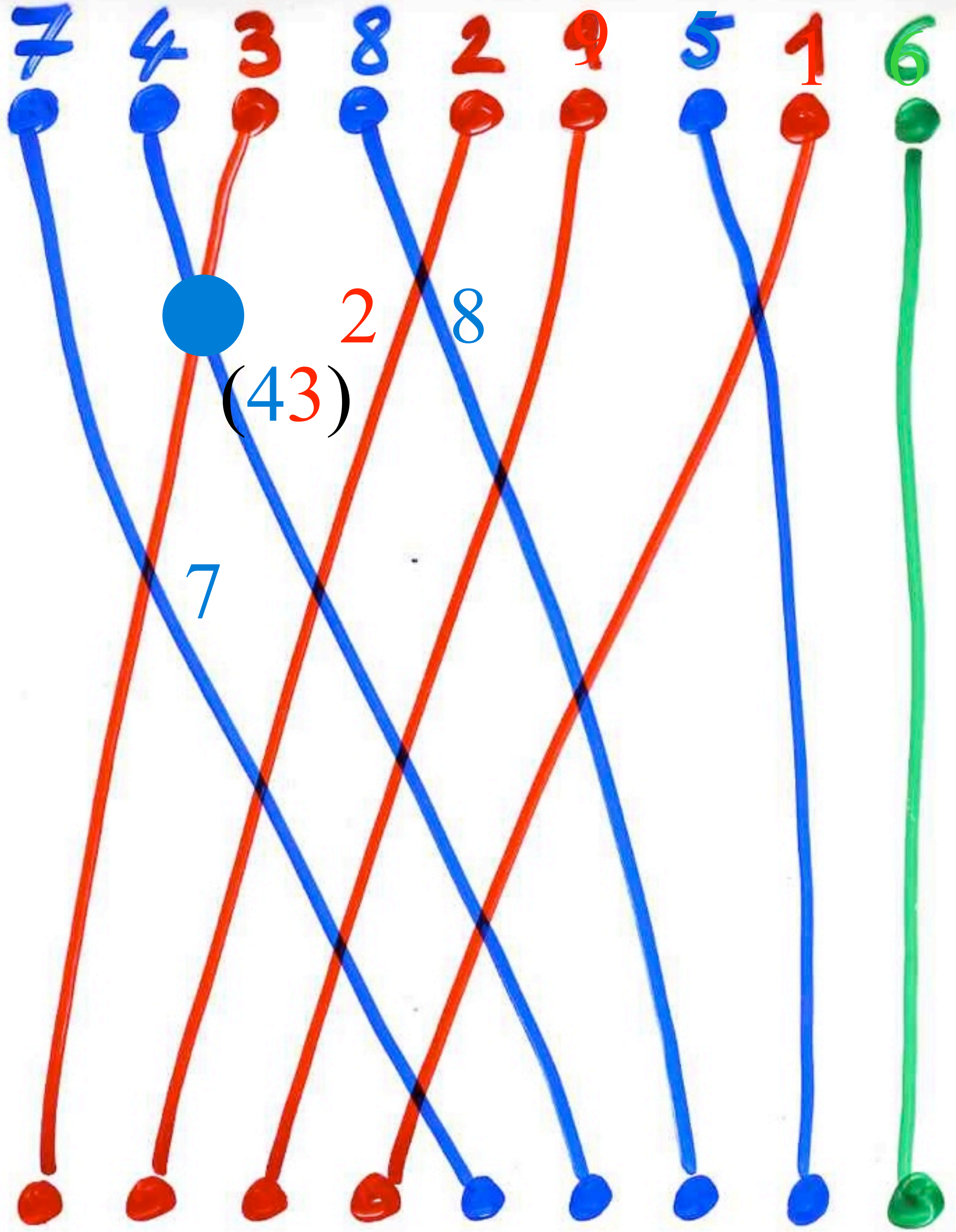

7 4 3 8 2 9 5 1 6
$(43)^2$
8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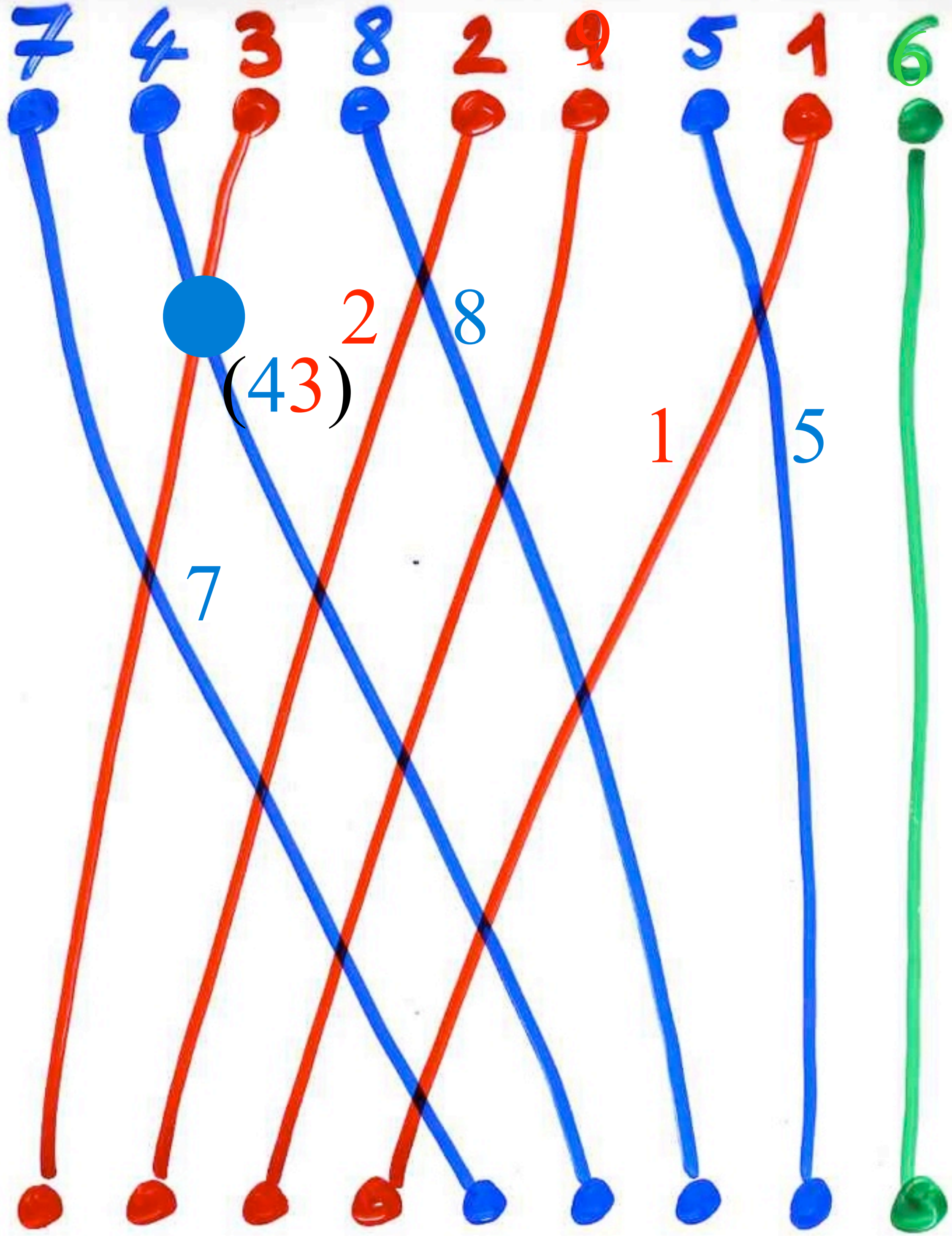
2
8
(43)
7



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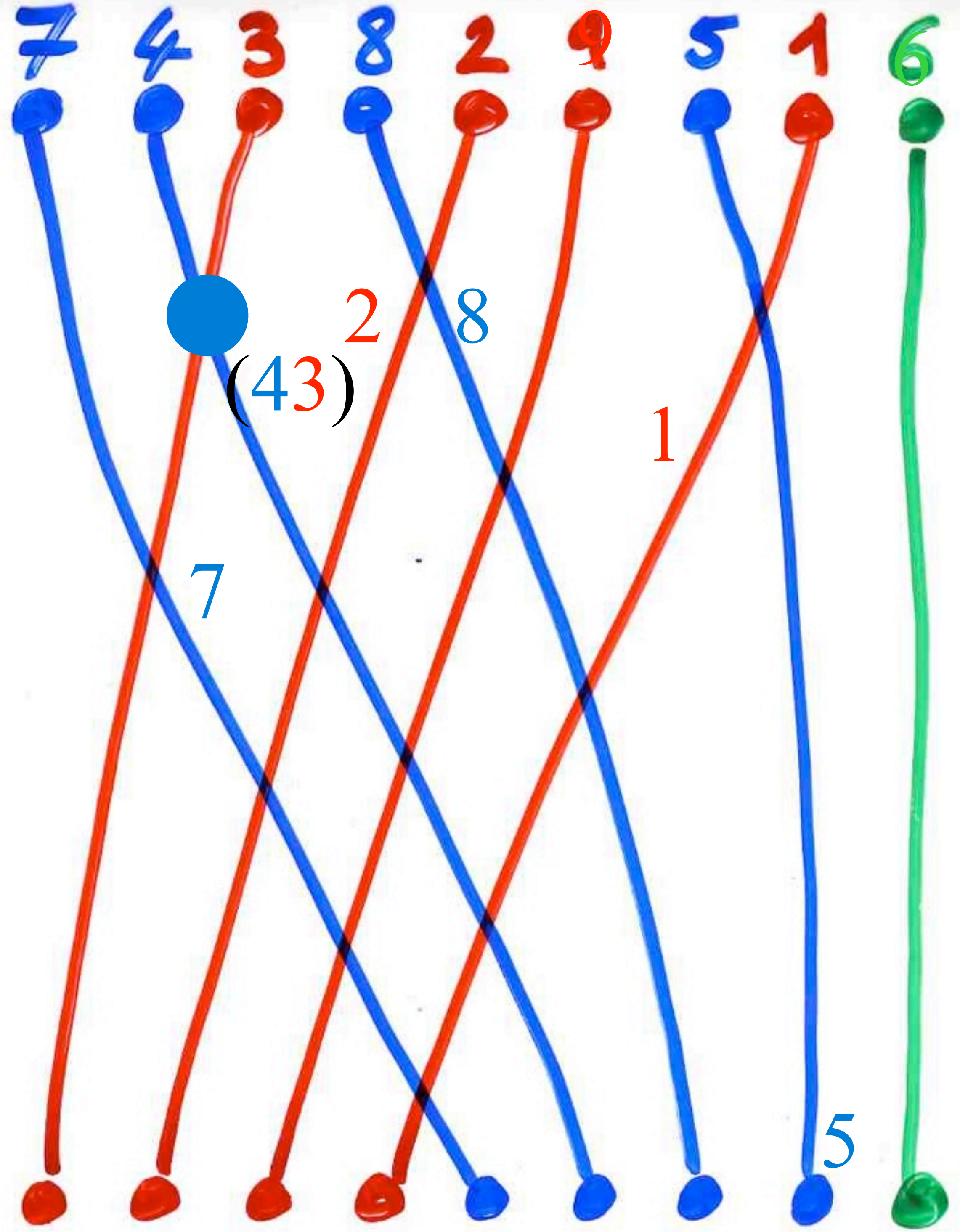
2
8
(43)
1
5
7



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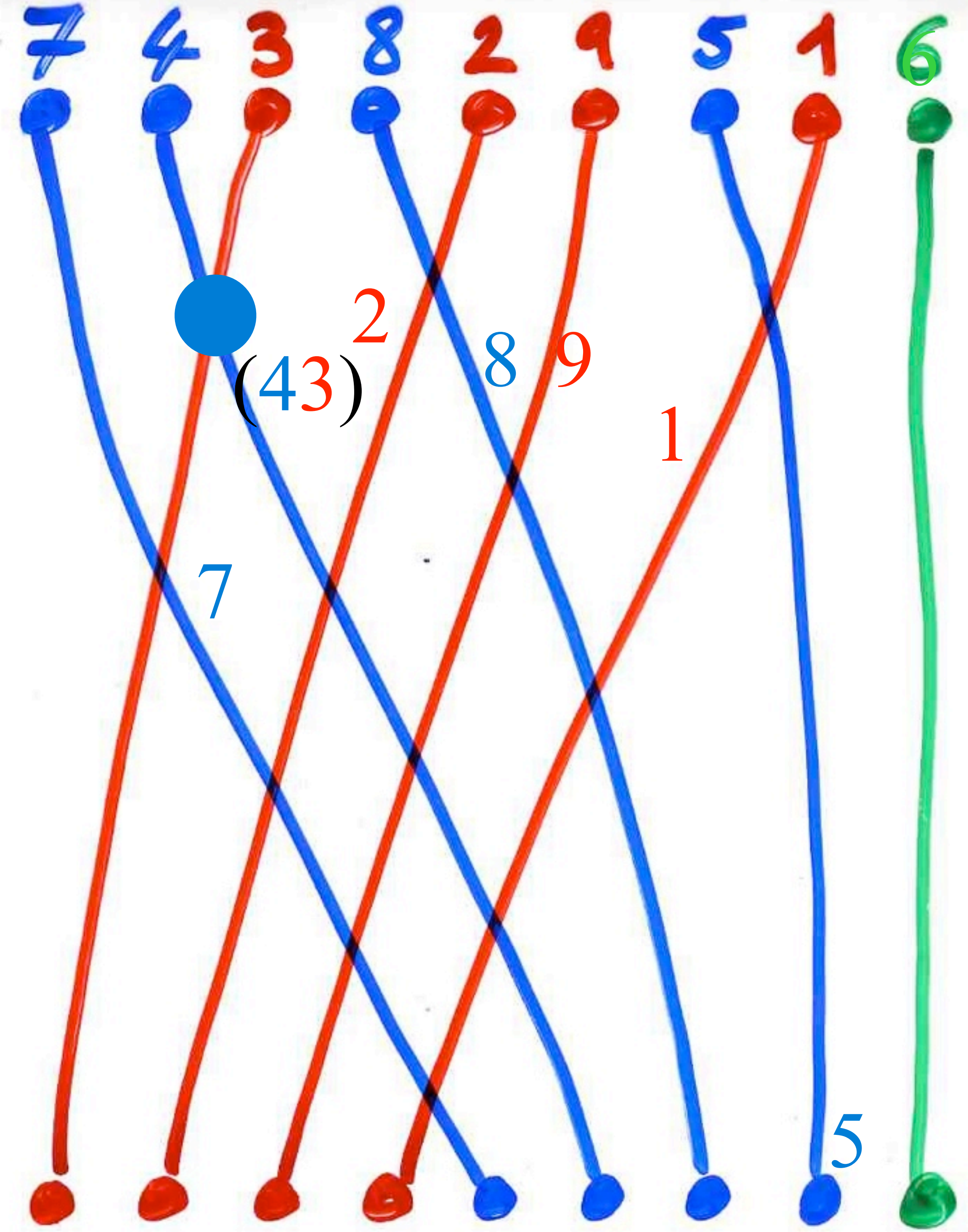
2
8
(43)
1
7
5



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(43)
2
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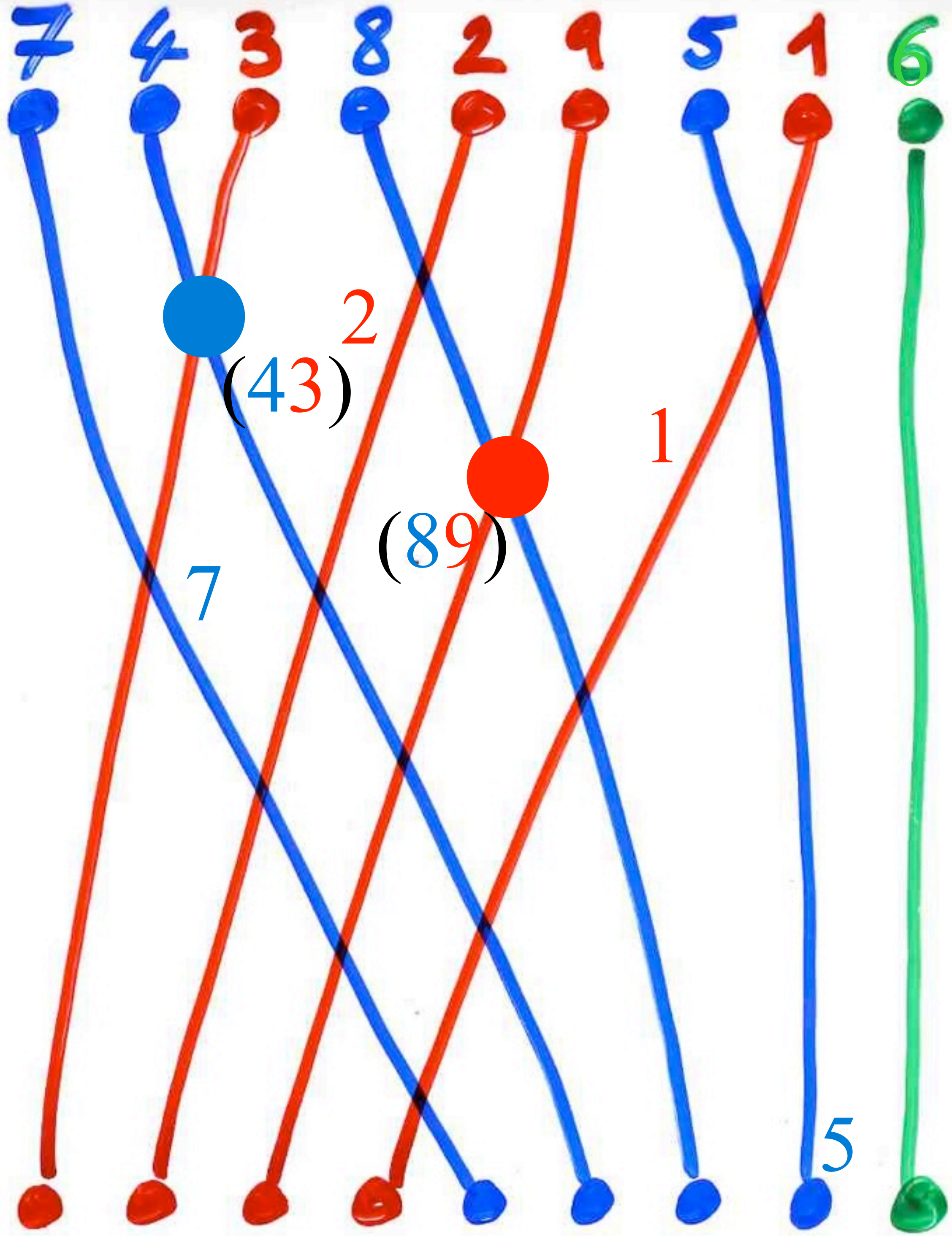
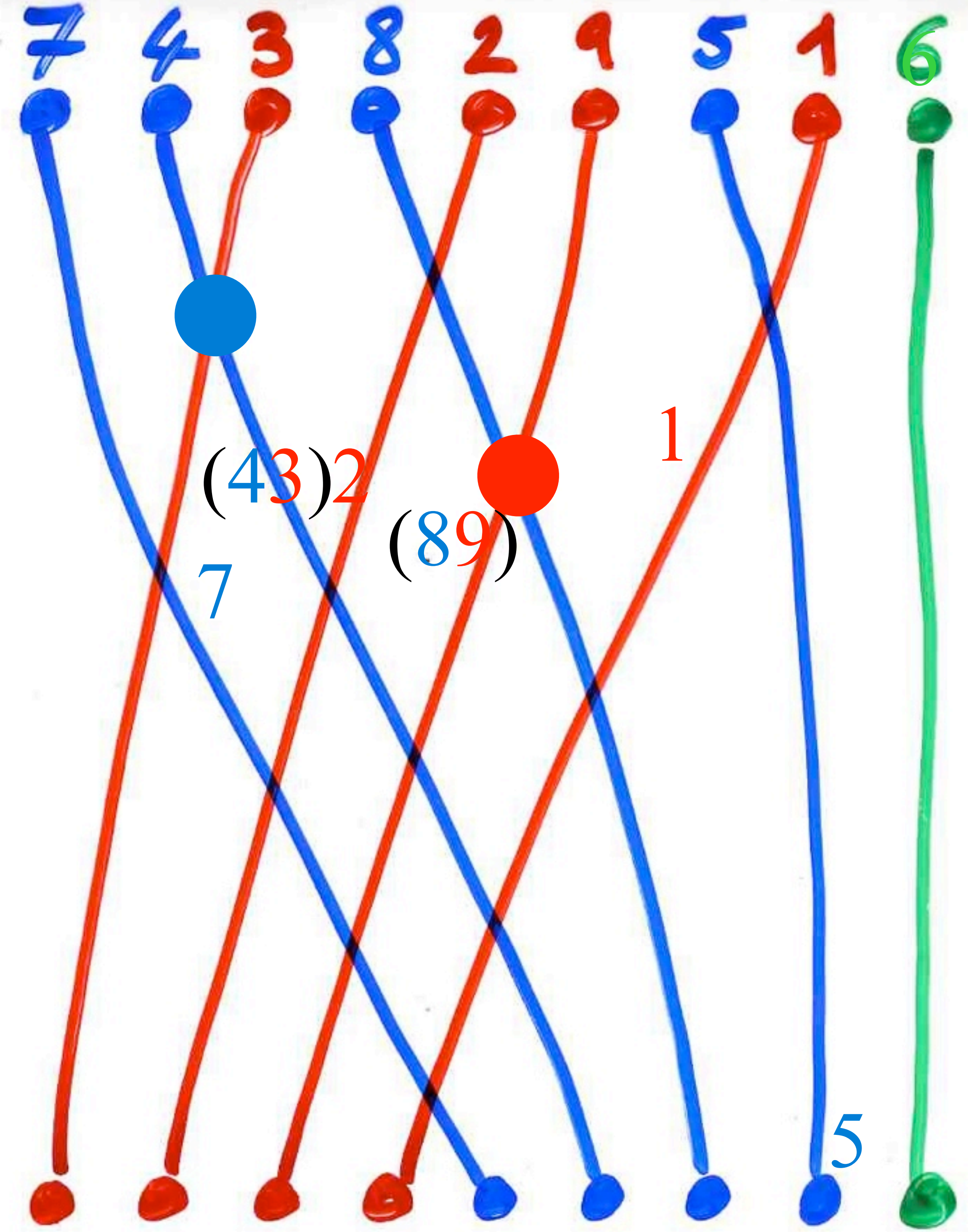
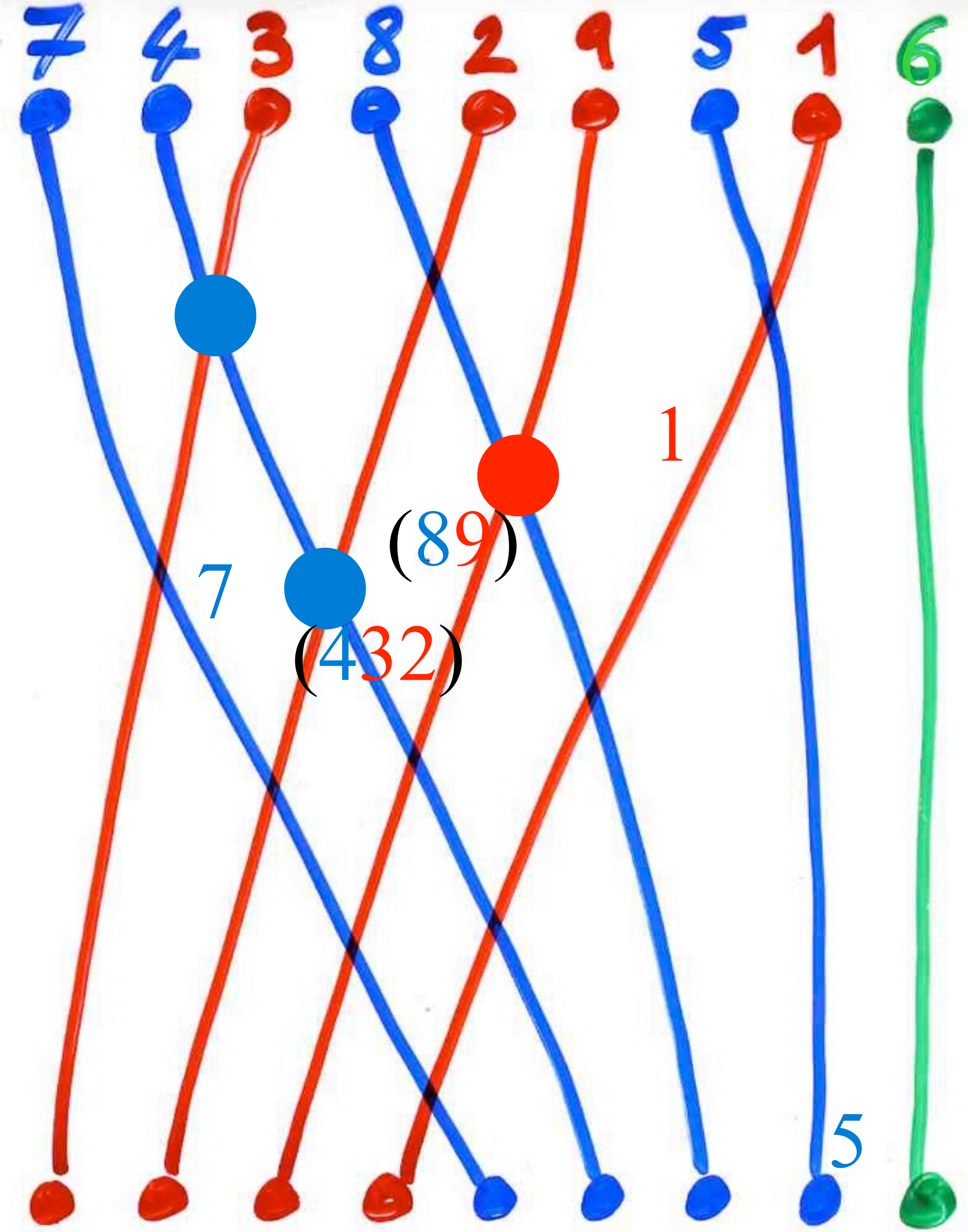
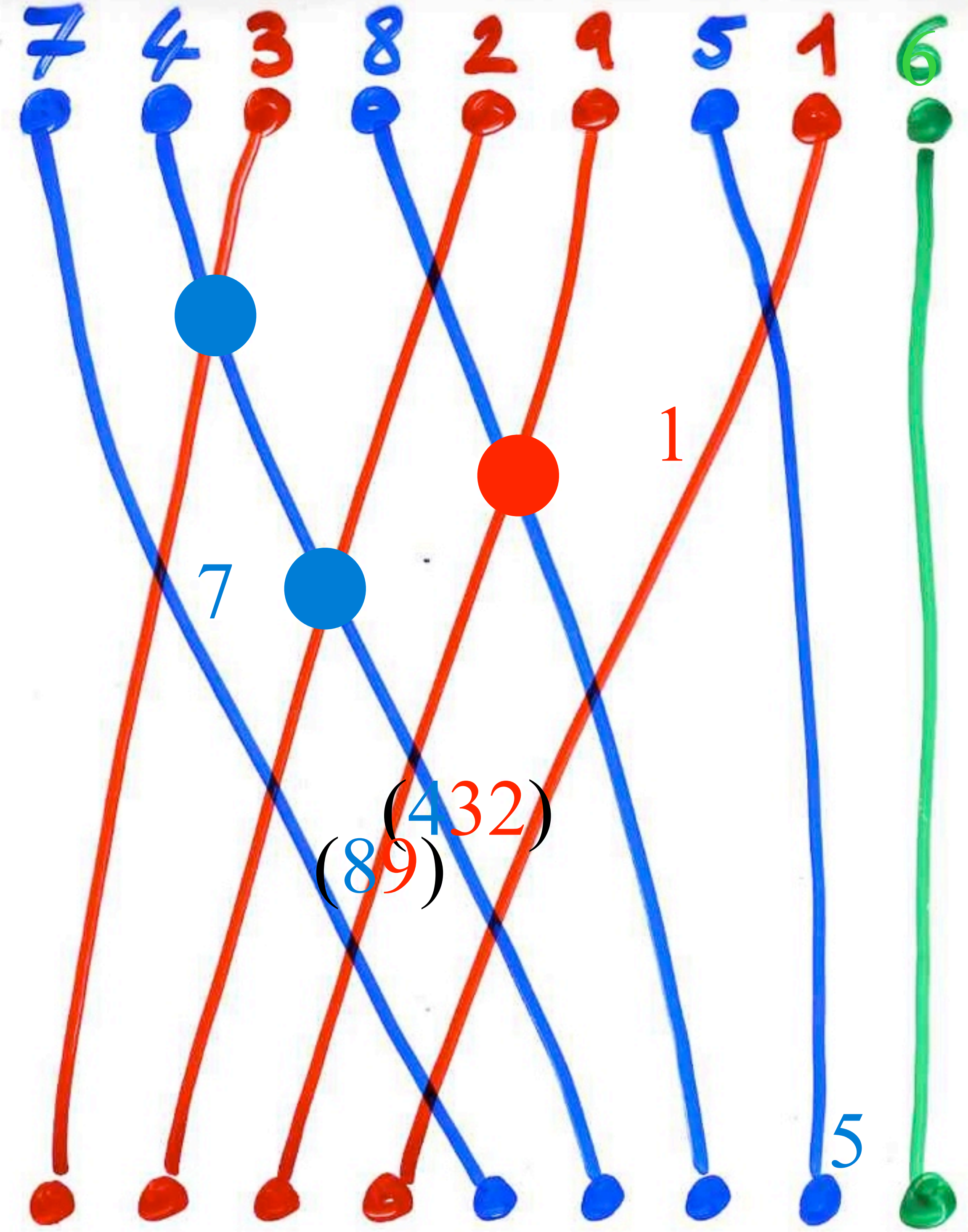


7 4 3 8 2 9 5 1 6
2
(43)
1
(89)
7
5

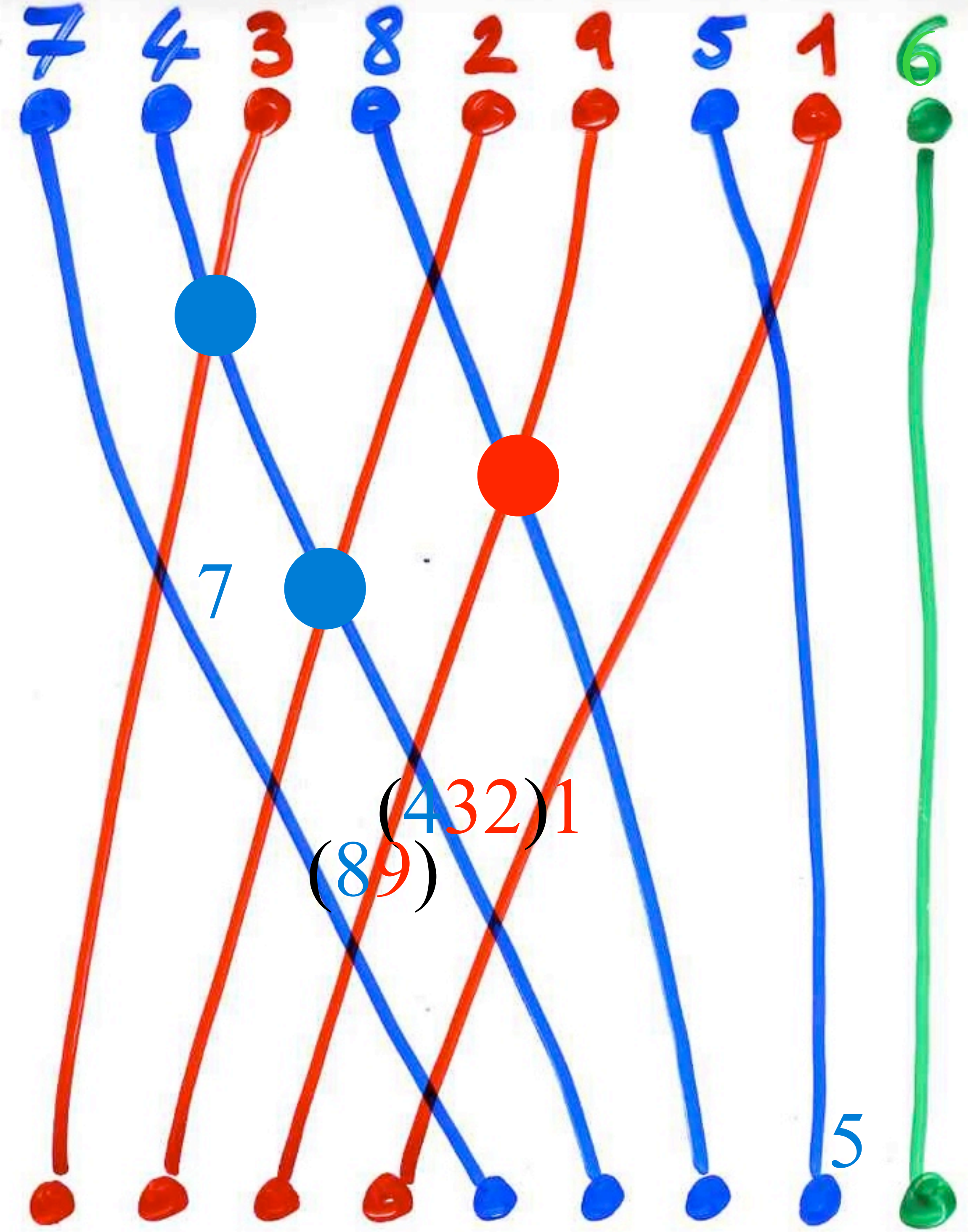


7 4 3 8 2 9 5 1 6
(43)2
(89)
1
7
5

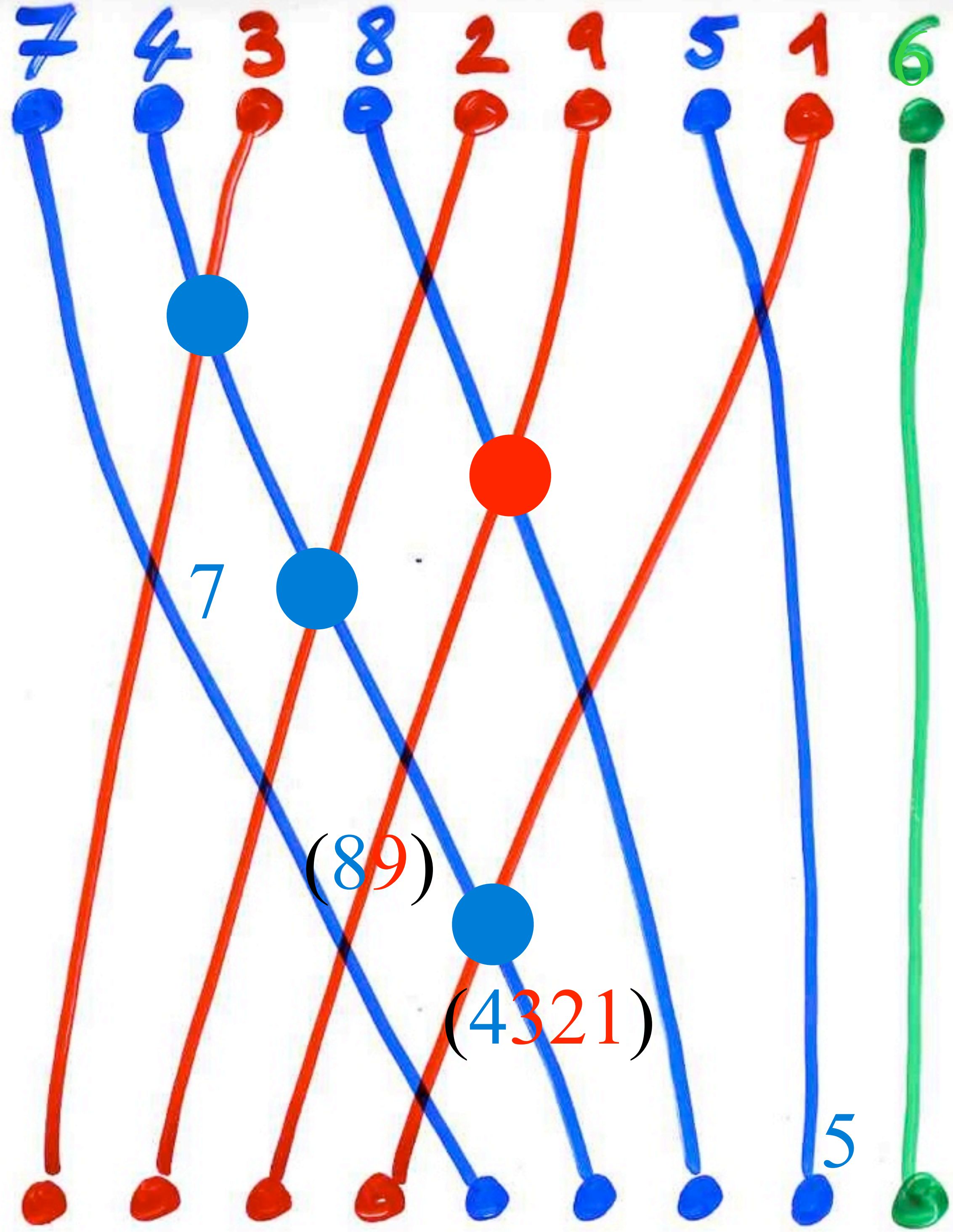

7 4 3 8 2 9 5 1 6
1
(89)
7
(432)
5



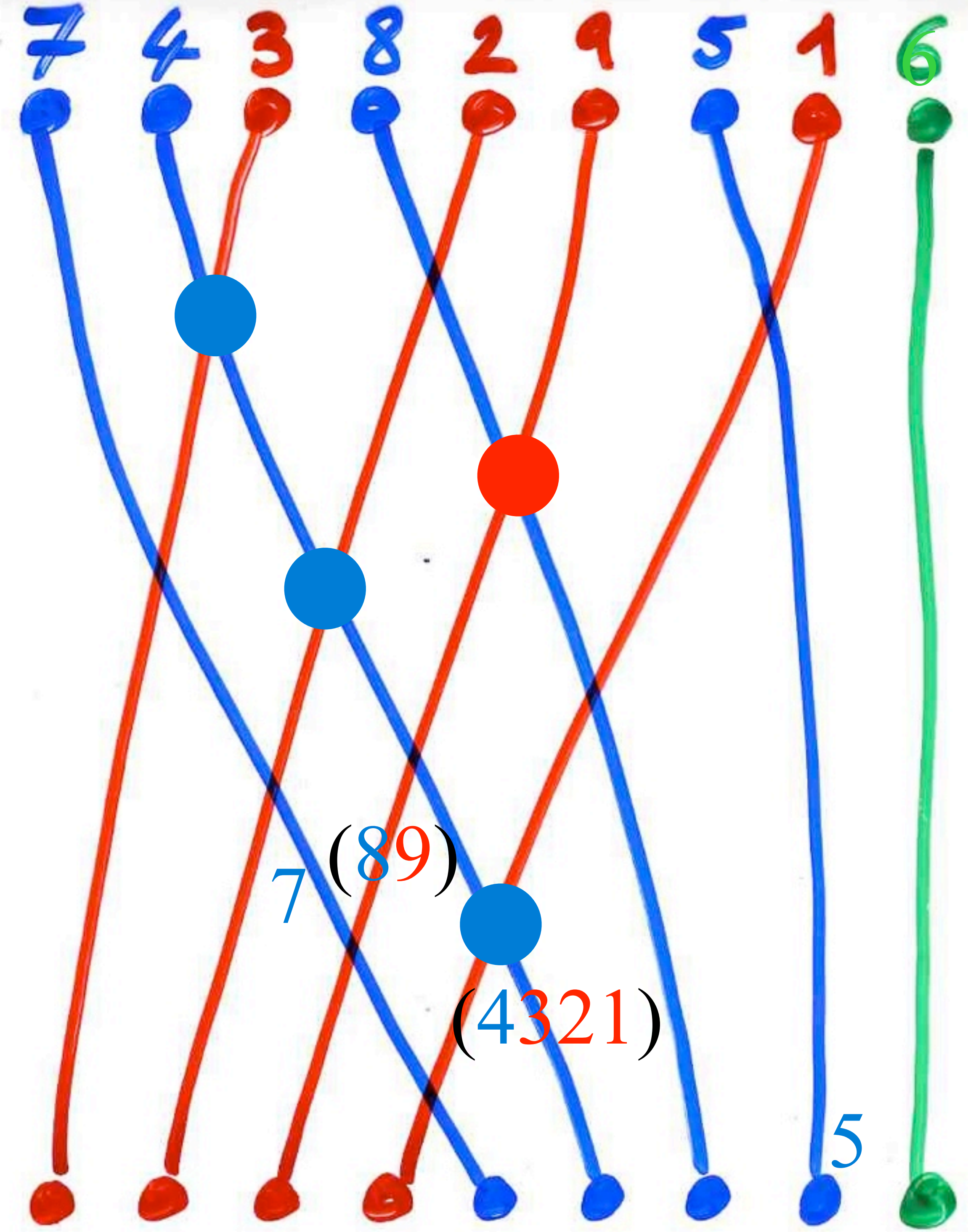
7 4 3 8 2 9 5 1 6
1
7
(432)
(89)
5



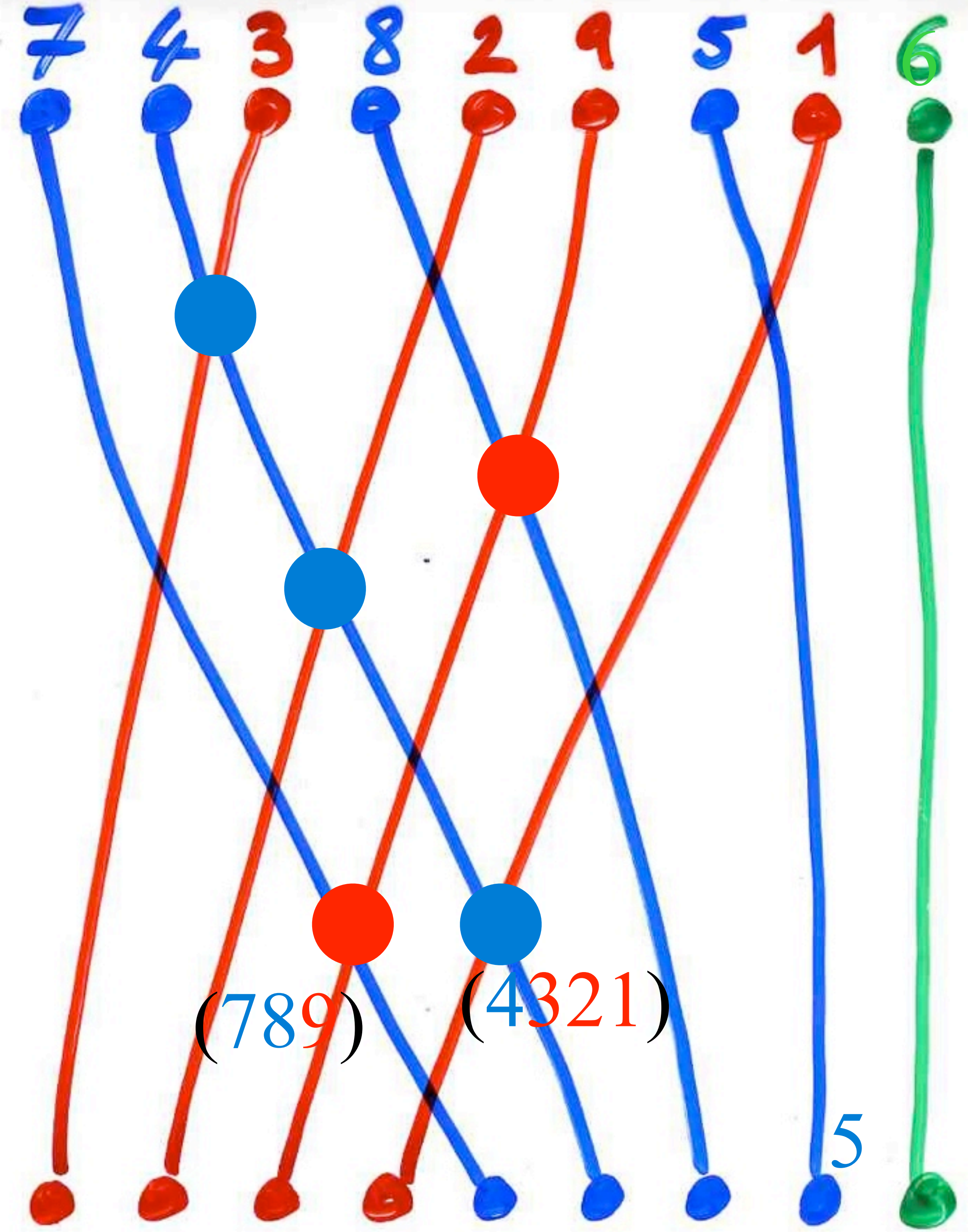
7 4 3 8 2 9 5 1 6
7
(432)1
(89)
5



7 4 3 8 2 9 5 1 6
7
(89)
(4321)
5

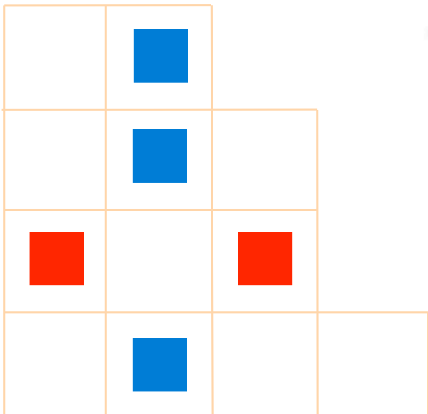


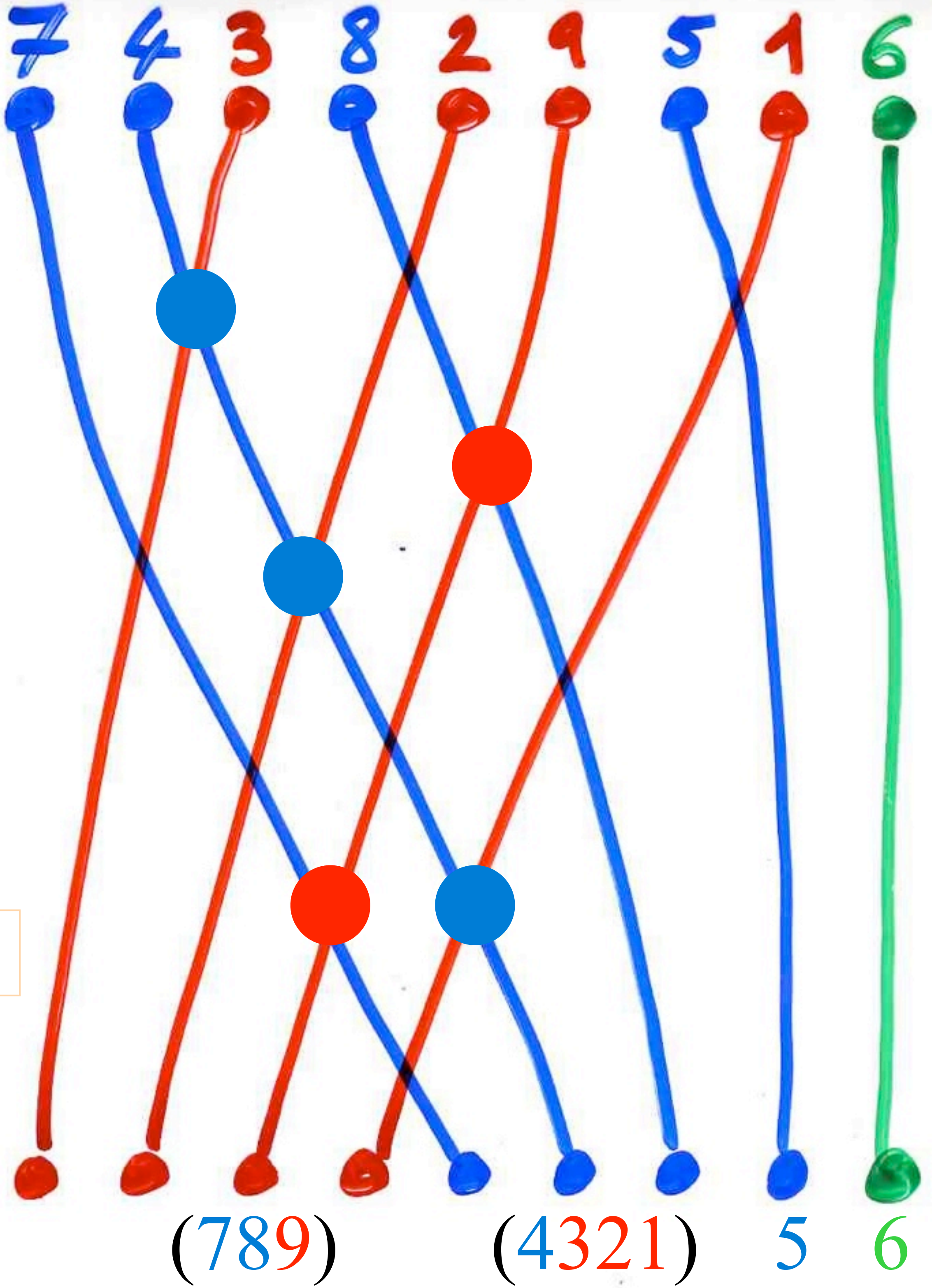
7 4 3 8 2 9 5 1 6
(89)
7
(4321)
5



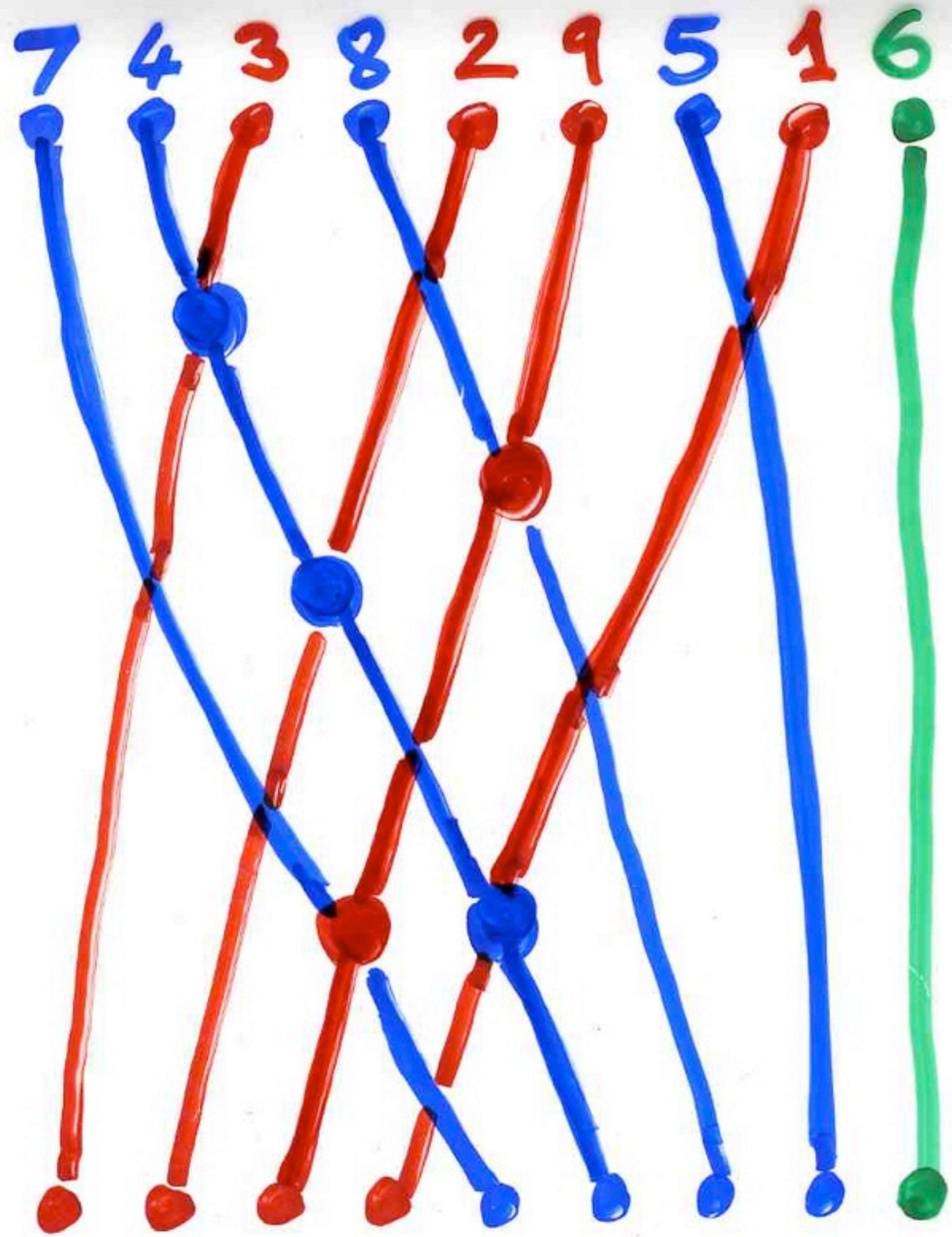
7 4 3 8 2 9 5 1 6
(789)
(4321)
5

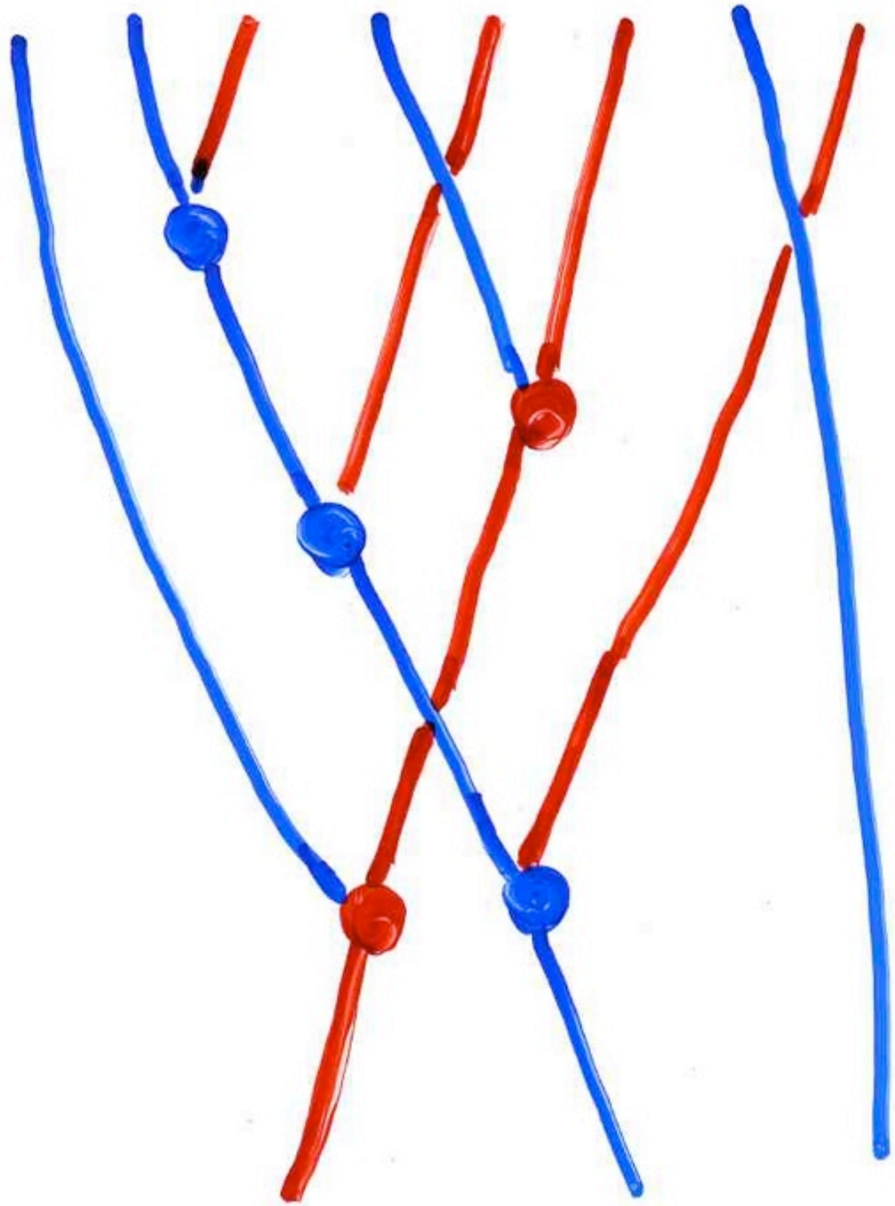
“exchange-fusion” algorithm

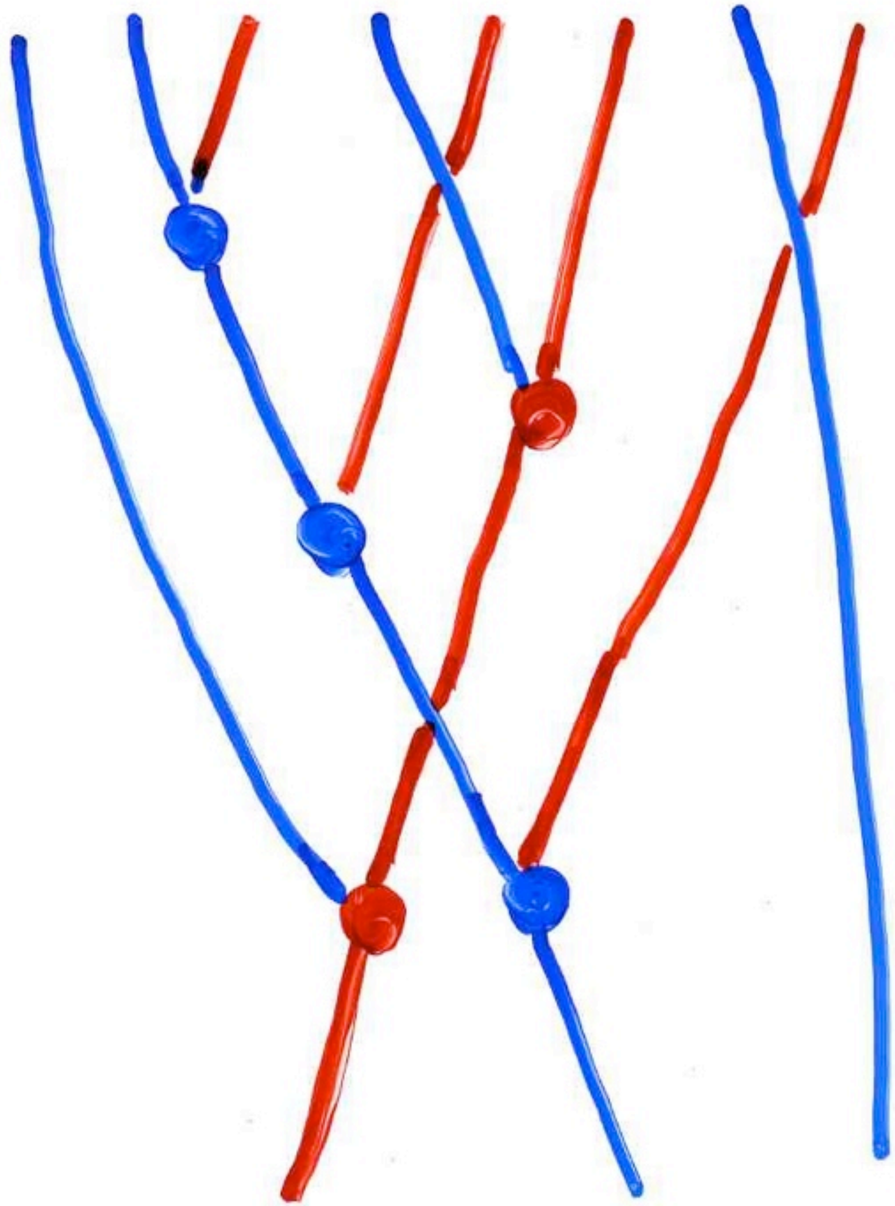


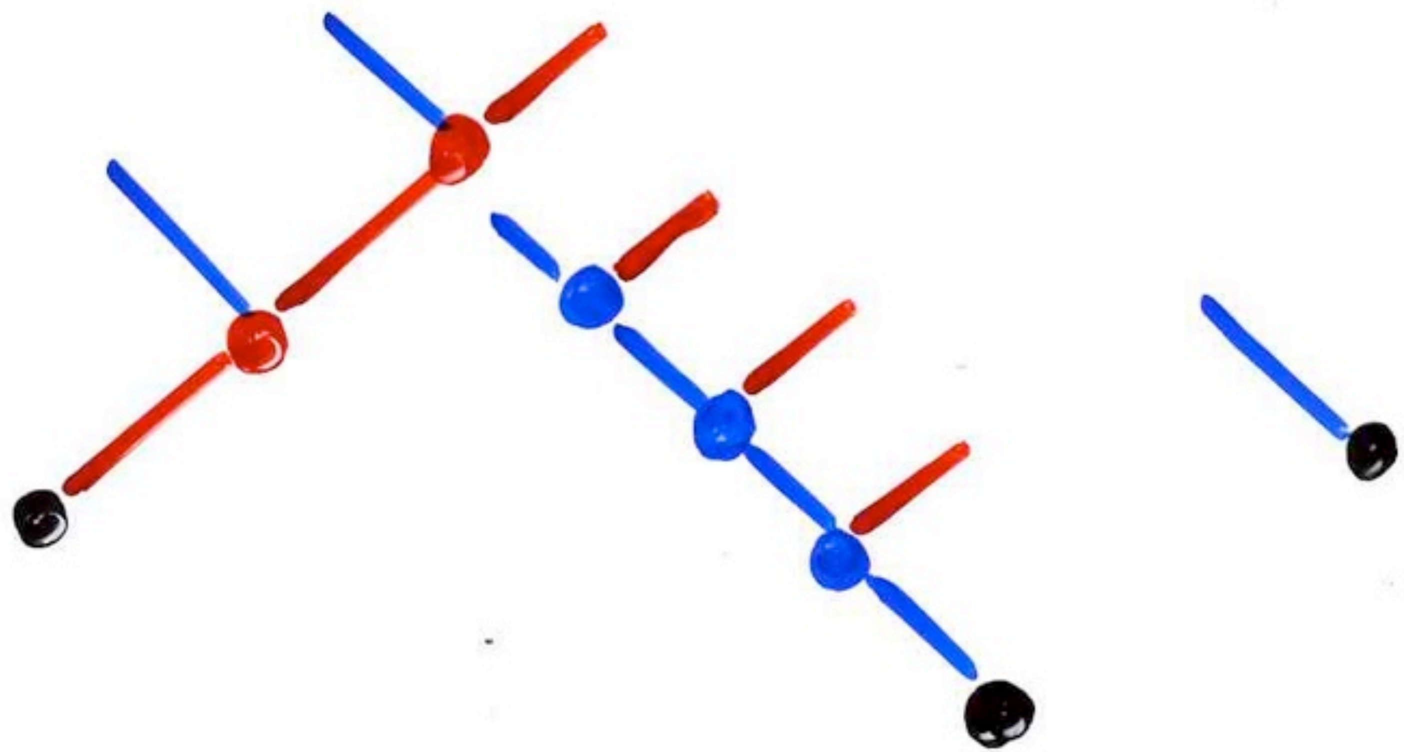


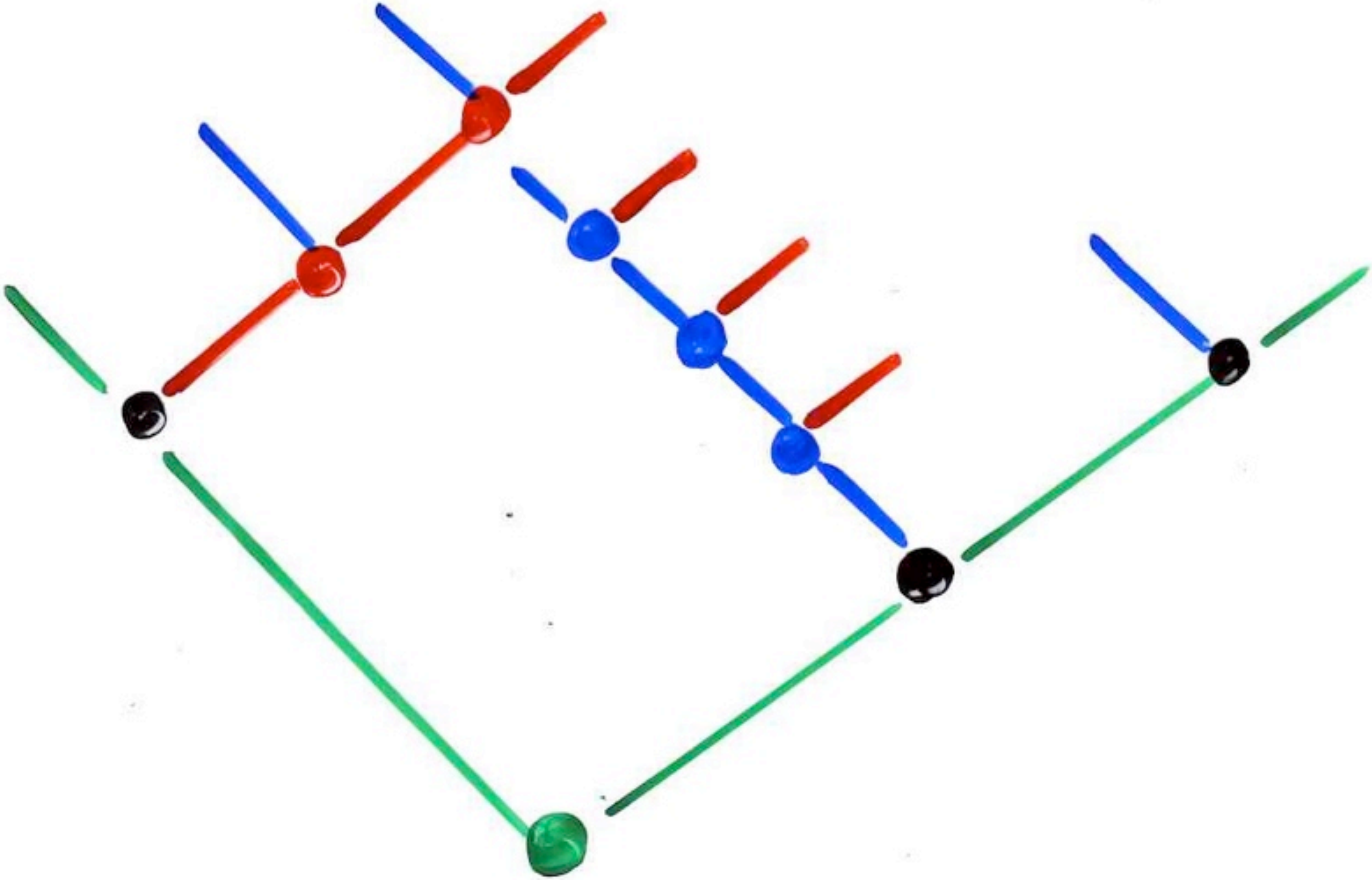
The inverse “exchange-fusion” algorithm

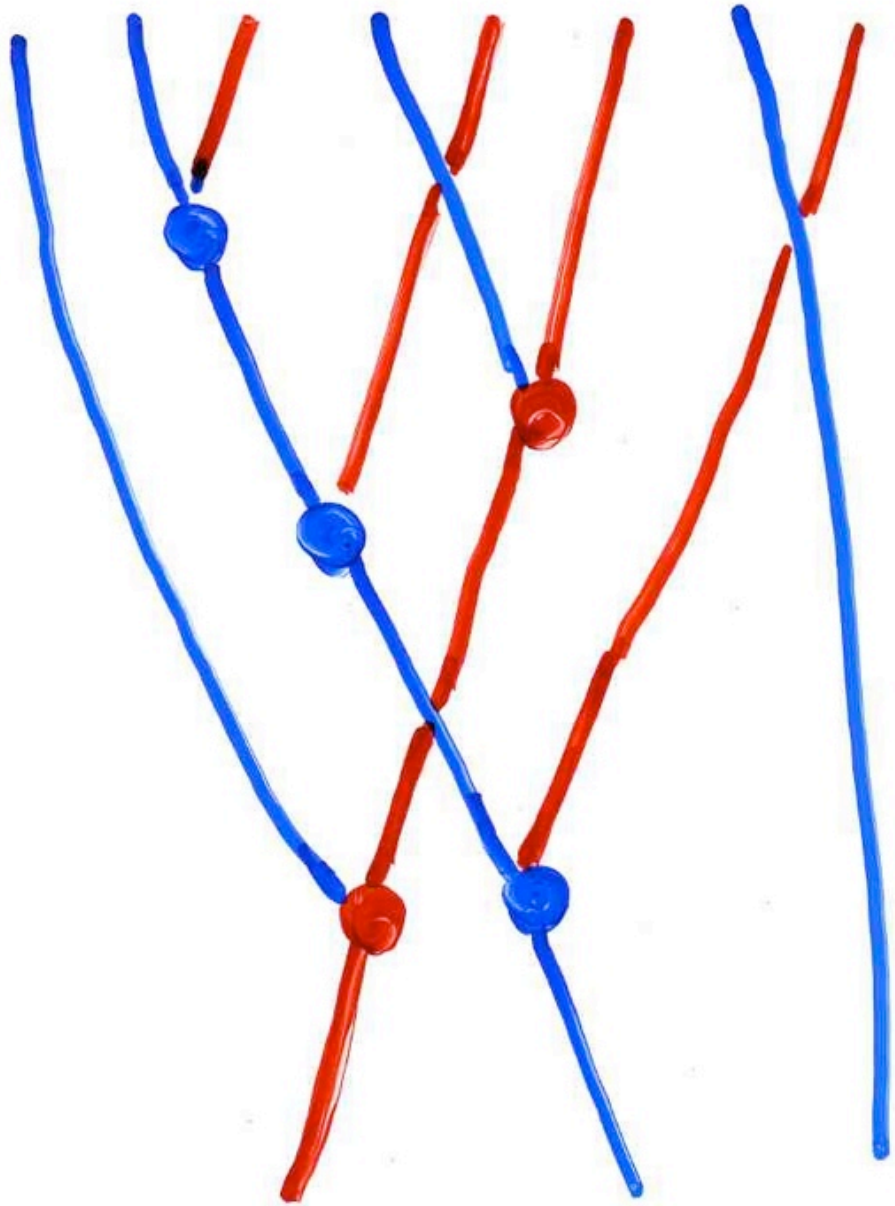

7 4 3 8 2 9 5 1 6

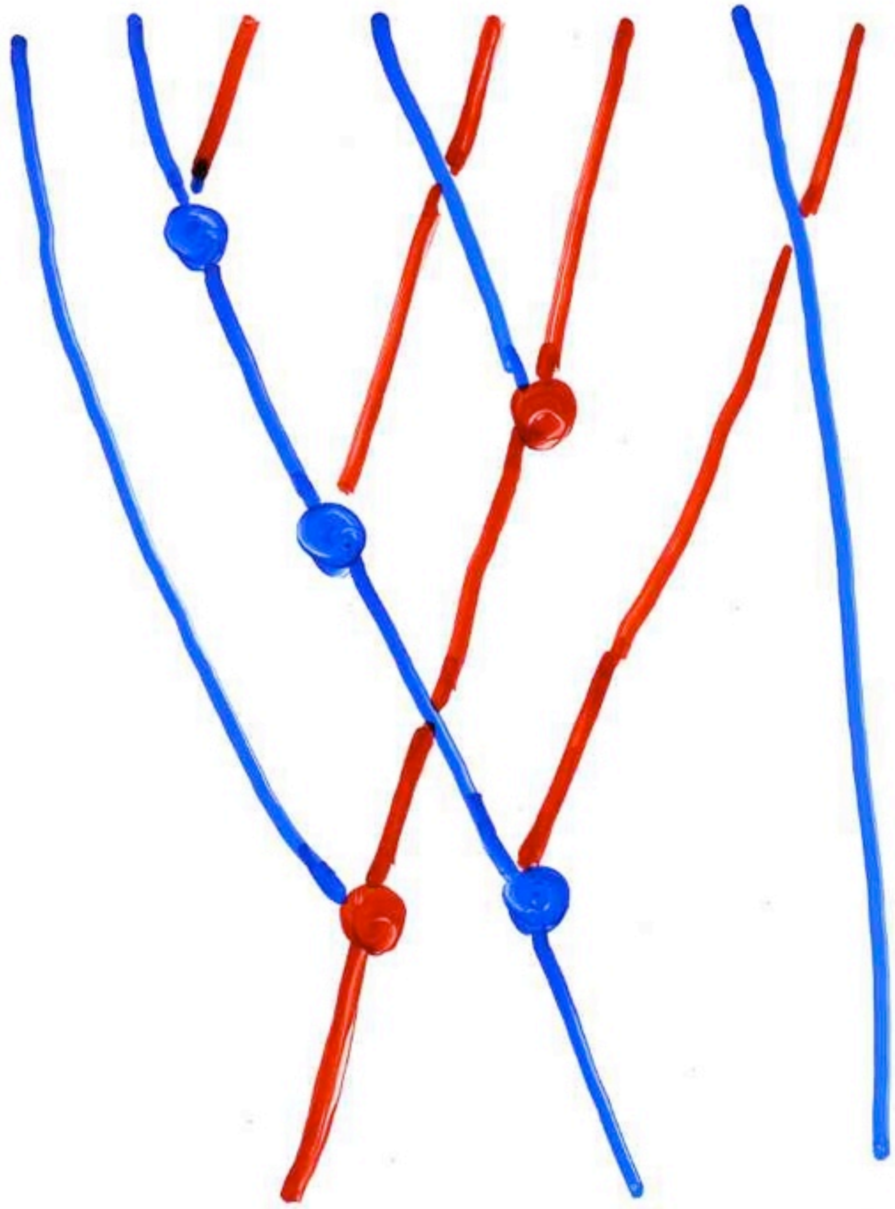




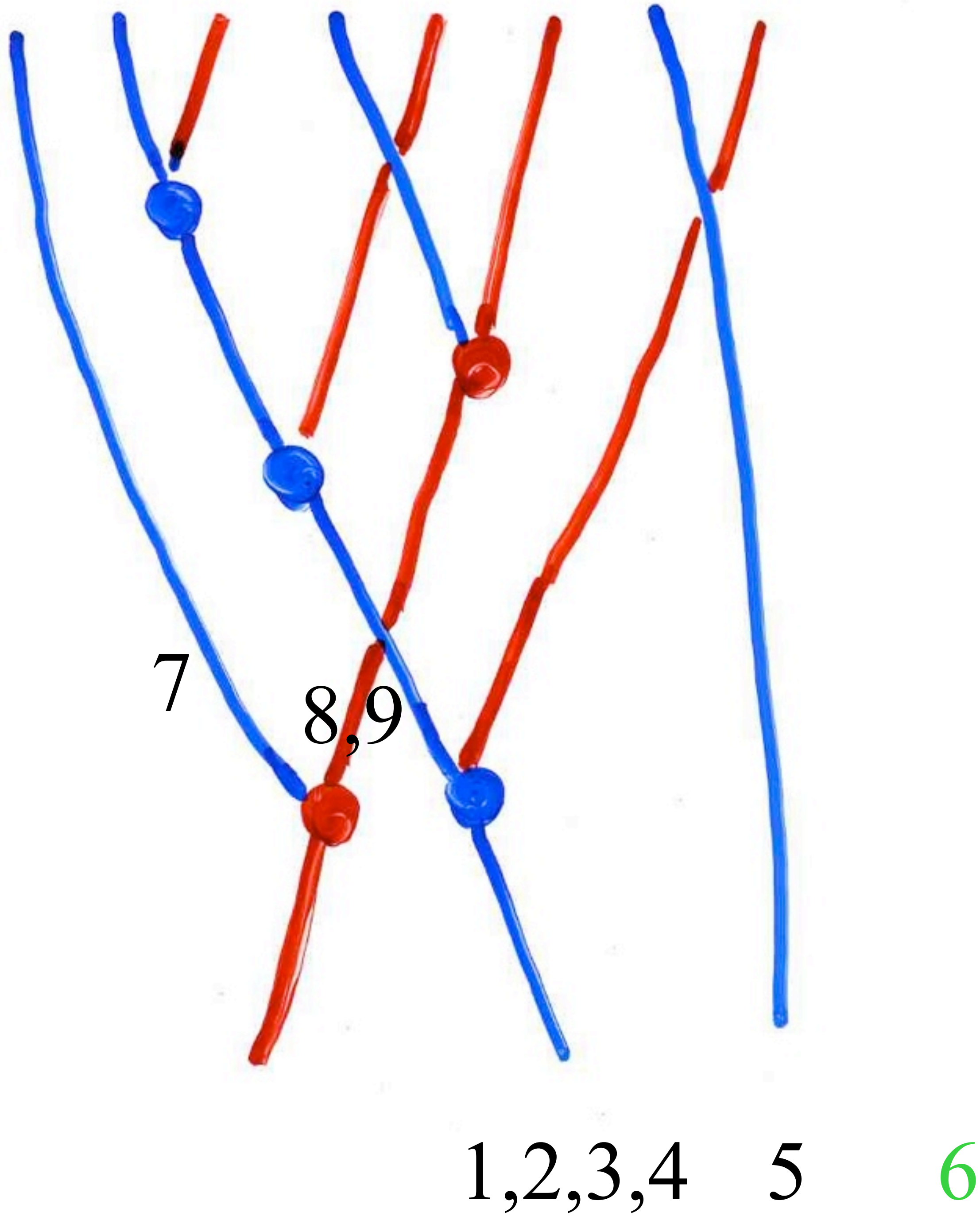


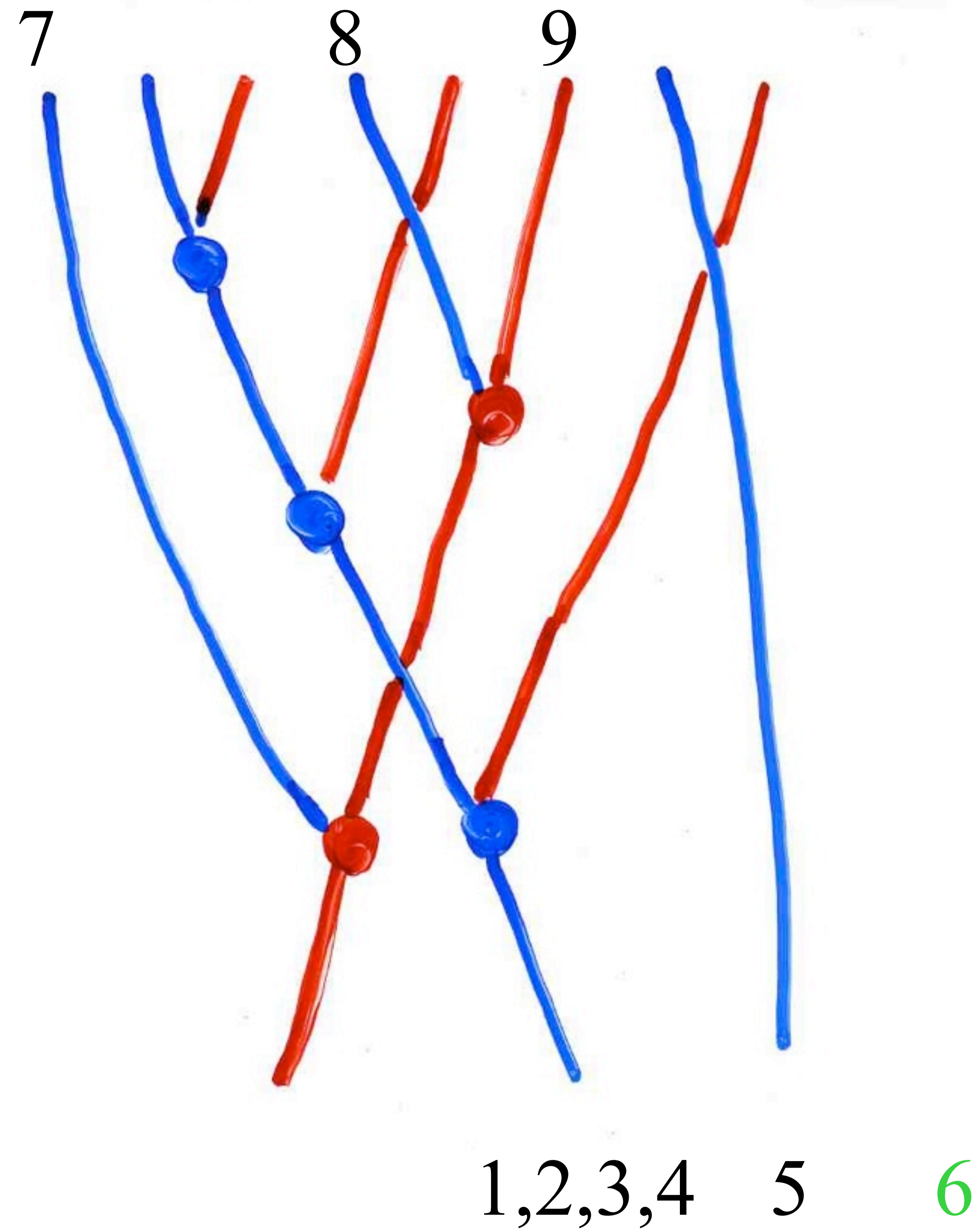


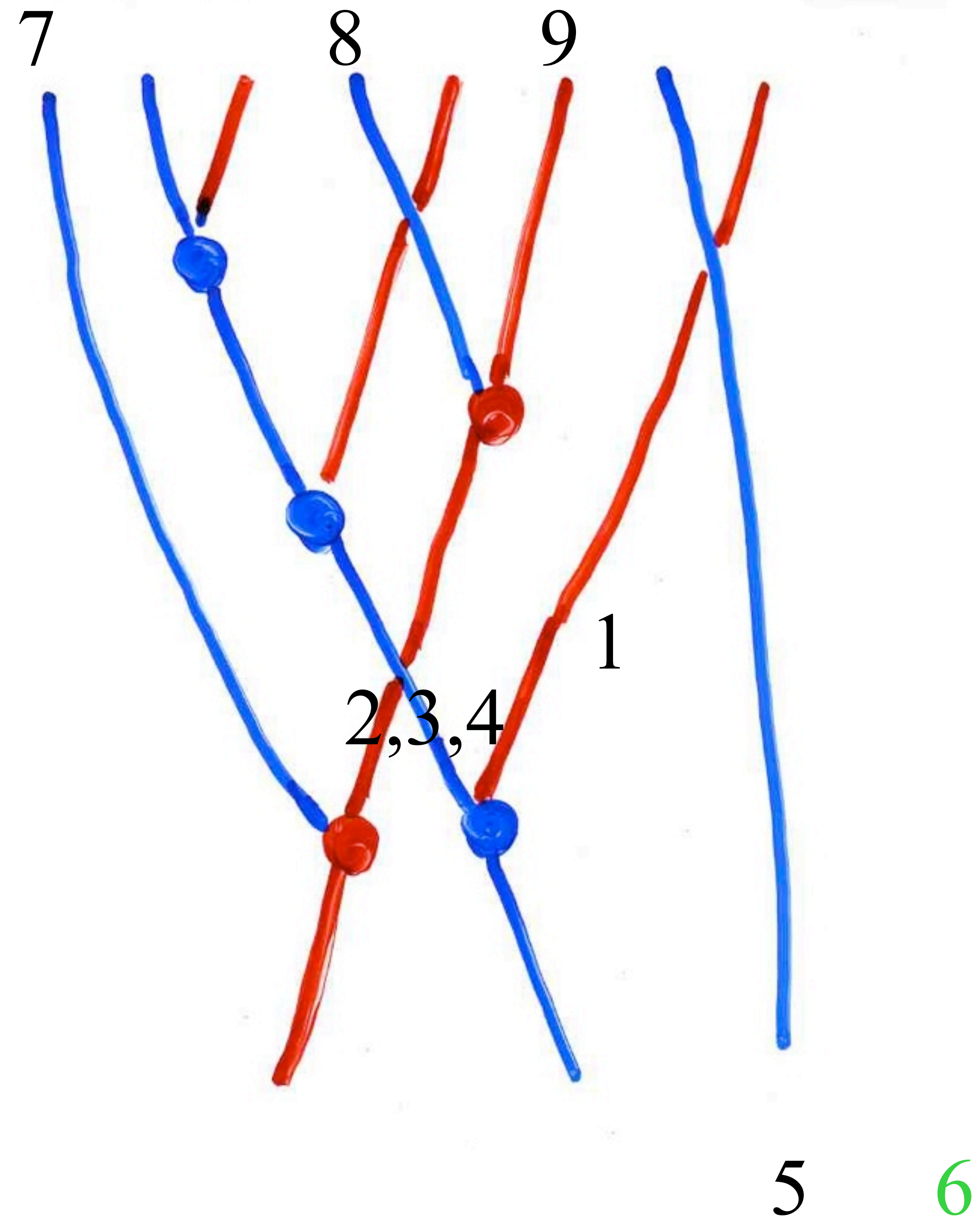




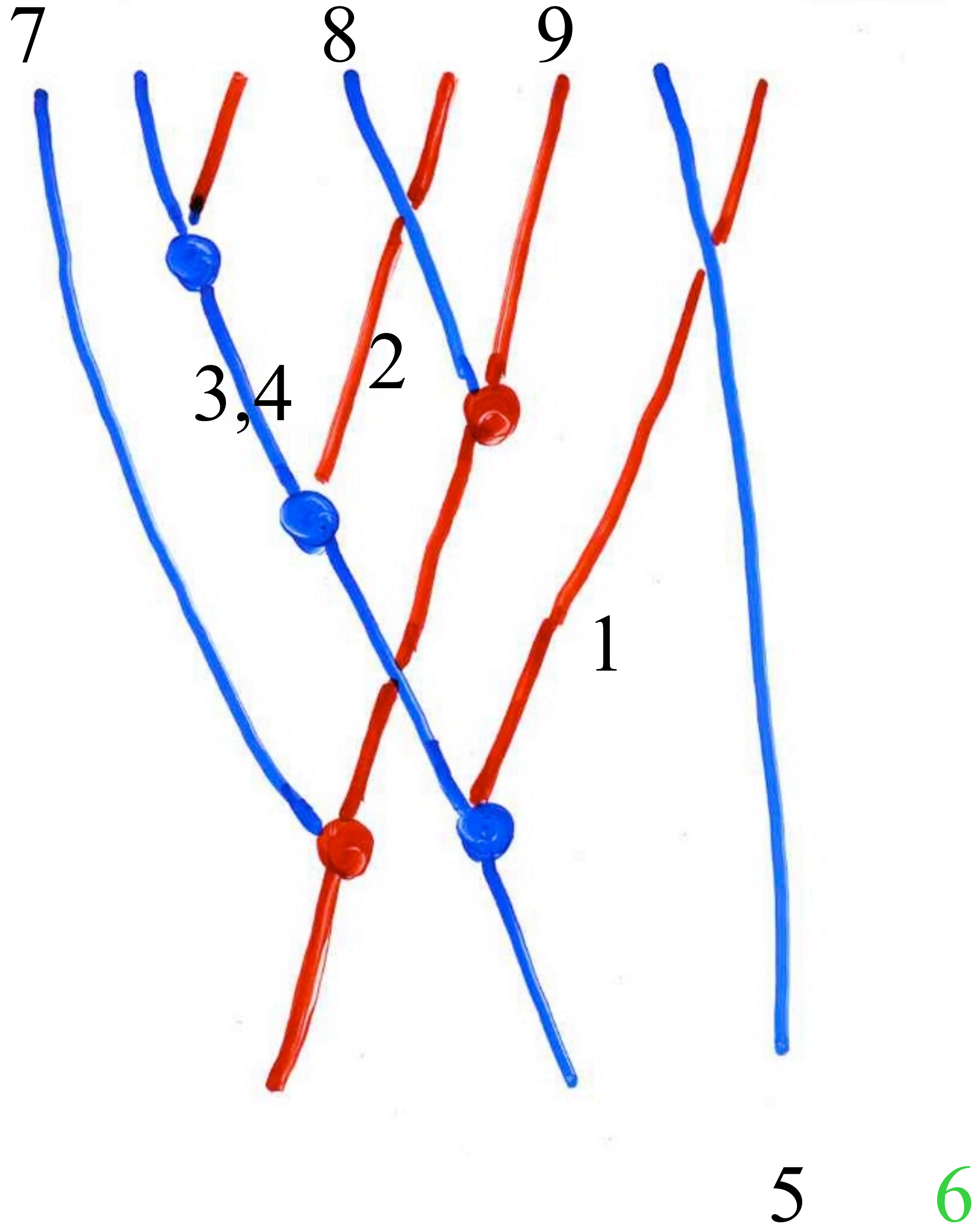
7,8,9 1,2,3,4 5 6

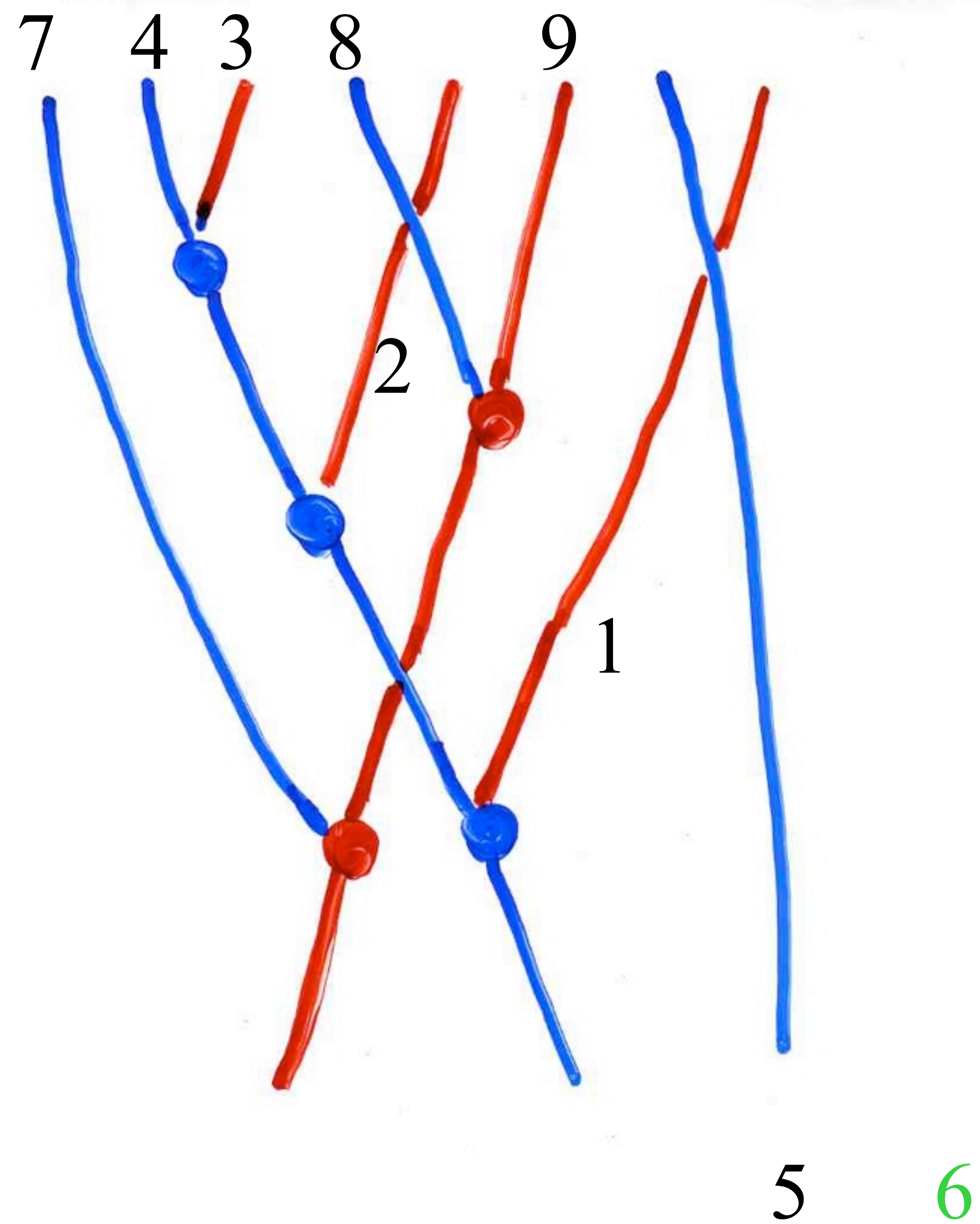

7
8,9
1,2,3,4
5
6

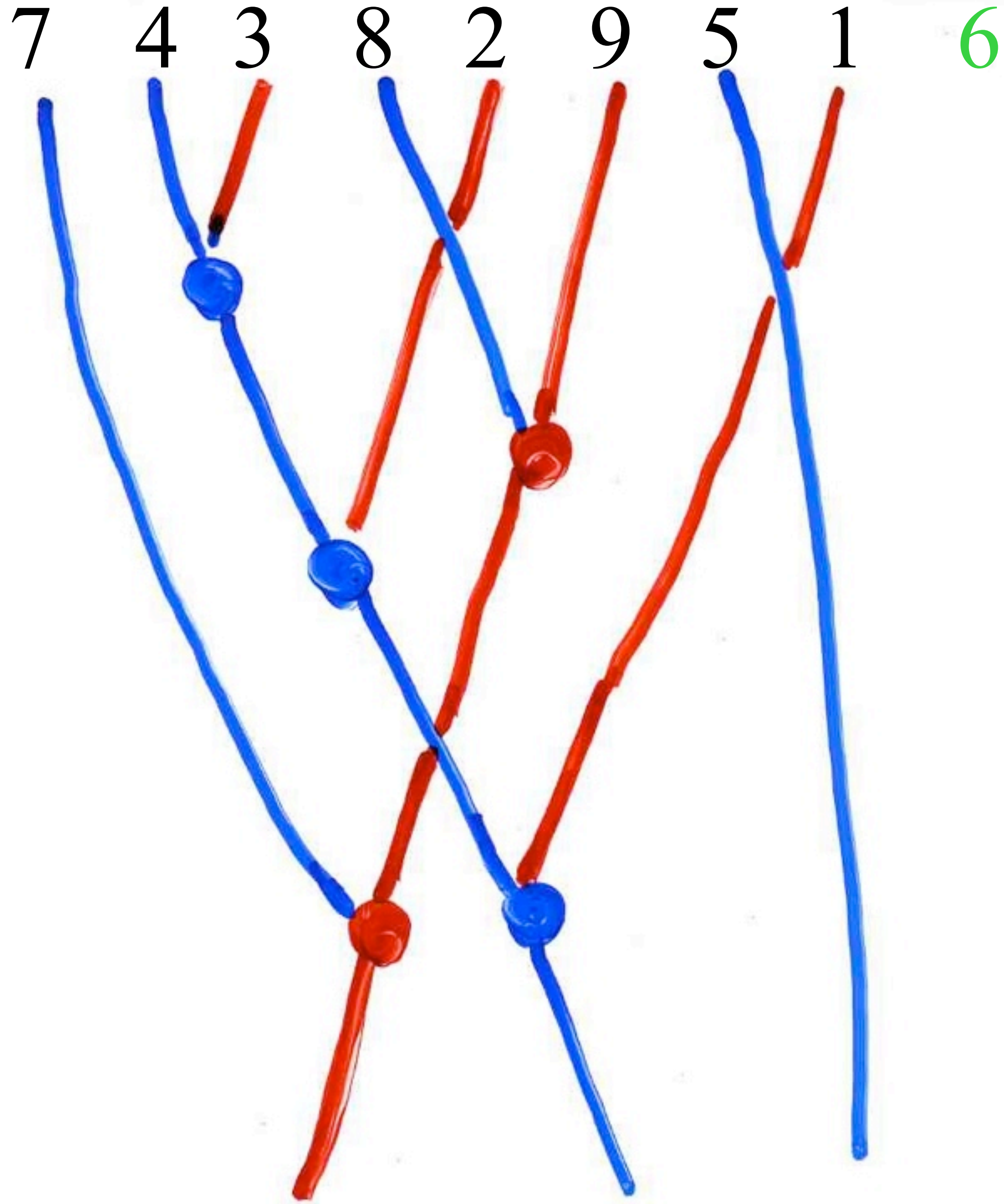

7
8
9
1,2,3,4
5
6



7
8
9
1
2,3,4
5
6


7
8
9
2
3,4
1
5
6


7
4
3
8
9
2
1
5
6



7 4 3 8 2 9 5 1 6

Genocchi sequence
of a permutation

Def. Genocchi sequence of a permutation

$$\sigma = \sigma(1) \quad \sigma(n)$$

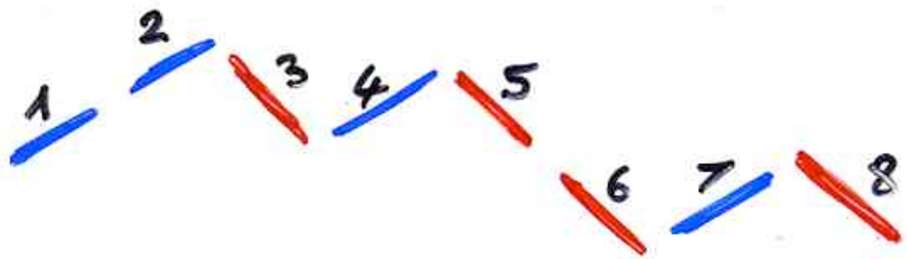
$$G(\sigma) = z_1 \ \dots \ z_{n-1}$$

$$z_x = \begin{cases} a & \text{(ascent)} \\ d & \text{(descent)} \end{cases} \qquad x = \sigma(i) \begin{array}{l} < \sigma(i+1) \\ > \sigma(i+1) \end{array}$$

$1 \leq x \leq n-1$ "value" "index"

convention : $\sigma(n+1)=0$ ($\sigma(n)$ is a descent)

ex : $\sigma = (8\ 5\ 3\ 2\ 7\ 9\ 1\ 4\ 6)$



9

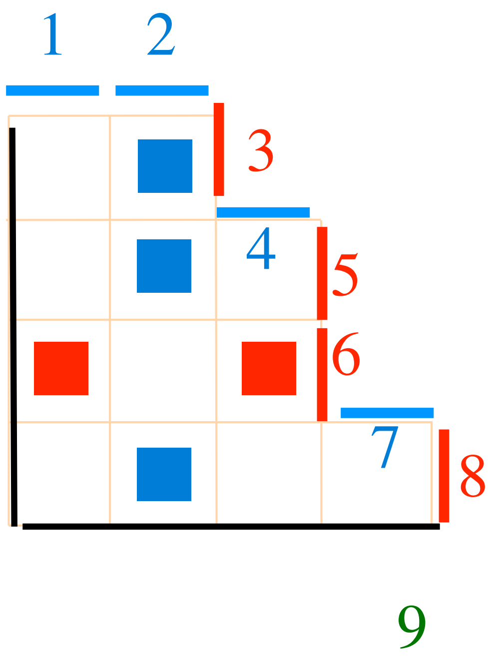
σ permutation

(valeur) x $\begin{cases} \text{avance} \\ \text{recul} \end{cases}$ ssi (indice) x $\begin{cases} \text{montée} \\ \text{descente} \end{cases}$

$\sigma^{-1}(x) < \sigma^{-1}(x+1)$

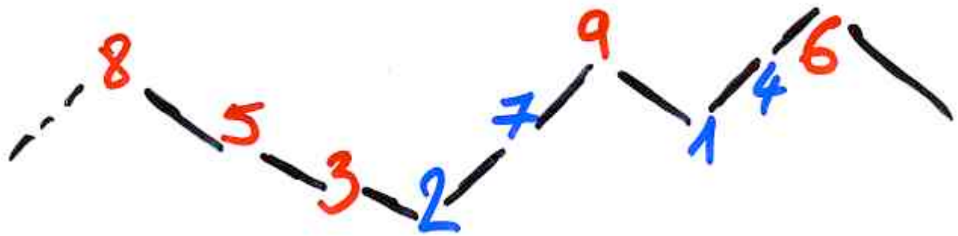
$\sigma^{-1}(x) > \sigma^{-1}(x+1)$

convention : $\sigma(n)$ descente



$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 4 & 3 & 8 & 2 & 9 & 5 & 1 & 6 \end{pmatrix}$$

$$\sigma^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 5 & 3 & 2 & 7 & 9 & 1 & 4 & 6 \end{pmatrix}$$



"Genocchi shape" of a permutation

alternating sequence $d\,a\,d\,a\,d \ldots a\,d\,a$

Prop. (Dumont, 1974)
The nb of permutations on $\{1, 2, .., 2n\}$ having an alternating Genocchi sequence is the Genocchi numbers G_{2n+2}

nombres de Genocchi

$$G_{2n} = 2(2^{2n} - 1)\, B_{2n}$$

Bernoulli

$$2^{2n} G_{2n+2} = (n+1)\, T_{2n+1}$$

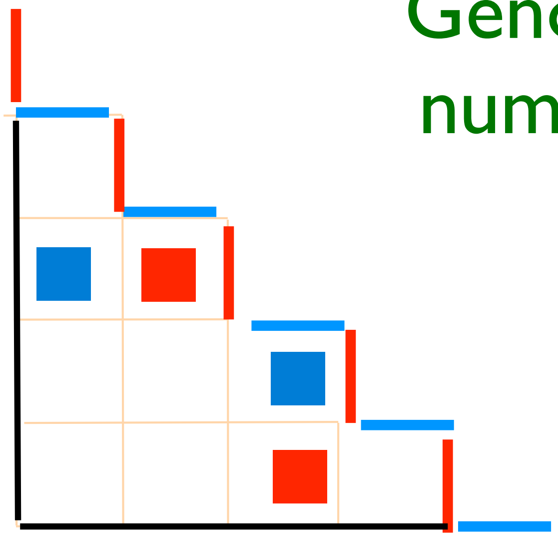
nombres de Genocchi

$$G_{2n} = 2(2^{2n} - 1) B_{2n}$$

Bernoulli

$$2^{2n} G_{2n+2} = (n+1) T_{2n+1}$$

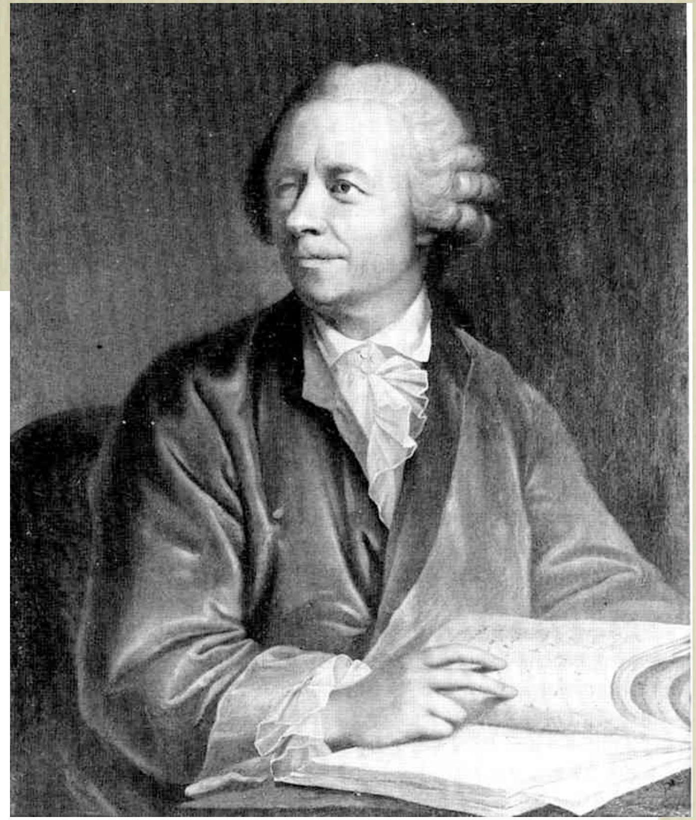
Genocchi numbers



alternating shape

Angelo Genocchi
1817 - 1889

Hinc igitur calculo inſtituto reperietur:



$$A = 1$$
$$B = 1$$
$$C = 3$$
$$D = 17$$
$$E = 155 = 5.31$$
$$F = 2073 = 691.3$$
$$G = 38227 = 7.5461 = 7.\frac{127.129}{3}.$$
$$H = 929569 = 3617.257$$
$$I = 28820619 = 43867.9.73 \quad \&c.$$

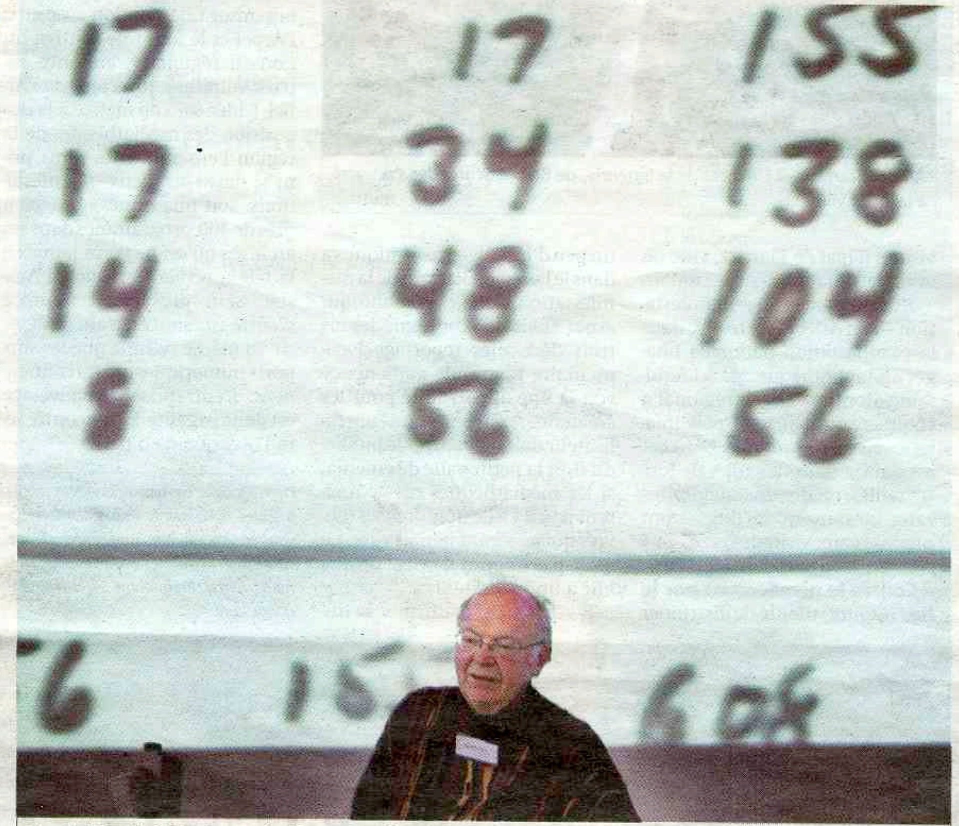
BORDEAUX 1. Le professeur Donald Knuth consacre sa vie à la programmation informatique, considérée comme un art. Il vient d'être sacré docteur honoris causa à Bordeaux

L'ermite de l'informatique

: Bernard Broustet

Une sommité de l'informatique mondiale a séjourné en Gironde ces derniers jours. Donald Knuth, 69 ans, a été sacré mardi docteur honoris causa de l'université Bordeaux 1, après avoir été lundi au centre d'une journée d'échanges qui réunissait une bonne partie du gratin français et européen de la recherche en informatique (1).

Depuis son premier contact, il y a un demi-siècle, avec un monumental et dinosaurien IBM 650, Donald Knuth n'a cessé d'être habité par la passion de l'informatique. Physicien, puis mathématicien de formation, ce géant affable et modeste a voué sa vie à ce qu'il appelle « l'art de la programmation informatique ». Car, à ses yeux, plus qu'une technique, c'est une forme d'activité qui requiert à la fois rigueur, intuition et sens esthétique. Les programmes informatiques réussis ont une sorte de beauté à laquelle même les non-spécialistes peuvent être sensibles.



Donald Knuth, à Bordeaux, le 29 octobre. À 69 ans, il animait une journée d'échanges avec le gratin européen de la recherche en informatique PHOTO LAURENT THEILLET

Une encyclopédie. Au long de sa carrière académique (pour l'essentiel à l'université californienne de Stanford), Donald Knuth a fait preuve d'une grande fécondité, en jouant notamment un rôle essentiel dans le développement de langages toujours utilisés par la communauté des mathématiciens. Mais, à 55 ans, le professeur Knuth a décidé de prendre sa retraite de Stanford. Il trouve que les fonctions administratives sont trop absorbantes pour lui permettre de mener à bien l'œuvre entamée à la fin des années 60 sous le titre de « Art of computer programming », sorte d'encyclopédie de l'algorithmique et de la programmation informatique. Donald Knuth a publié, il y a quelque temps déjà, les trois premiers volumes de cette gigantesque somme, traduite en russe, en japonais, en polonais, etc. mais pas en français. Le quatrième tome est pour bientôt. Et Donald Knuth se dit décidé à poursuivre sa tâche tant qu'il en aura la force. Ses ouvrages, dont les ventes cumulées au fil des ans approchent le million d'exemplaires, visent essentiellement les informaticiens et créateurs de programmes. Une communauté certes minoritaire à travers le monde, mais qui se trouve investie d'une mission considérable. En quelques décennies, l'écriture informatique a aidé à résoudre d'innombrables problèmes. « Mais il y en a tant d'autres qui attendent des solutions, notamment dans le domaine médical », affirme le professeur émérite de Stanford.

Un chèque de 2,56 dollars. Pour mener à bien sa tâche, Donald Knuth s'est imposé une vie d'ermite. D'ordinaire, sa journée débute par la bibliothèque ou la piscine. Après quoi, il passe tout le reste de son temps à sa table de travail, dimanche compris. Il n'a plus d'e-mail depuis le début des années 90, considérant que le courrier électronique représente une perte de temps, dès lors qu'on veut aller au fond des choses et non pas rester à leur surface. Une secrétaire lui fait passer les messages considérés comme les plus urgents. Pour le reste, Donald Knuth demande qu'on lui écrive par courrier ordinaire ou par fax, dont il prend parfois connaissance avec des mois de retard. Il s'oblige, en revanche, à tenir aussi scrupuleusement que possible sa promesse d'envoyer un chèque de 2,56 dollars à tout lecteur ayant détecté une erreur dans un de ses livres. Par ailleurs, pour se détendre, il pratique l'orgue, appris dans sa prime jeunesse auprès de son père qui partagea sa vie entre la musique et l'enseignement.

L'orgue de Sainte-Croix. Donald Knuth n'est pas fermé aux choses de ce monde. Sur son site Internet, à la rubrique « Questions qui ne me sont pas fréquemment posées », il demande entre autres : « Pourquoi mon pays a-t-il le droit d'occuper l'Irak ? », « Pourquoi mon pays ne soutient-il pas une Cour internationale de justice ? » Mais cet homme de conscience ne se veut pas militant, pas plus qu'il n'aspire au vedettariat et à la richesse. « Beaucoup de gens, dit-il, ont tendance à considérer que l'informatique, c'est surtout des histoires de business, d'entreprise. Ce n'est pas mon cas. » Sortant de sa semi-réclusion, Donald Knuth s'est donc laissé convaincre d'accepter les hommages de l'université de Bordeaux, après celles de Harvard, d'Oxford, de Tübingen. Il a eu le coup de foudre pour la beauté et l'agrément de la ville. Et il n'oubliera sans doute pas de sitôt l'orgue illustre de l'église Sainte-Croix (2), sur lequel il a eu le bonheur d'exercer son talent.

(1) Ces journées étaient organisées par le Laboratoire bordelais de recherche en informatique (Labri).

(2) Thierry Semenoux, professeur d'orgue au conservatoire de Bordeaux, a joué dans ce domaine un rôle de cicérone auprès de Donald Knuth.

some parameters

The maximum letter of the blocks of letters reaching the ground level are:

- for the columns of T (red threads), the left-to-right maximum elements of the values of the permutation s less than the last letter $s(n+1)$,
- for the rows of T (blue threads), the right-to-left maximum elements of the values of the permutation s bigger than the last letter

(3 proofs comming 3 different methodologies: by P. Nadeau , O.Bernardi and xgv)

This gives an interpretation of the two parameters on alternative tableaux:
- number of “open” columns (i.e. columns without a red cell)
- number of “open” rows (i.e. rows without a blue cell)

total order

$\{1, 2, \ldots, n\}$

$\sigma = 7\ 2\ 3\ 9\ 6\ 8\ 5\ 1\ 4$

word

left-to-right

right-to-left

minimum elements

$\sigma = 7\ 2\ 3\ 9\ 6\ 8\ 5\ \boxed{1}\ \boxed{4}$

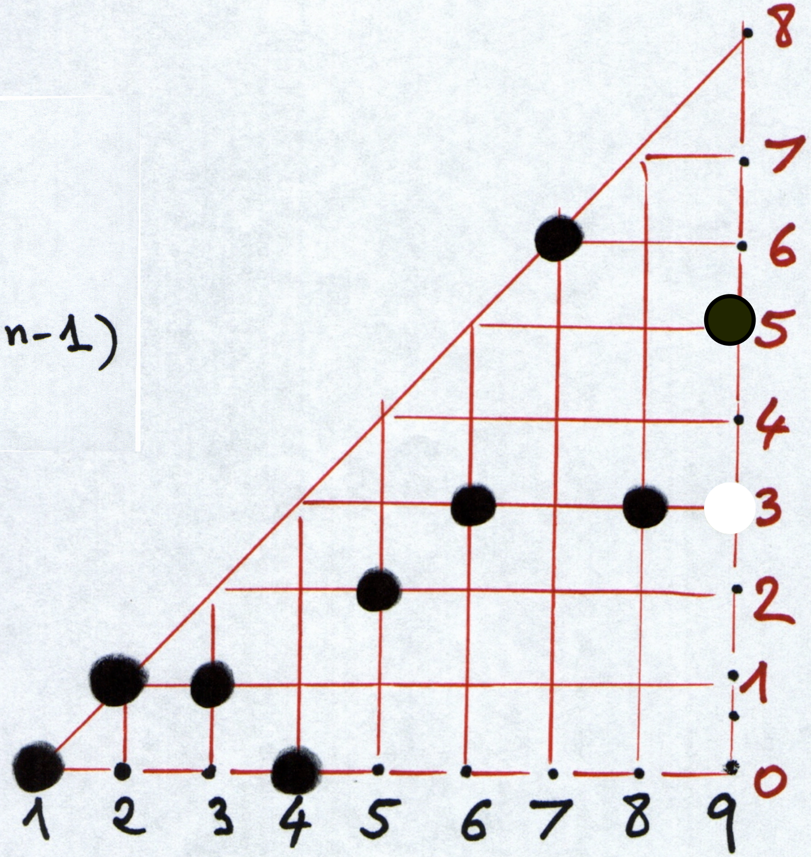
left-to-right
right-to-left minimum elements

$$\sigma = 7\ 2\ 3\ 9\ 6\ 8\ 5\ \boxed{1}\ \boxed{4}$$

Stirling numbers $s_{n,k}$

$$\sum_{k \geq 1} s_{n,k} x^k = x(x+1) \dots (x+n-1)$$

$$xy(x+y)(x+1+y)\dots(x+n-2+y)$$

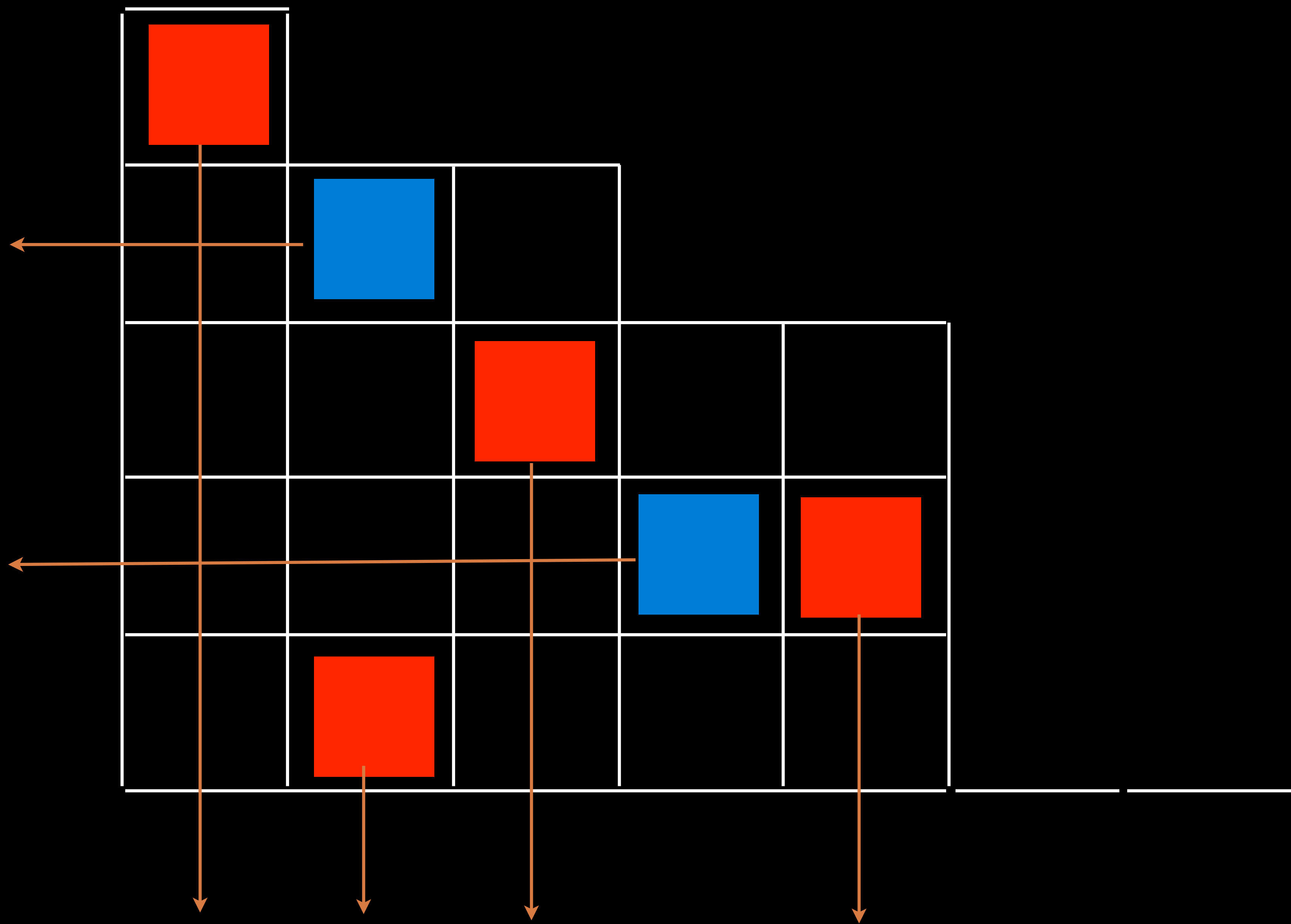


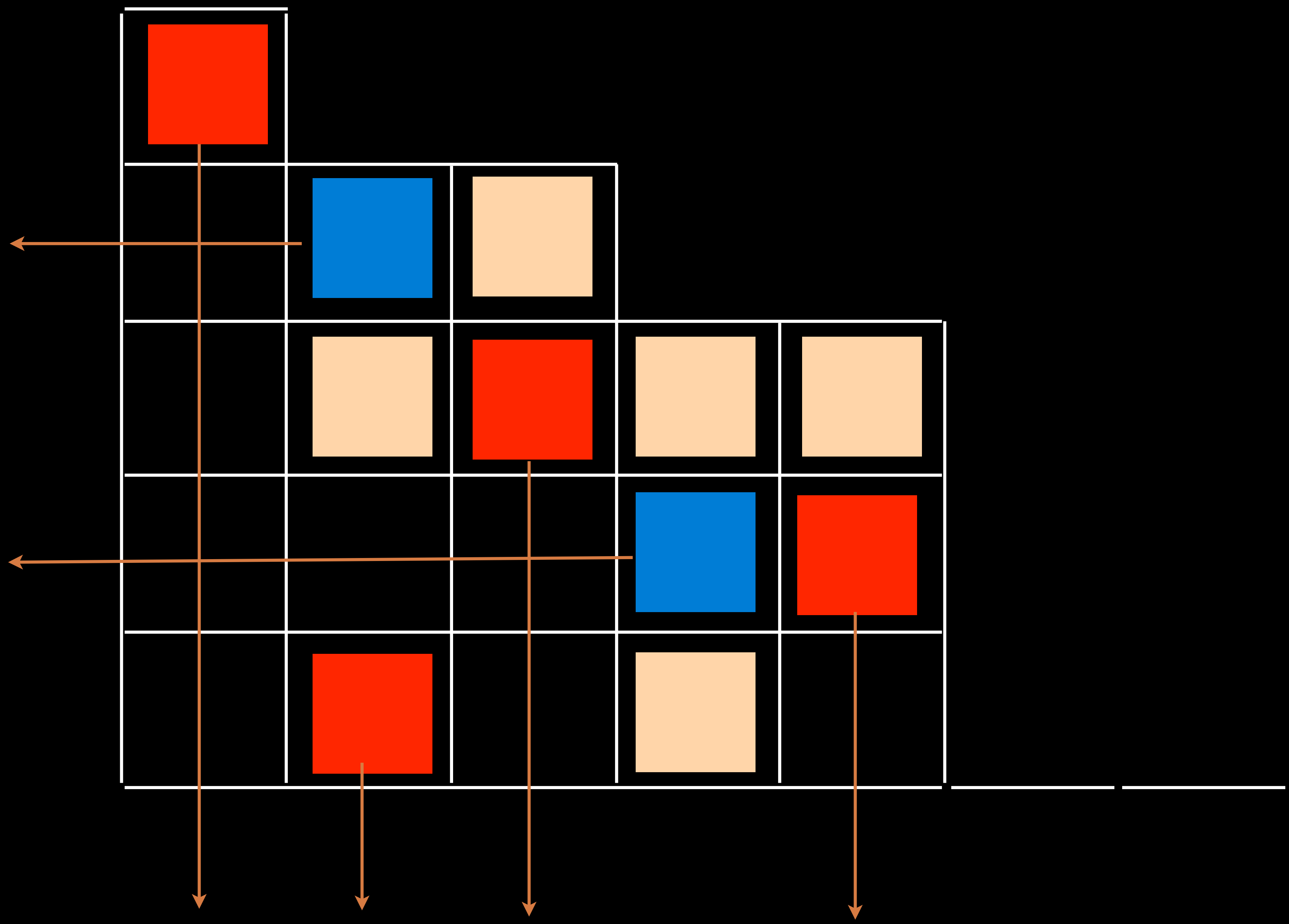
Def - Crossing of an alternative tableau:
a non-colored cell which is
neither at the left of a blue cell
neither below a red cell

crossing of alternating tableau

$\updownarrow$

exchange in the "exchange-fusion" algorithm





The number of crossings of the alternative tableau has been be characterized by O.Bernardi on the corresponding permutation s.

It is the number of pairs (x,y), $x=s(i)$, $y=s(j), 1\le i<j\le n+1$, such that there exist two integers $k, l\ge 0$ such that: the set of the values $x+1, x+2, \ldots, x+k$, $y+1, \ldots, y+l$ are located between x and y (in the word s), and $x+k+1$ is located (in s) at the right of y and $y+l+1$ is located (in s) at the left of x (with the convention of $n+2$ at the left of all the values).

Permutations

with no subsequence of the type

$$\cdots\ (y+1)\ \cdots\ x\ \cdots\ y\ \cdots\ (x+1)\ \cdots$$

ex: $\sigma = 6\ 4\ 5\ 3\ 9\ 7\ 8\ (10)\ 1\ 2$

Prop. (O. Bernardi, 2008)
The number of such permutations on n elements is C_n Catalan number

From work of Corteel, Nadeau, Steingrímsson, Williams

we know that parameter "number of crossing" in alternating tableaux:

same distribution as "q-analog of Laguerre histories"

Laguerre histories

Combinatorial theory
of orthogonal polynomials

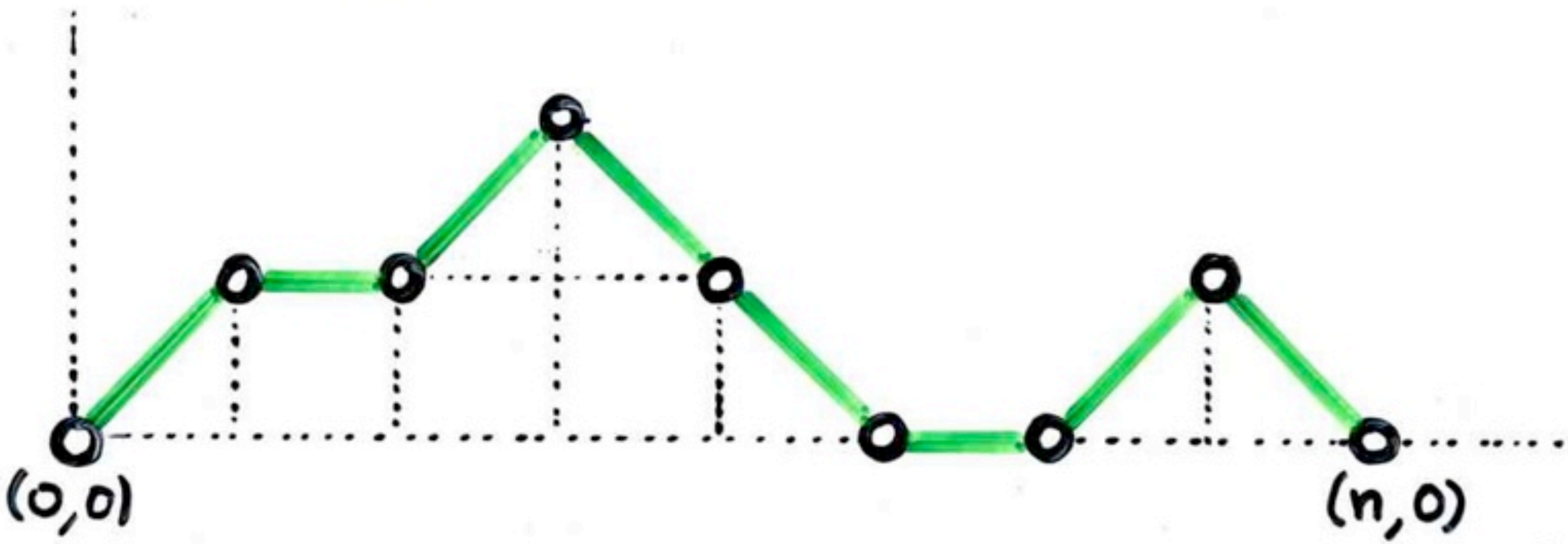
Bijection Permutations $n+1$

Histoires de Laguerre $(\gamma_c ; f)$ n

Bijection **Permutations** $n+1$

Histoires de **Laguerre** $(\gamma_c \,;\, f)$ n

Chemin de **Motzkin** $n\cancel{+1}$

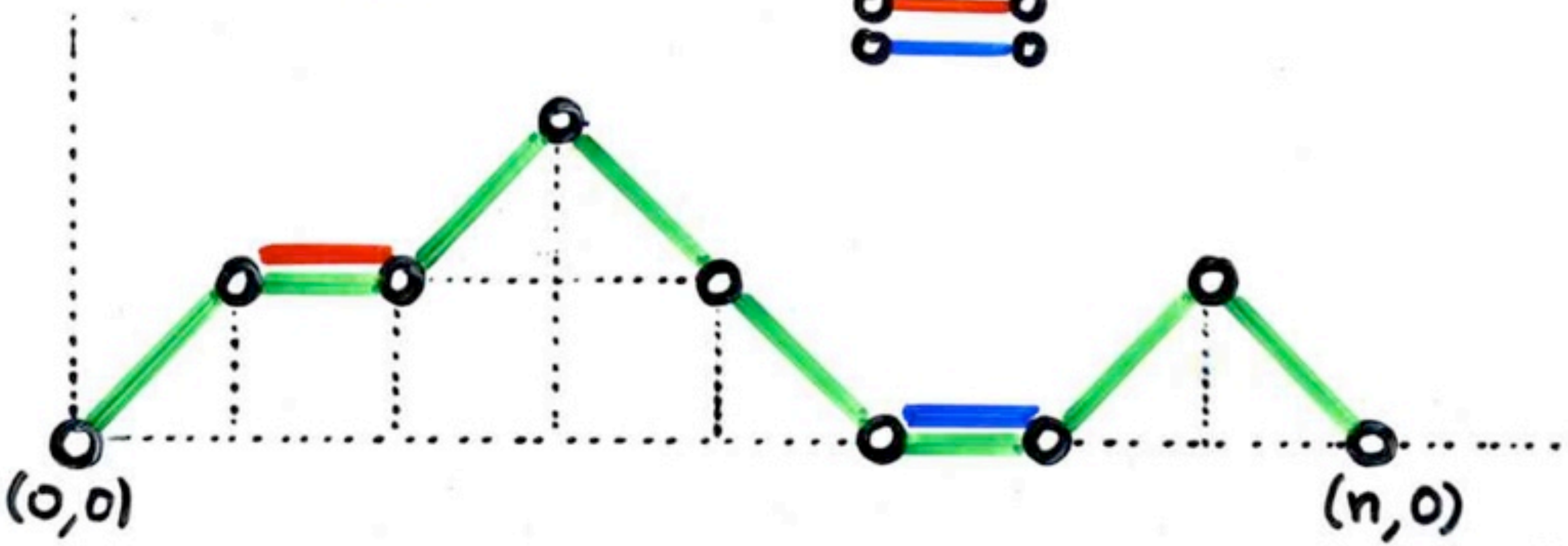


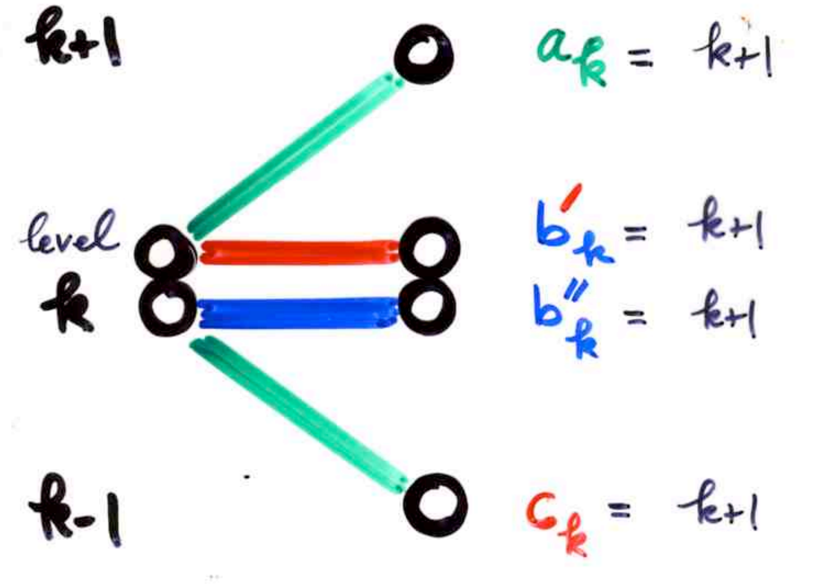
Bijection **Permutations** $n+1$

Histoires de Laguerre $(\gamma_c \,;\, f)$ n

Chemin de Motzkin $n-1$

2 couleurs paliers





Permutations $\quad n+1$

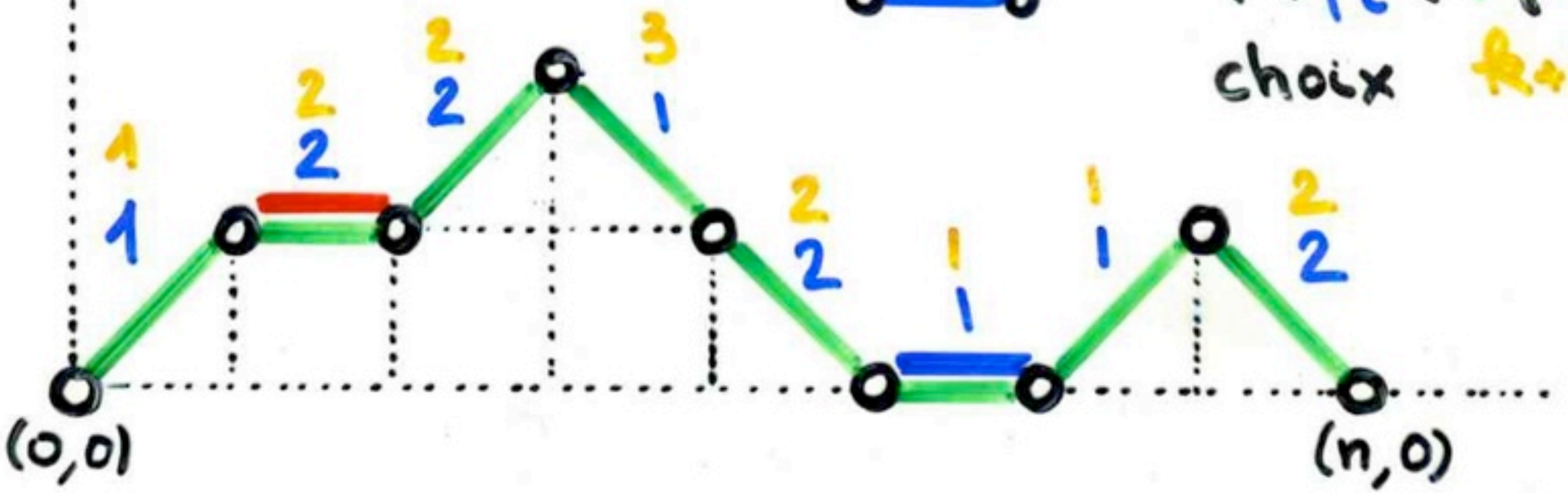
$(\gamma_c \, , \, f) \quad n$

2 couleurs
paliers

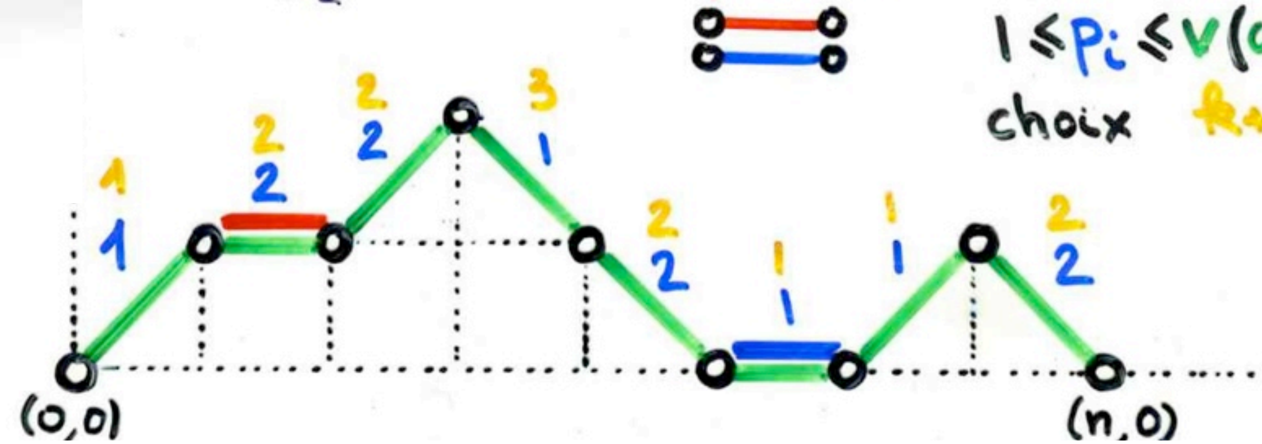
$f = (p_1, \dots, p_n)$

$1 \leq p_i \leq v(\omega_i)$

choix $k+1$



$h = (\omega_c \,;\, (p_1, \dots, p_n))$



$1 \leq p_i \leq v(\omega_i)$

choix $k+1$

x	ω_c	pos	v
1	↗	1	1
2	—	2	2
3	↗	2	2
4	↘	1	3
5	↘	2	2
6	—	1	1
7	↗	1	1
$n=$ 8	↘	2	2
9	•		

␣

␣ 1 ␣

␣ 1 ␣ 2

␣ 1 ␣ 3 ␣ 2

4 1 ␣ 3 ␣ 2

4 1 ␣ 3 5 2

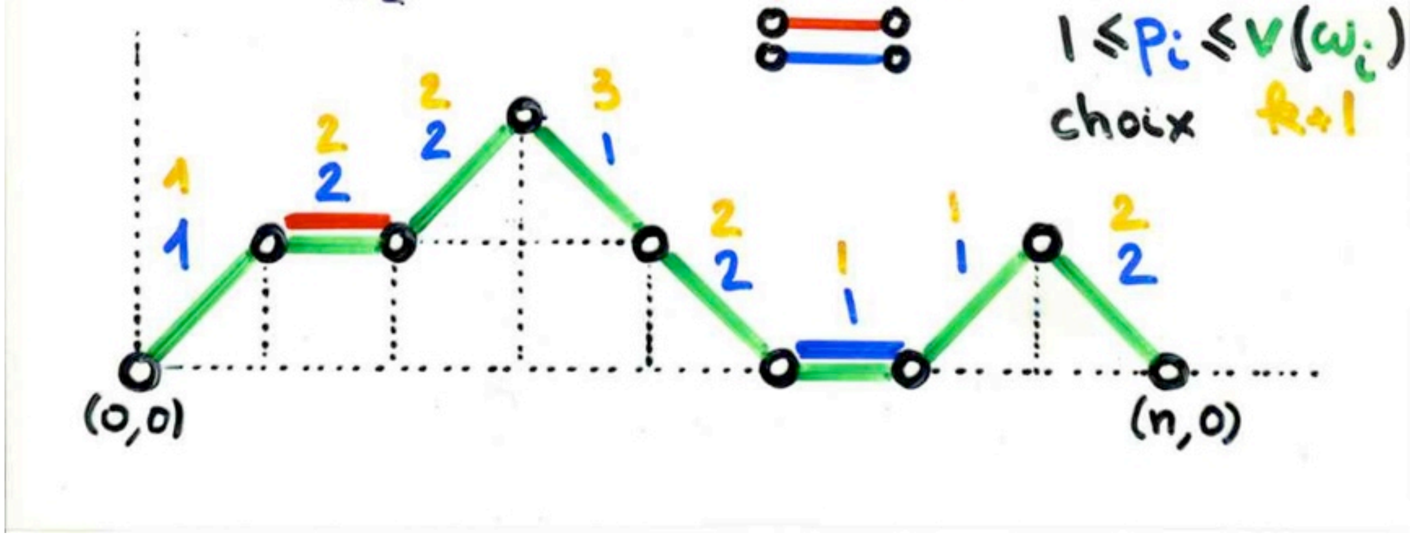
4 1 6 ␣ 3 5 2

4 1 6 ␣ 7 ␣ 3 5 2

4 1 6 ␣ 7 8 3 5 2

4 1 6 9 7 8 3 5 2 $= \sigma \in \mathfrak{S}_{n+1}$

“q-analogue”
of Laguerre
histories



choices function

1	2	3	4	5	6	7	8
1	2	2	1	2	1	1	2
0	1	1	0	1	0	0	1

q-Laguerre : q^4

␣
␣ 1 ␣
␣ 1 ␣ 2
␣ 1 ␣ 3 ␣ 2
4 1 ␣ 3 ␣ 2
4 1 ␣ 3 5 2
4 1 6 ␣ 3 5 2
4 1 6 ␣ 7 ␣ 3 5 2
4 1 6 ␣ 7 8 3 5 2
4 1 6 9 7 8 3 5 2 $= \sigma$
$\in \mathfrak{S}_{n+1}$

Lemme $\underset{\pi\cdot\theta}{}$ $h = (\omega_c \,;\, (p_1, \dots, p_n)) \in \mathcal{L}_n$ Laguerre history

$\downarrow$

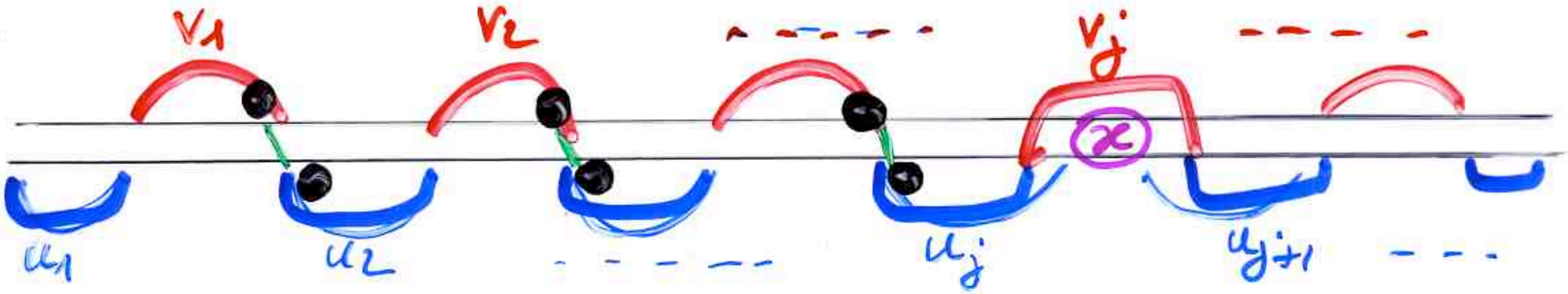
permutation $\sigma \in \mathfrak{S}_{n+1}$

$p_x = j$ est aussi :

$j = 1 +$ nb de triplets (a, b, x) ayant le "motif" $(31\text{-}2)$ c.à.d :

$a = \sigma(i)$, $b = \sigma(i+1)$, $x = \sigma(\ell)$

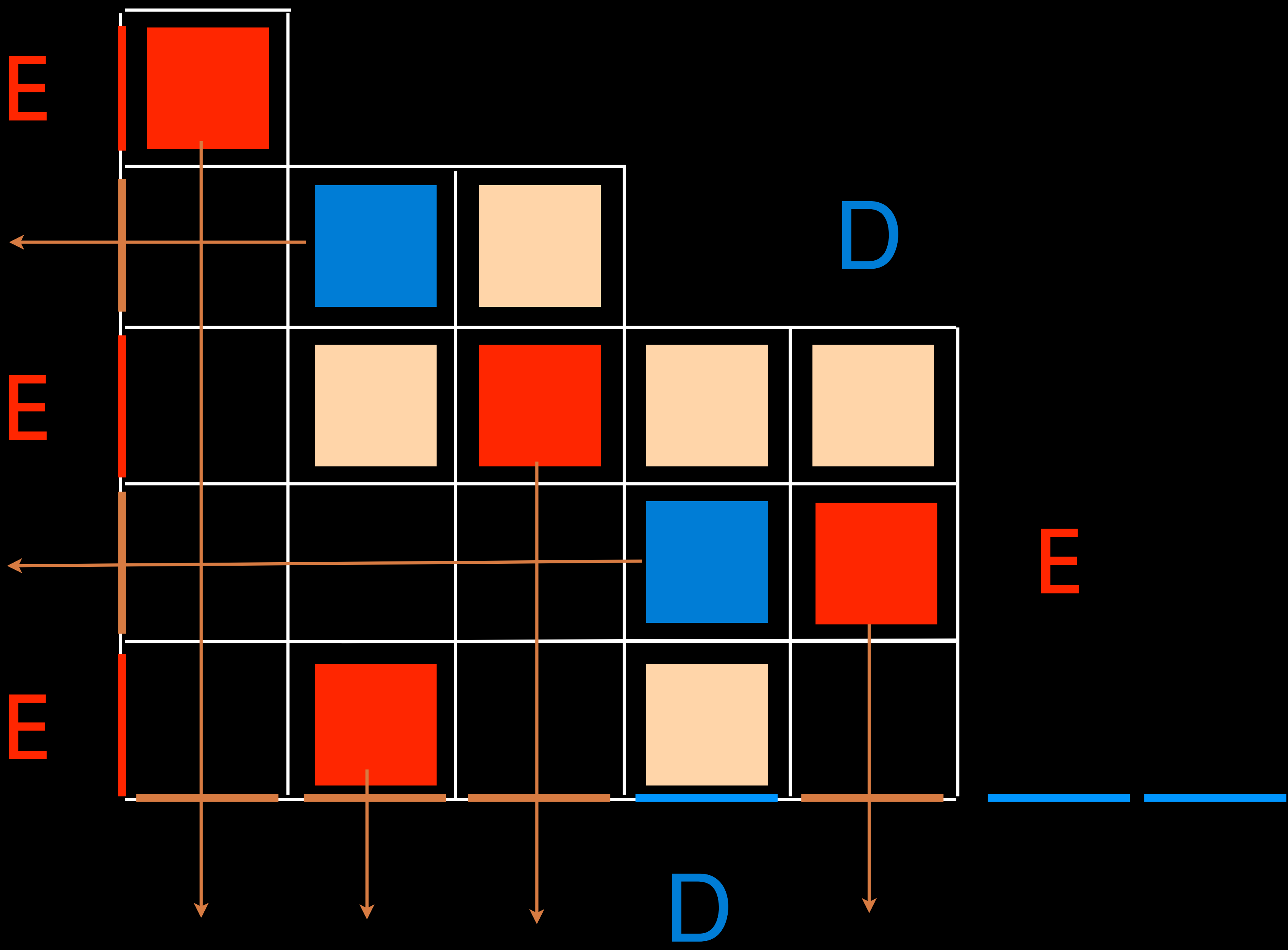
$i < i+1 < \ell$ $\qquad b < x < a$



"q-analogue" of Laguerre histories

The cellular Ansatz

From algebra $DE = ED + D + E$
to bijections


E
D
E
E
E
D

representation
of the
operators
E and D

$$DE = ED + E + D$$

V vector space generated by $\mathcal{B}$ basis

$\mathcal{B}$ alternating words two letters $\{\circ, \bullet\}$
(no occurrence of $\circ\circ$ or $\bullet\bullet$)

4 operators A, S, J, K

4 operators A, S, J, K, $\qquad u \in B$

$$\langle u | A = \sum_{\substack{\text{letter } \circ \\ \text{of } u}} v \quad , \quad v \text{ obtained by : } \circ \rightarrow \circ\bullet\circ$$

$$\langle u | S = \sum_{\circ \text{ of } u} v \qquad v \text{ obtained by : } \circ \rightarrow \bullet$$

$$(\text{and } \bullet\bullet \rightarrow \bullet \quad \bullet\bullet\bullet \rightarrow \bullet)$$

$$\langle u | J = \sum_{\circ \text{ of } u} v \qquad v, \quad \circ \rightarrow \bullet\circ \quad (\text{and } \bullet\bullet \rightarrow \bullet)$$

$$\langle u | K = \sum_{\substack{\circ \\ \text{of } u}} v \qquad v, \quad \circ \rightarrow \circ\bullet \quad (\text{and } \bullet\bullet \rightarrow \bullet)$$

$$\circ\bullet\circ\bullet \,|\, S \;=\; \bullet\circ\bullet \;+\; \circ\bullet$$

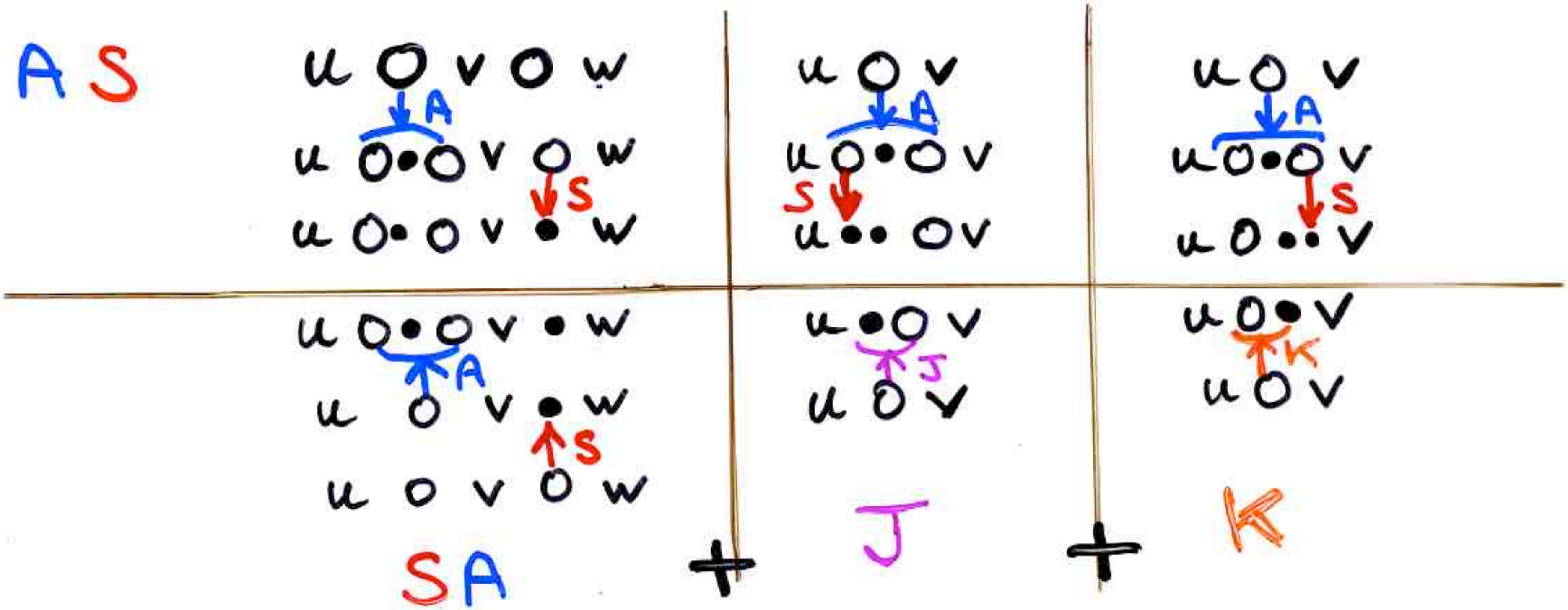
Lemma.

$$AS = SA + J + K$$

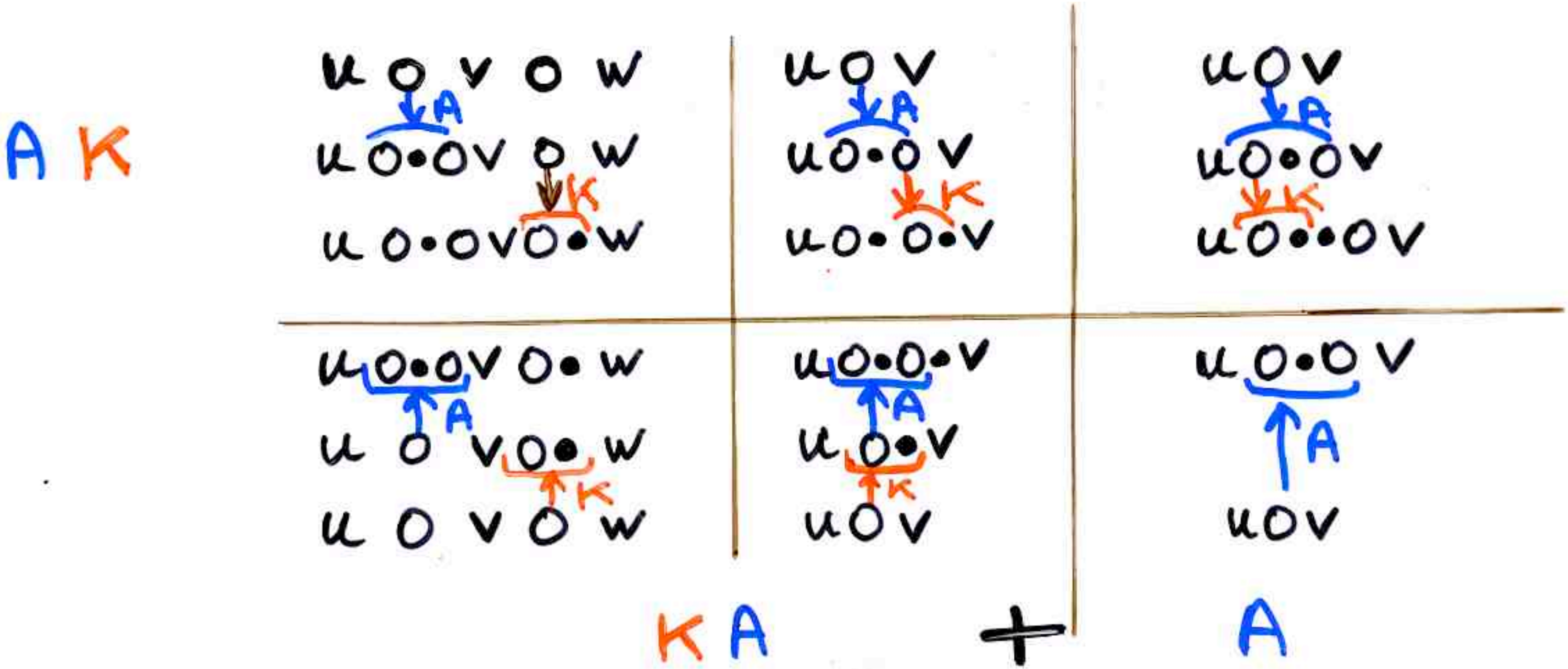
$$AK = KA + A$$

$$JS = SJ + S$$

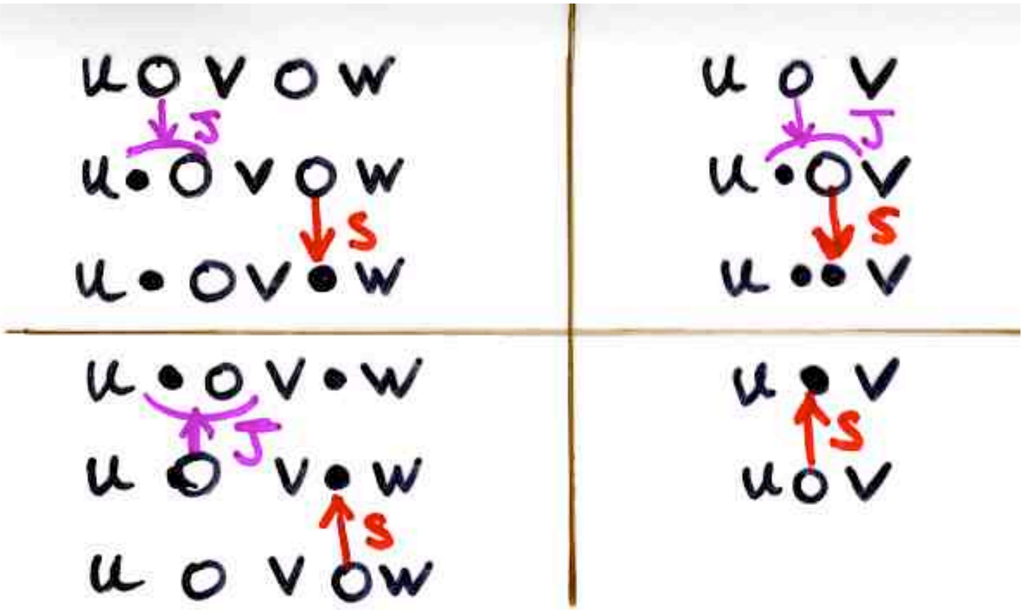
$$JK = KJ$$



A S
SA + J + K

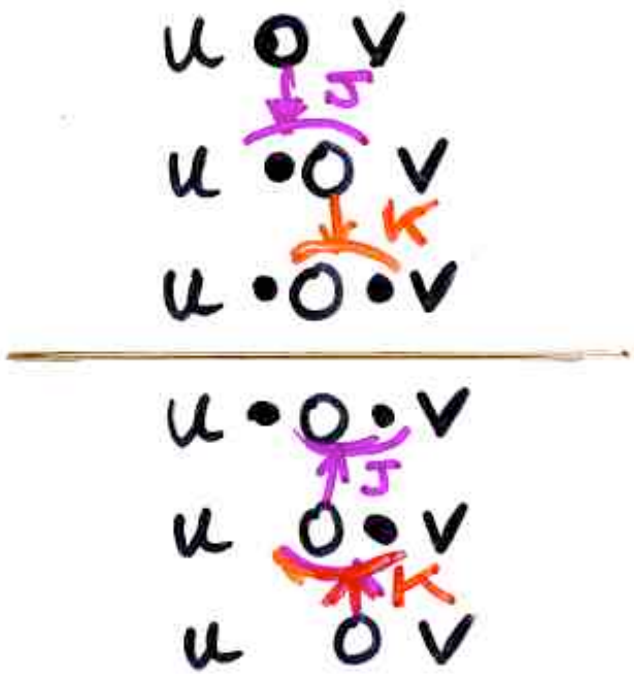

A K
K A
+
A

$J\,S$



$S\,J \;+\; S$

$J\,K$



$K\,J$

Lemma.

$$AS = SA + J + K$$

$$AK = KA + A$$

$$JS = SJ + S$$

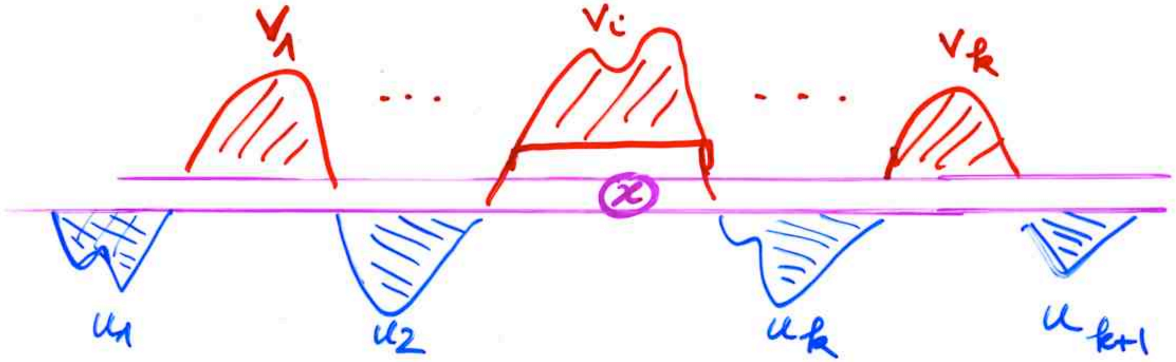
$$JK = KJ$$

$$D = A + J$$

$$E = S + K$$

$$D = A + J$$
$$E = S + K$$

$$
\begin{aligned}
DE &= (A+J)(S+K) \\
&= AS + AK + JS + JK \\
&= \underbrace{(SA + KA + SJ + KJ)}_{\underbrace{(S+K)(A+J)}_{ED}} + \underbrace{J + K + A + S}_{E + D}
\end{aligned}
$$



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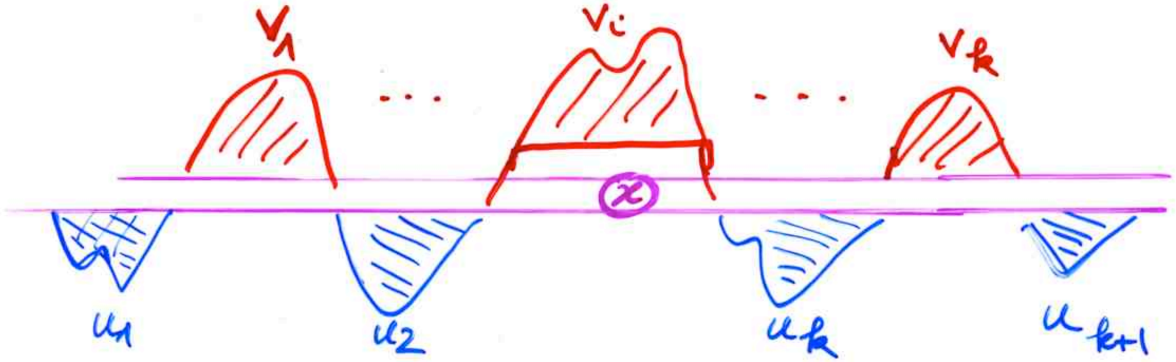
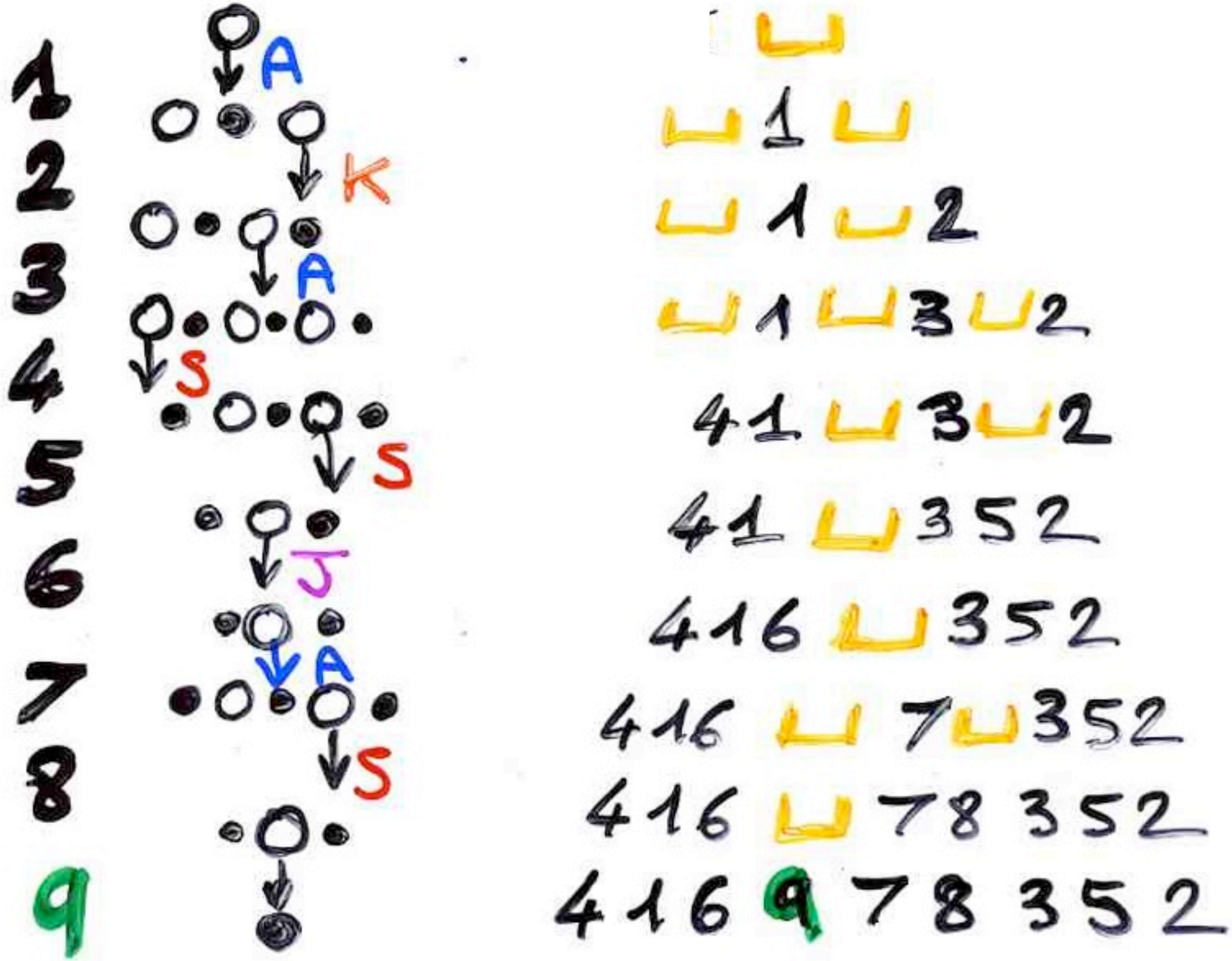
1	␣ 1 ␣
2	␣ 1 ␣ 2
3	␣ 1 ␣ 3 ␣ 2
4	4 1 ␣ 3 ␣ 2
5	4 1 ␣ 3 5 2
6	4 1 6 ␣ 3 5 2
7	4 1 6 ␣ 7 ␣ 3 5 2
8	4 1 6 ␣ 7 8 3 5 2
9	4 1 6 9 7 8 3 5 2


v_1
v_i
v_k
x
u_1
u_2
u_k
u_{k+1}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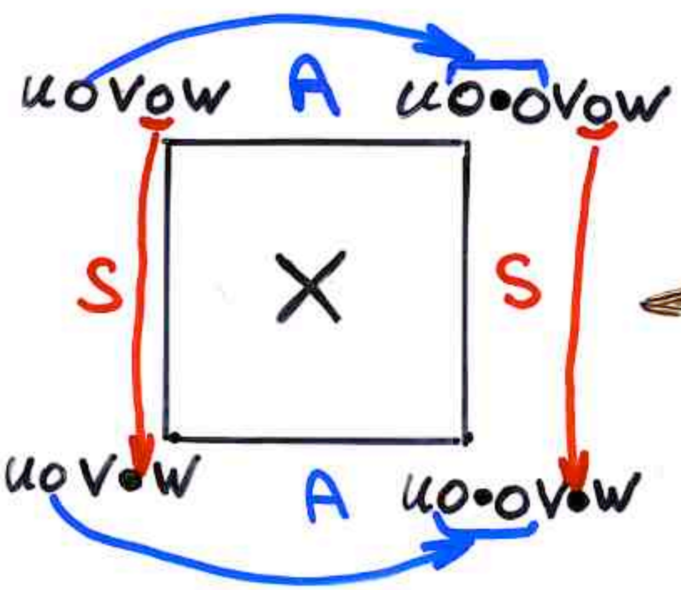
4 1 6 9 7 8 3 5 2

Cellular Ansatz

bijection

permutations --- alternative tableaux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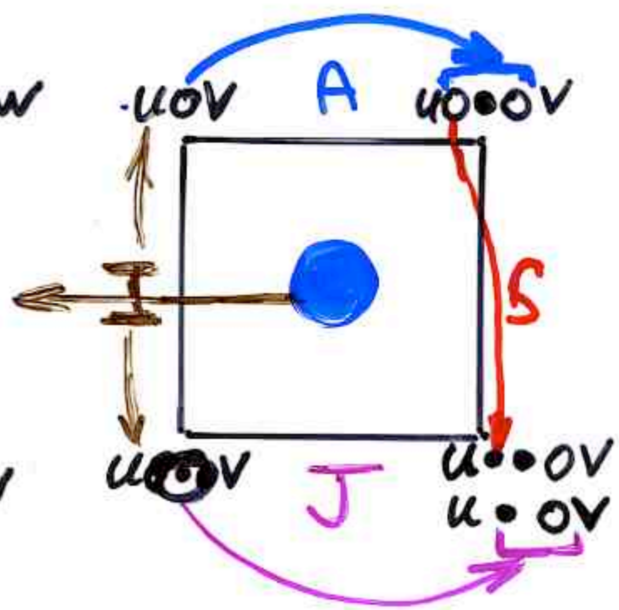
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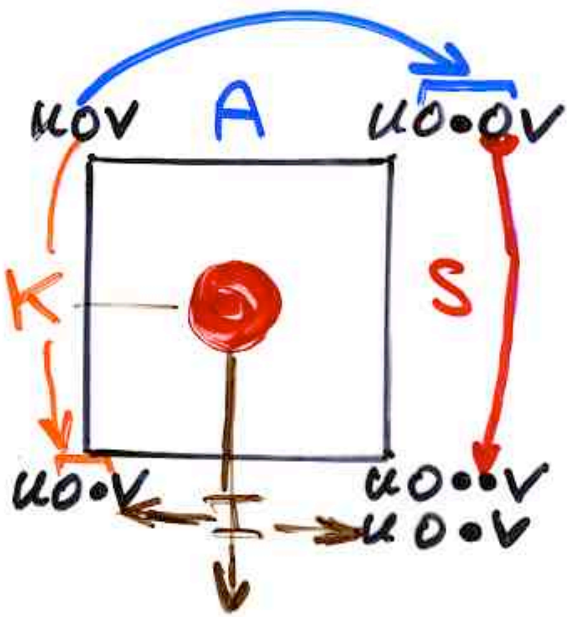
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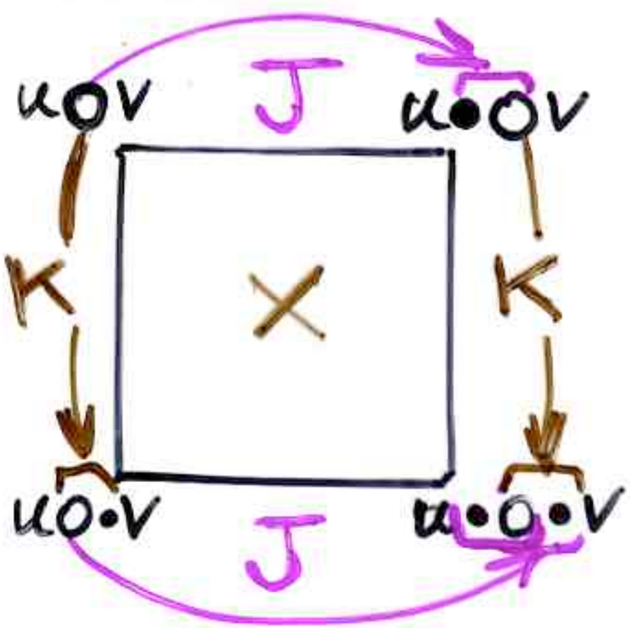
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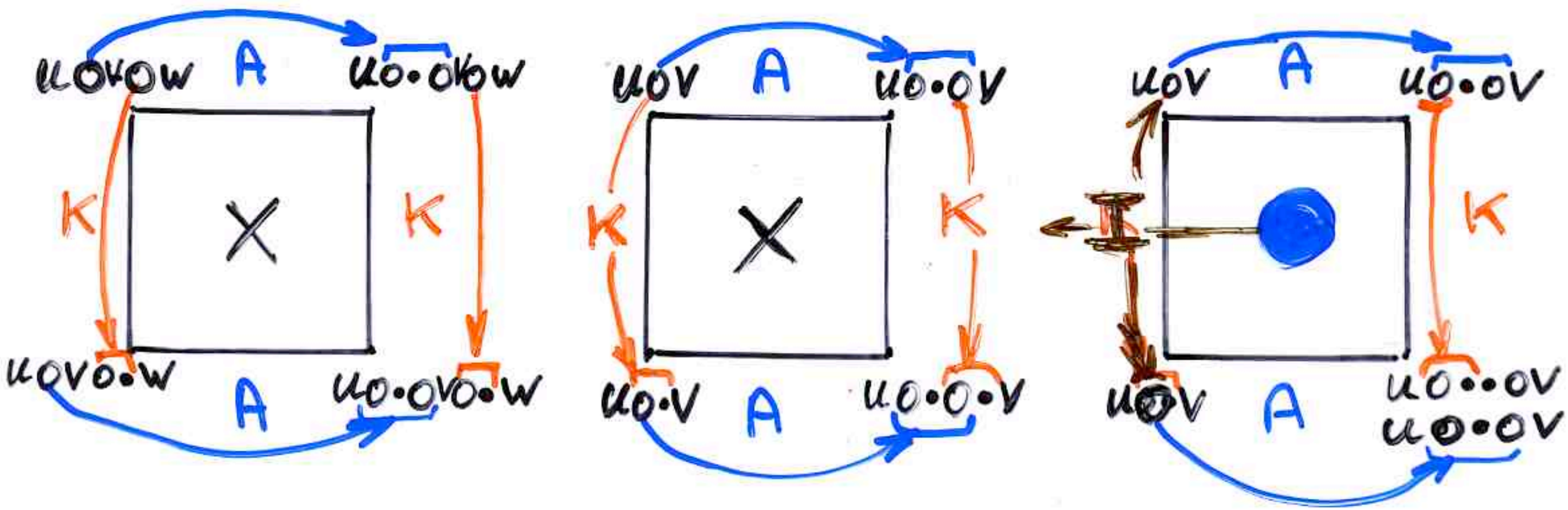

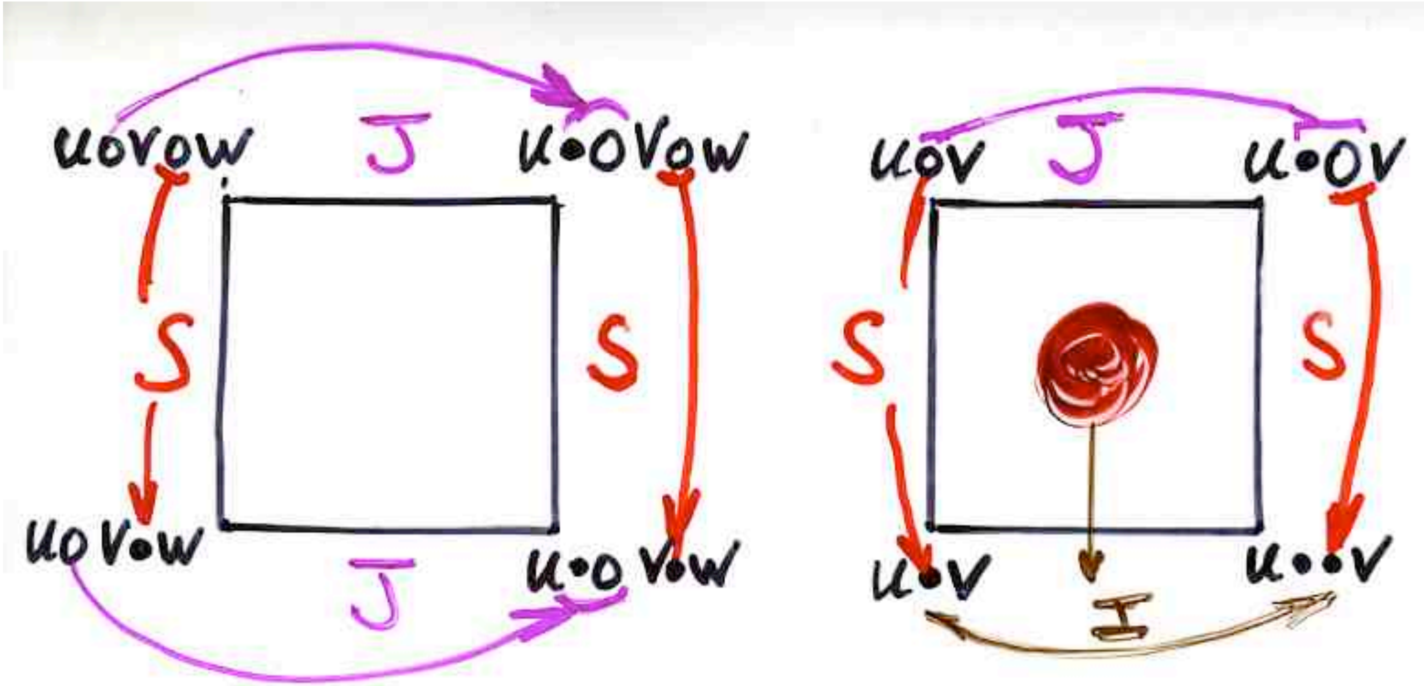

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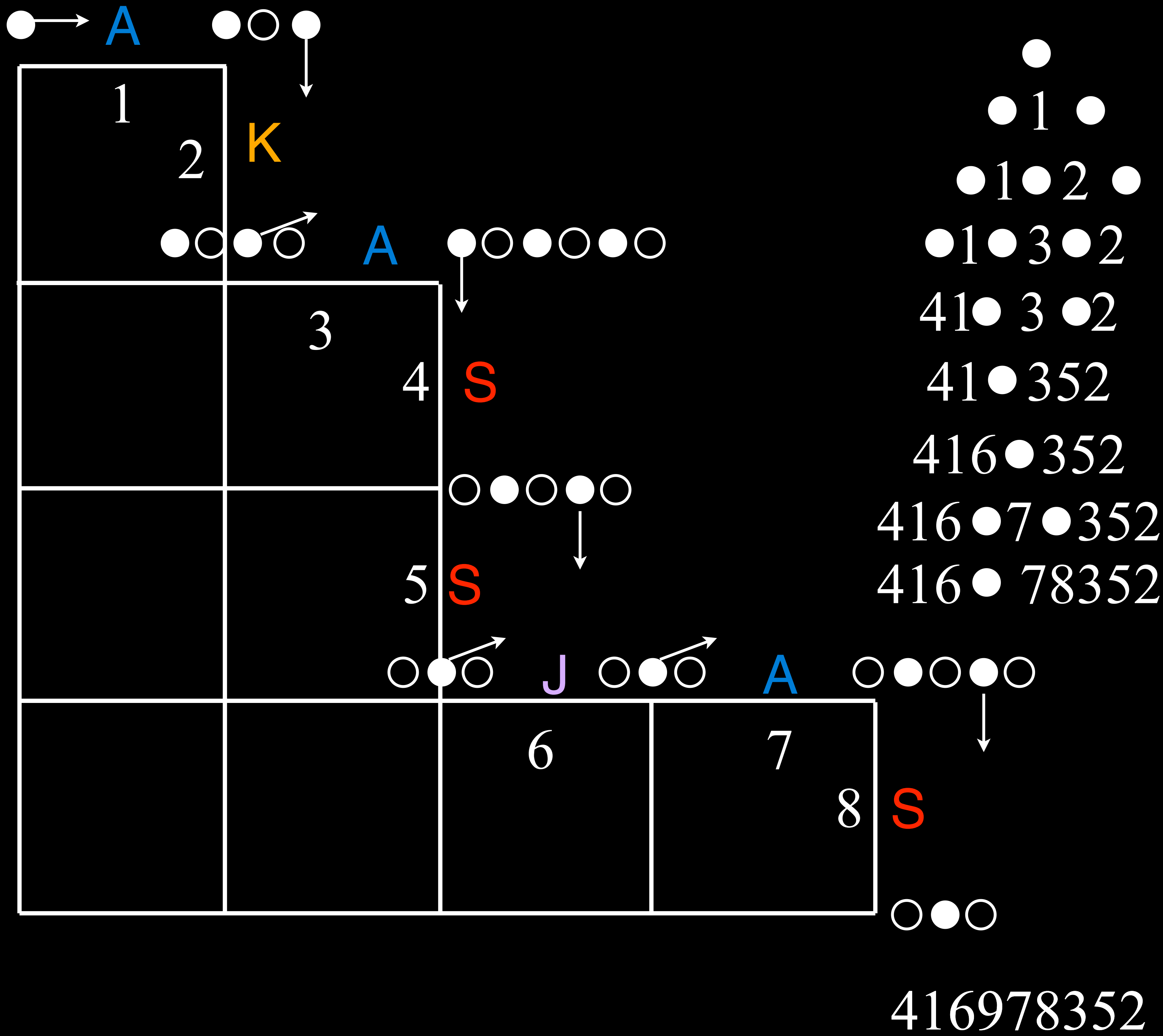
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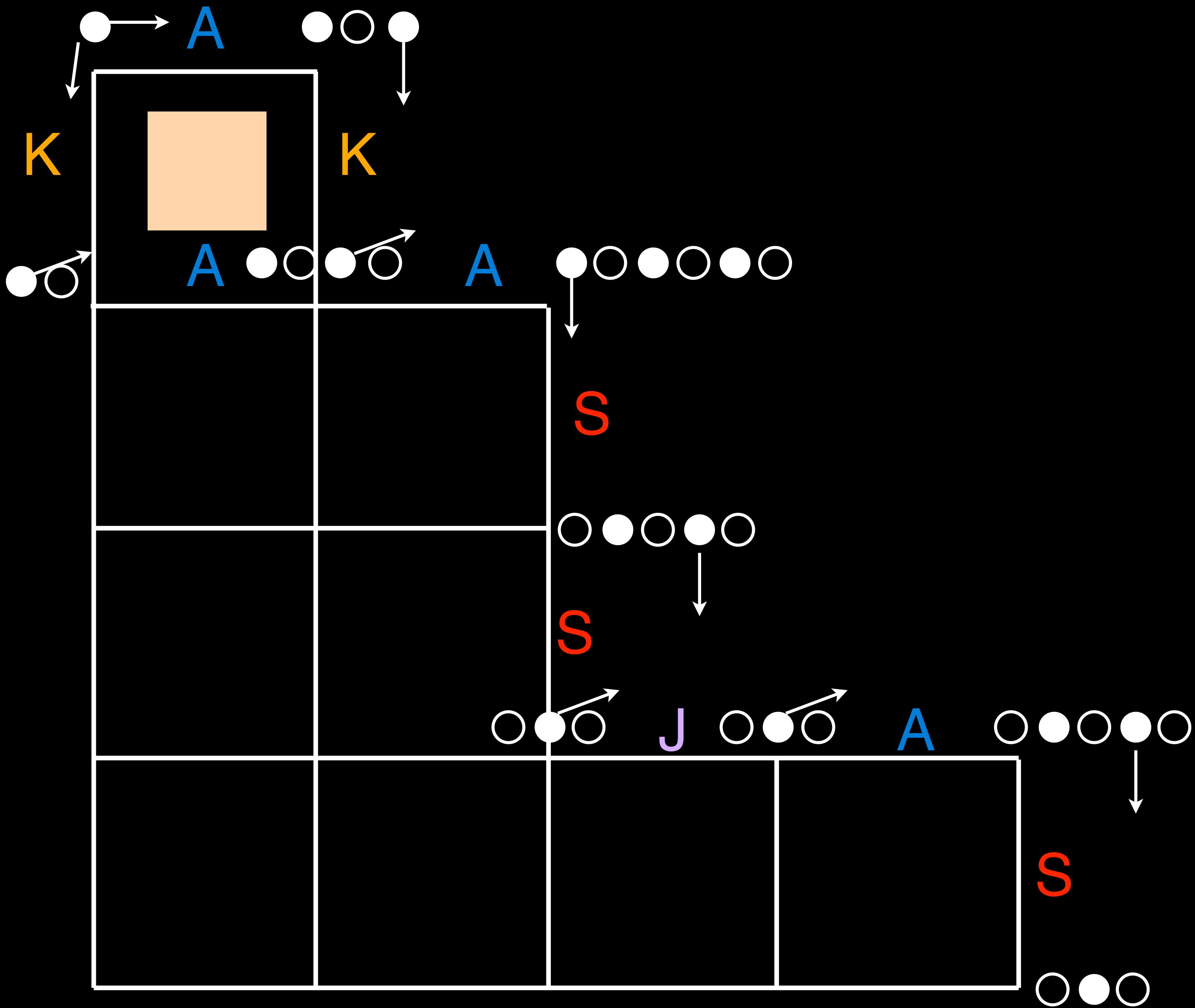
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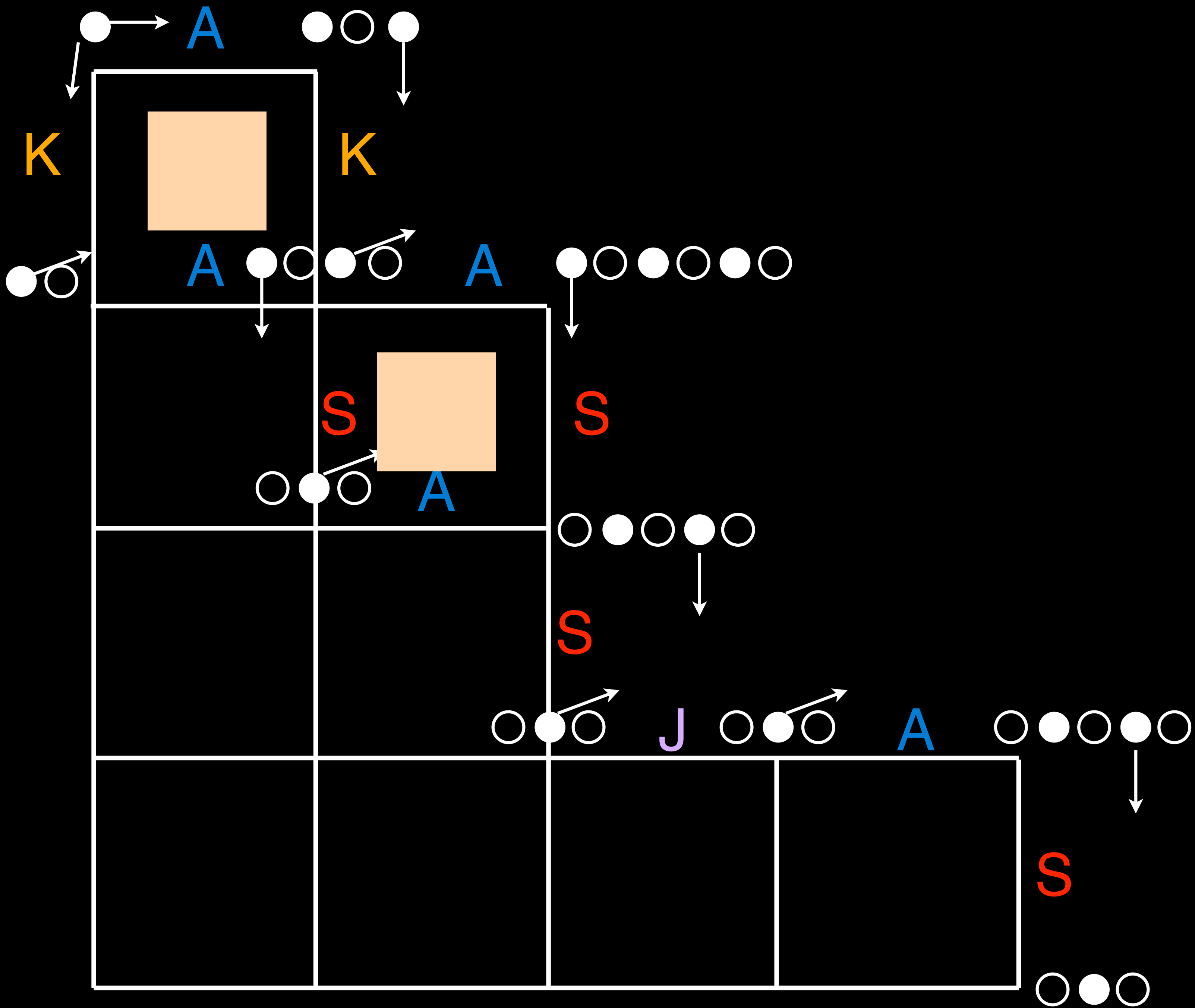
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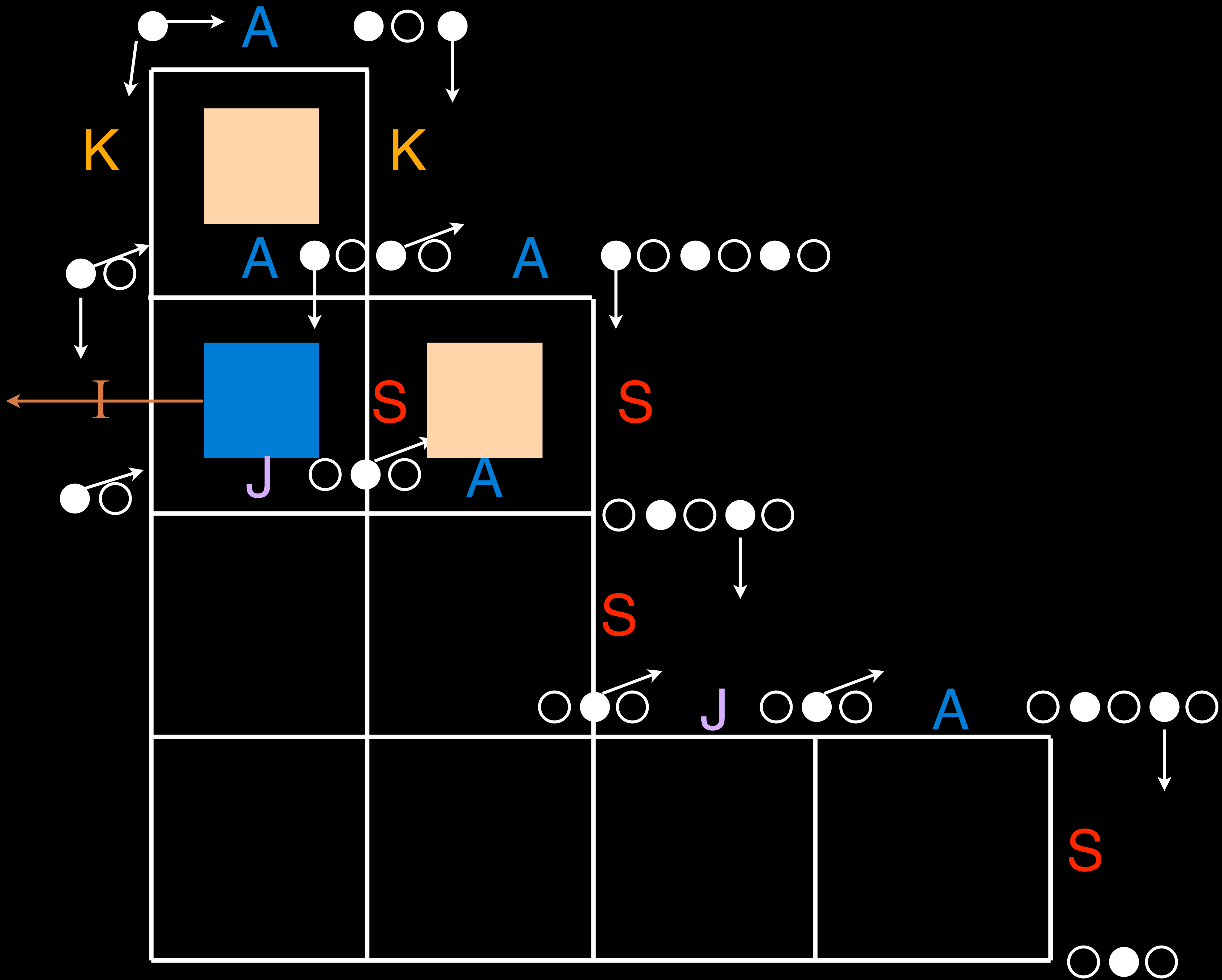
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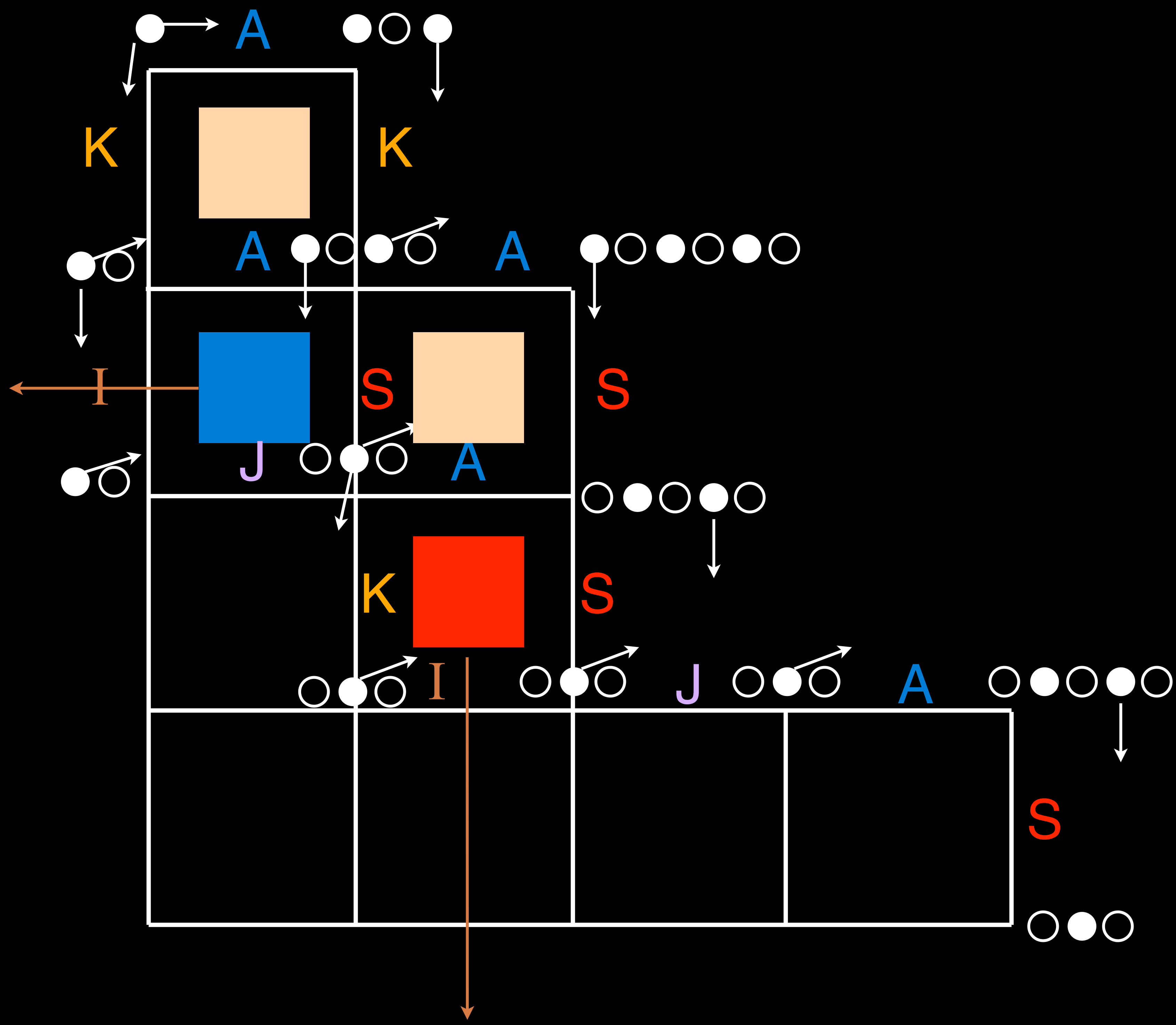

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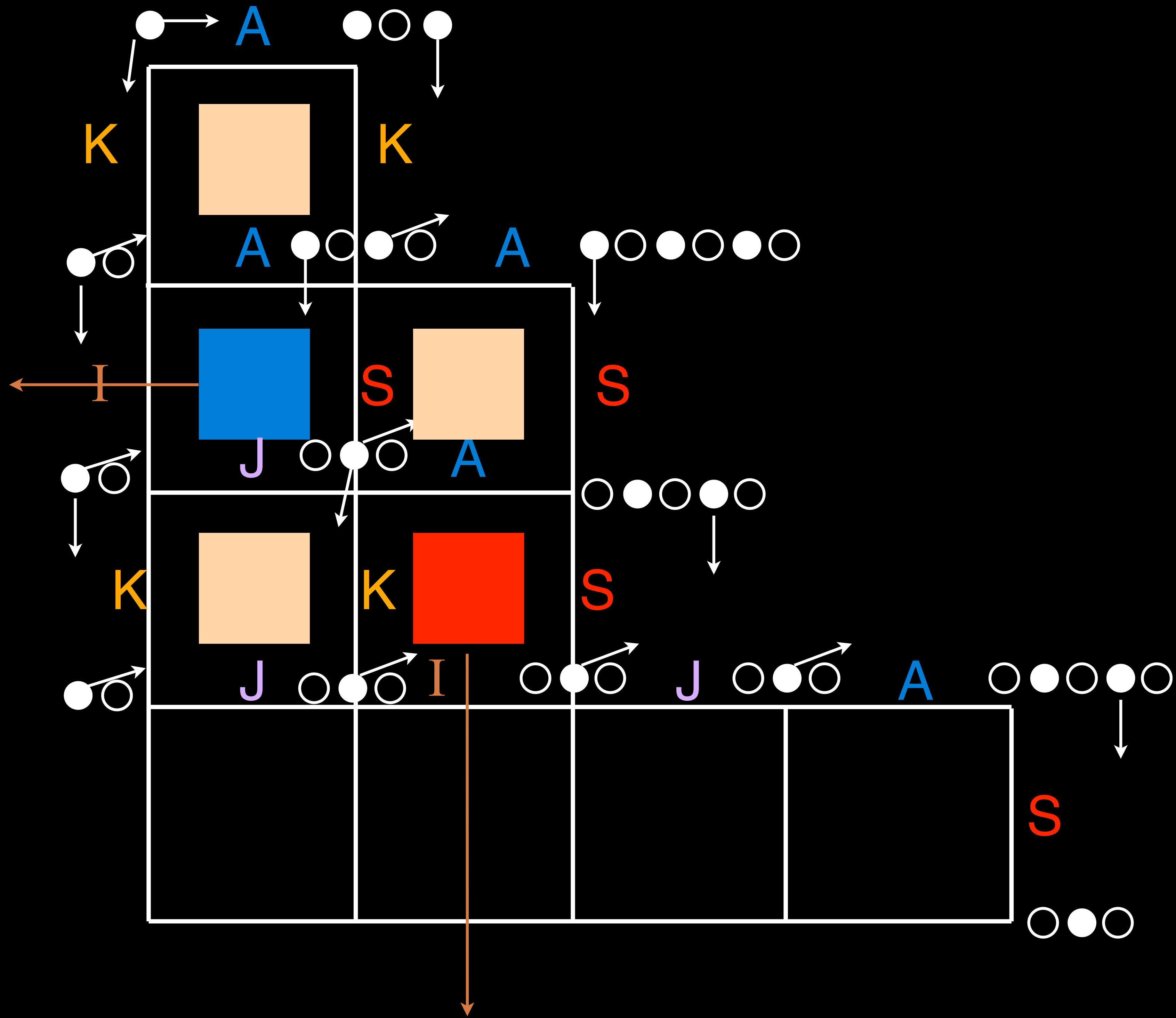
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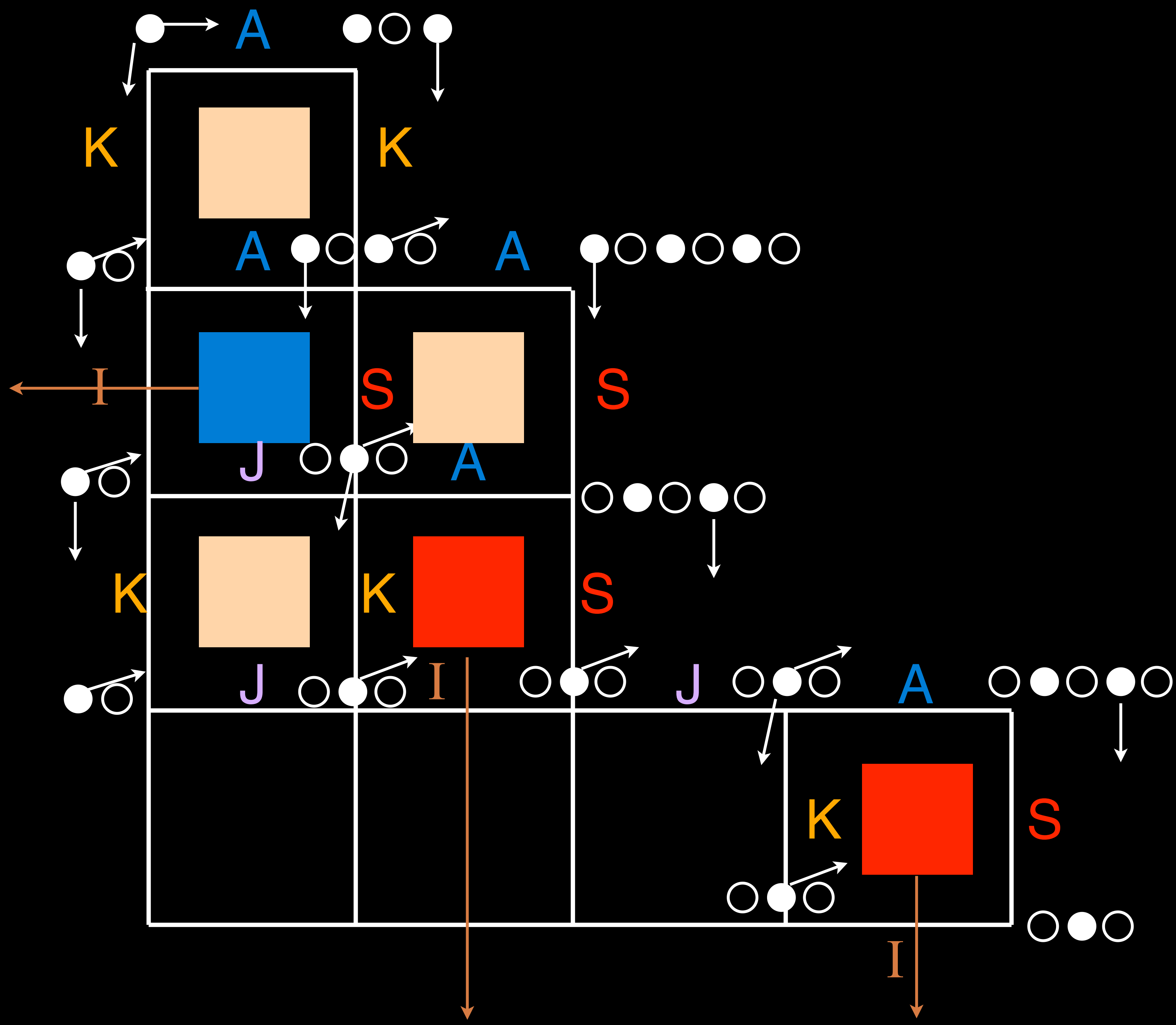
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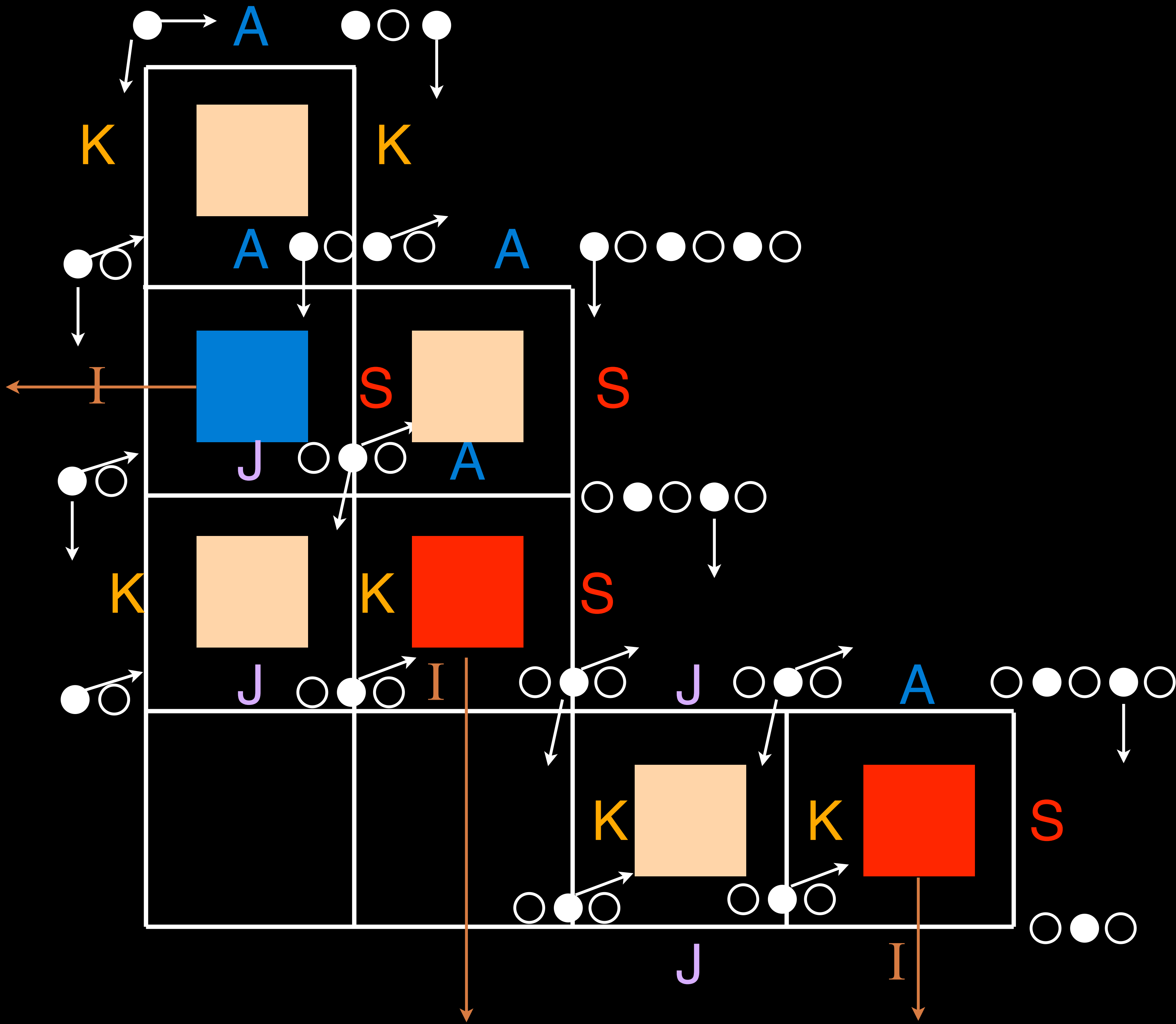
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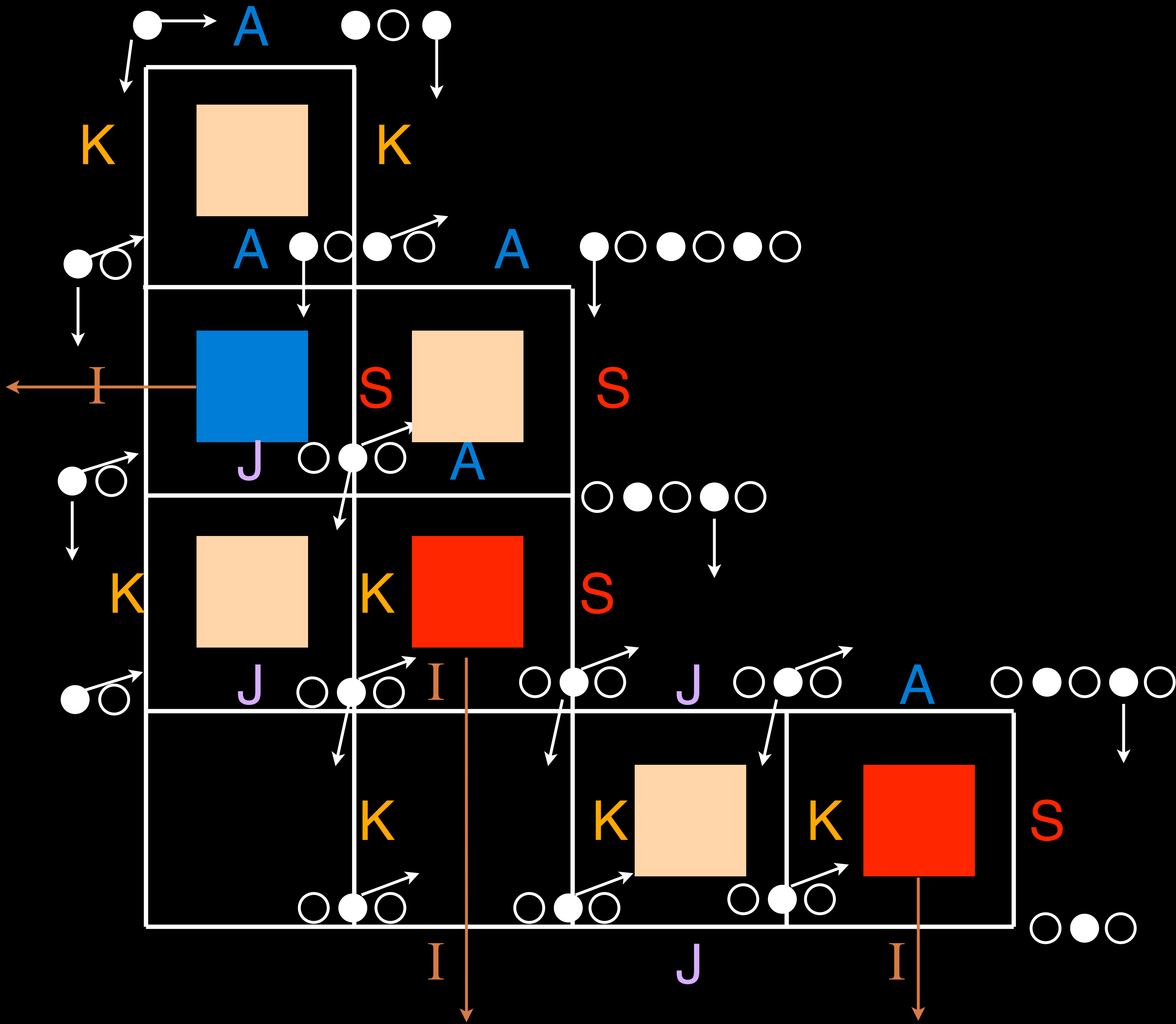

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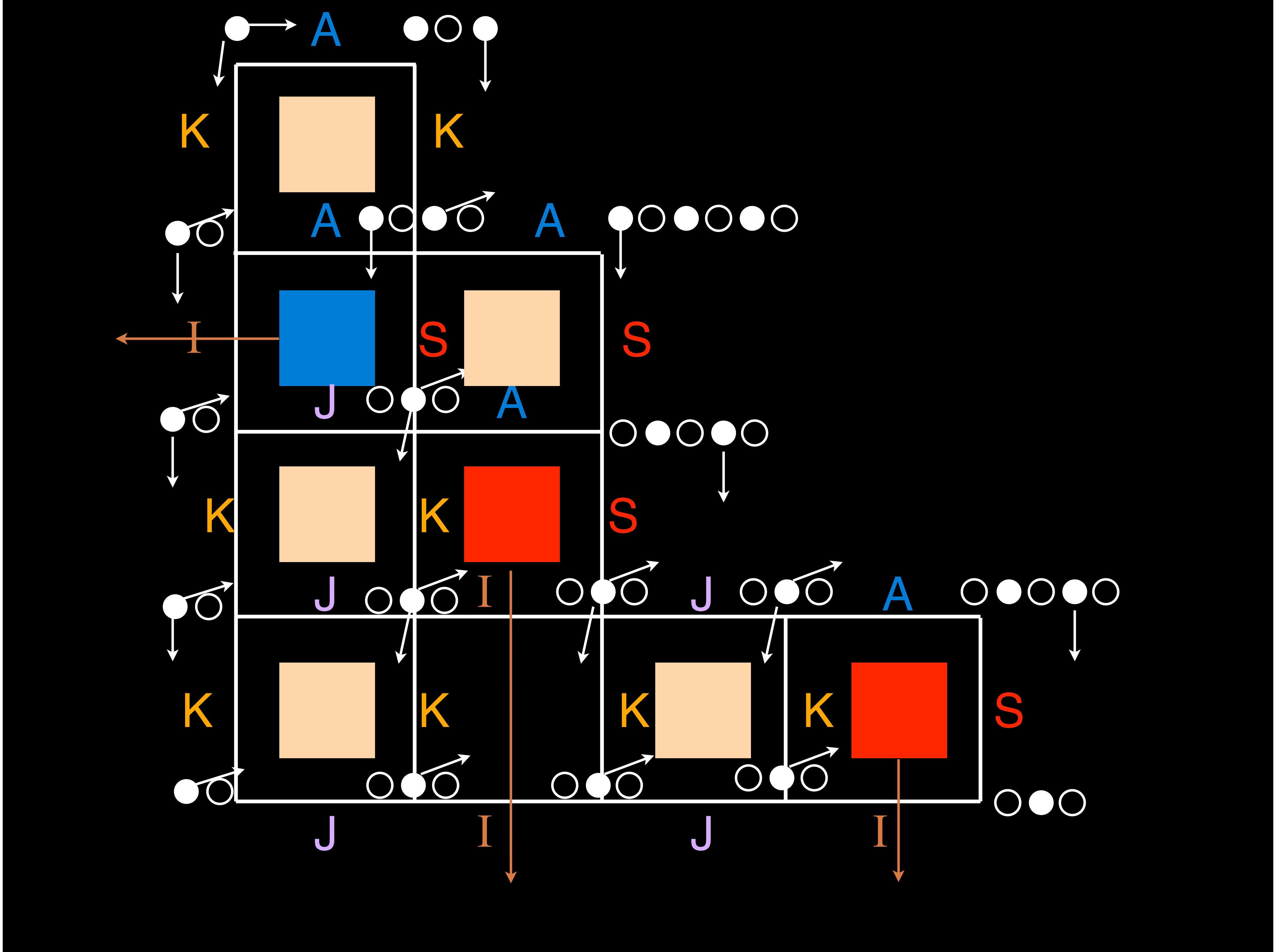

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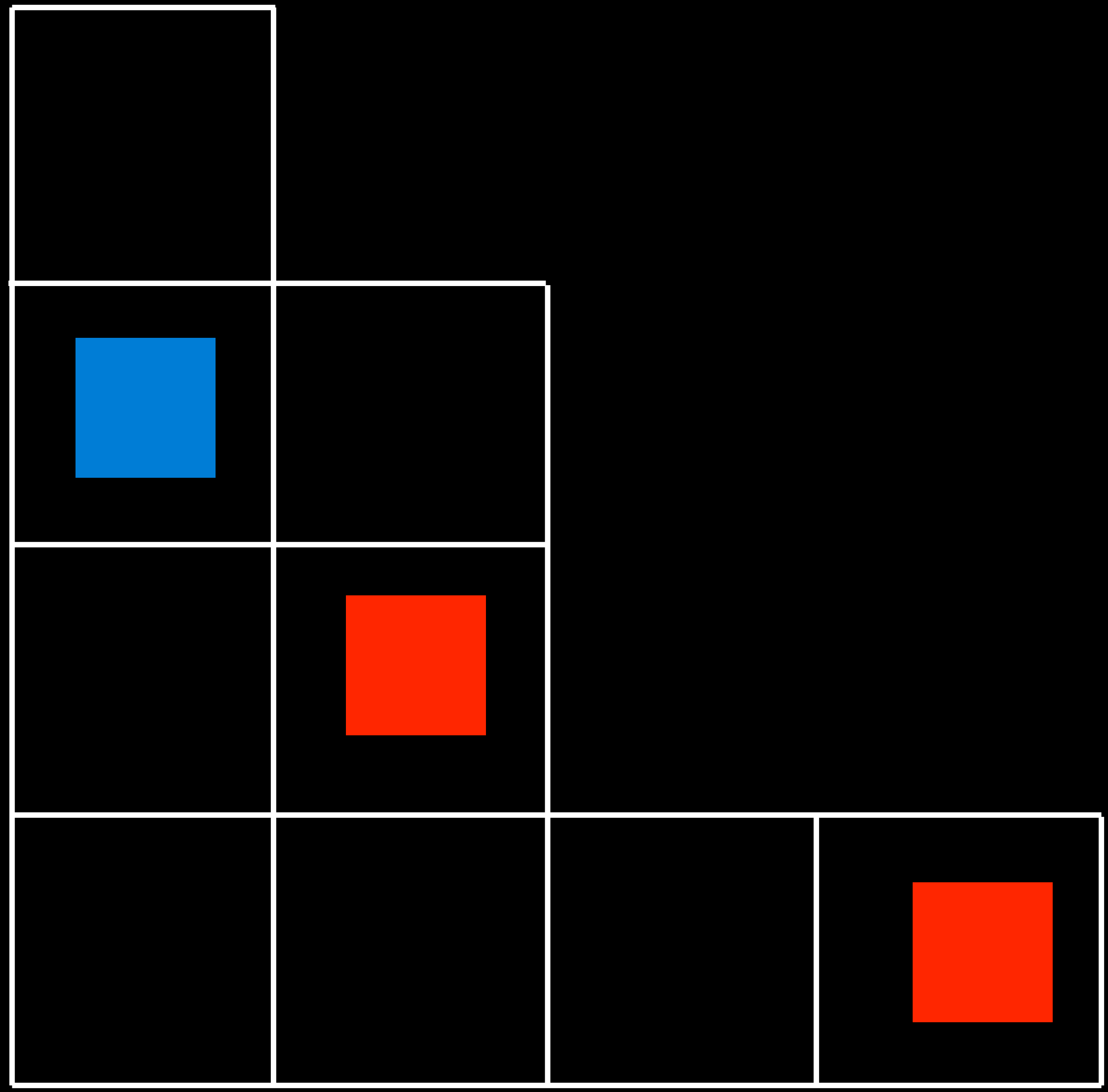

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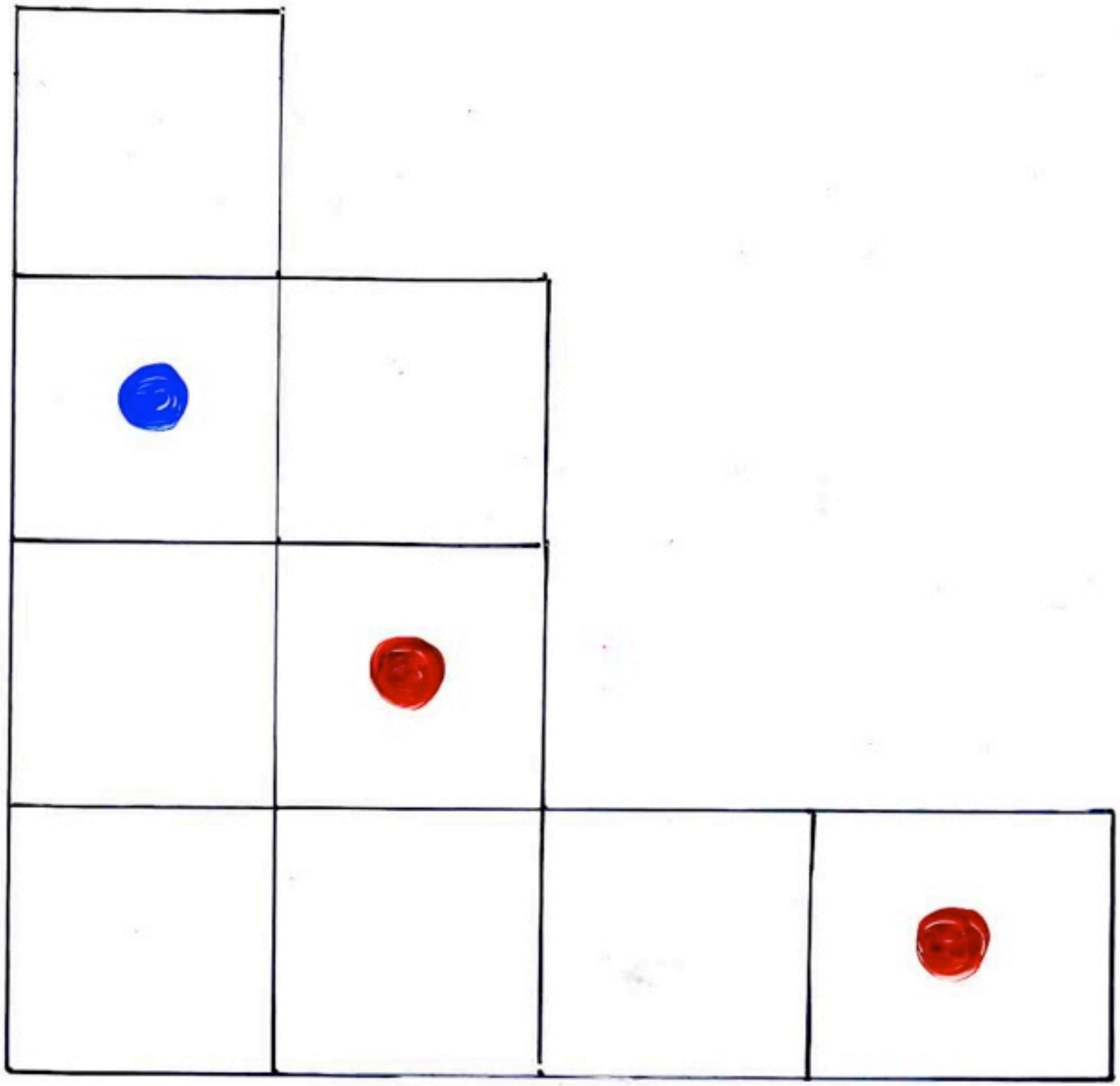
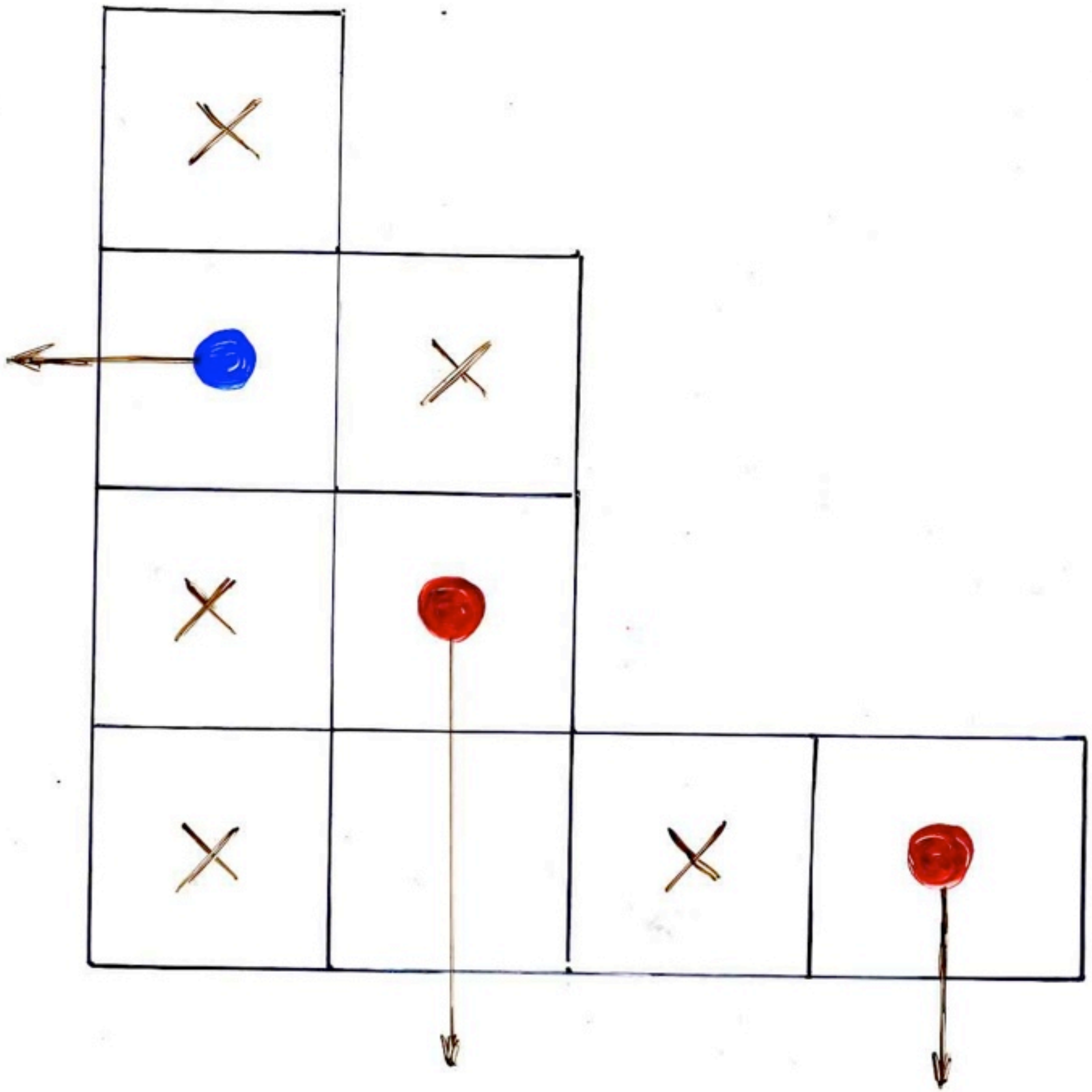
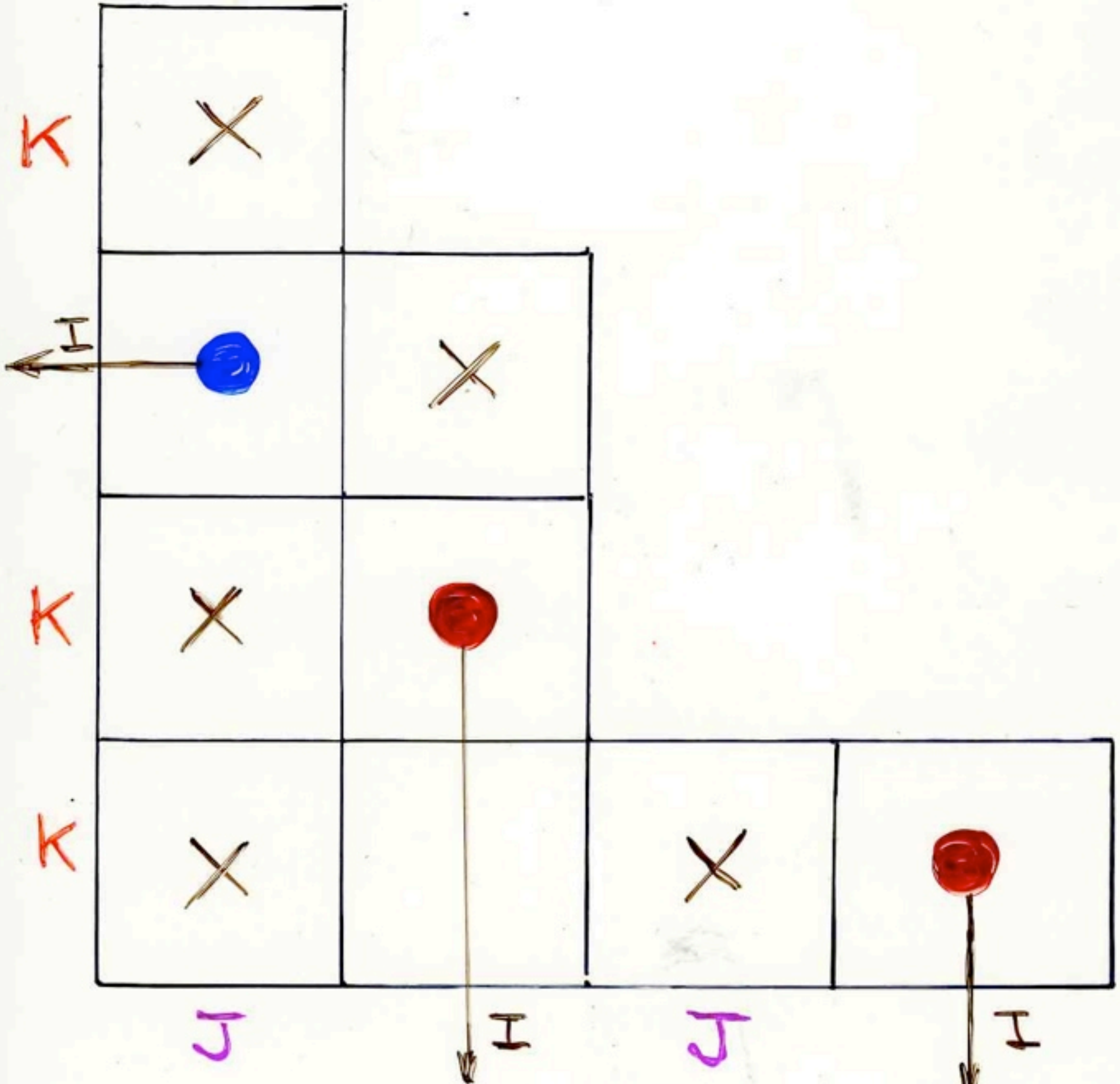
the inverse bijection

permutations --- alternative tableaux



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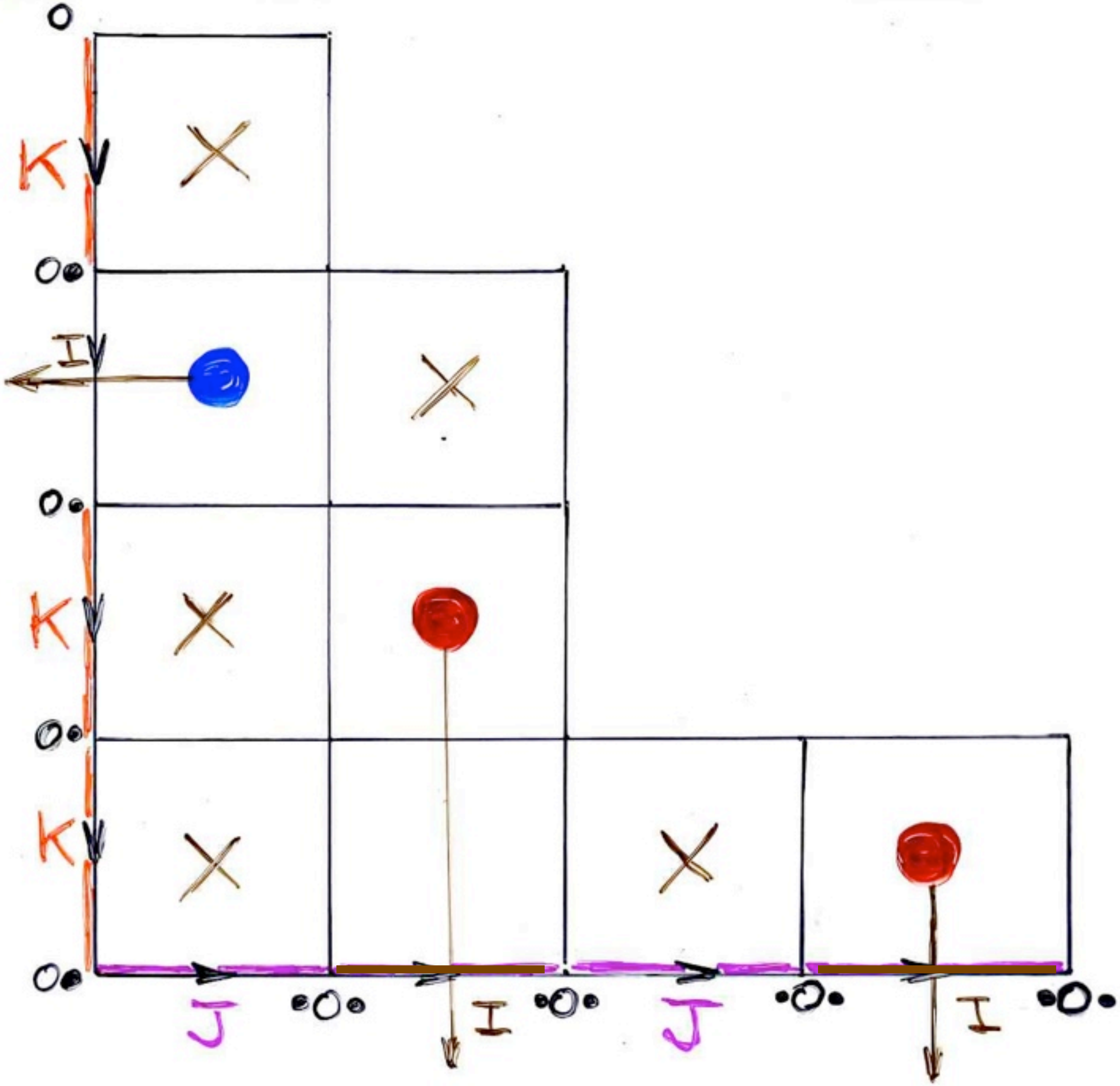

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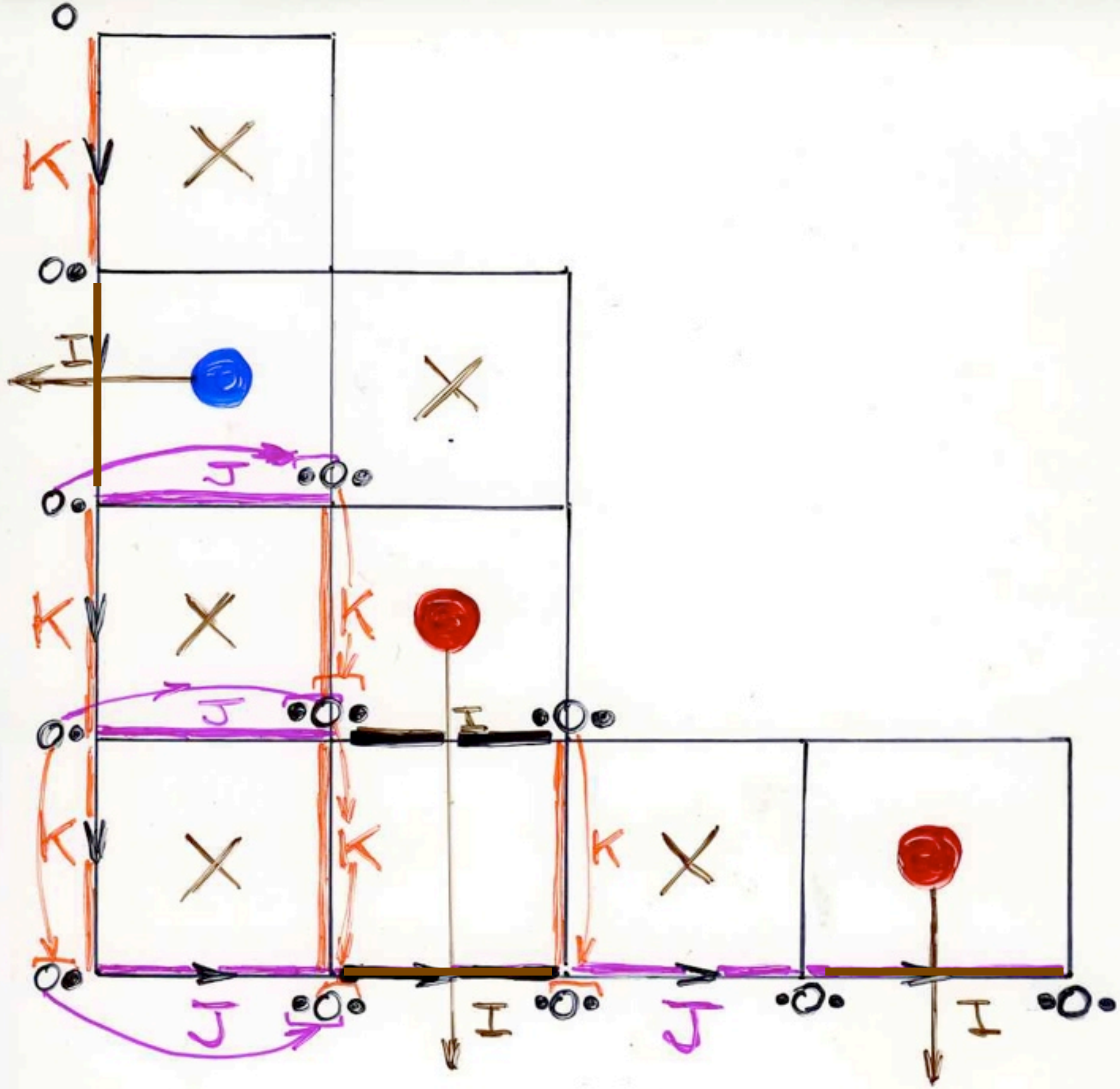

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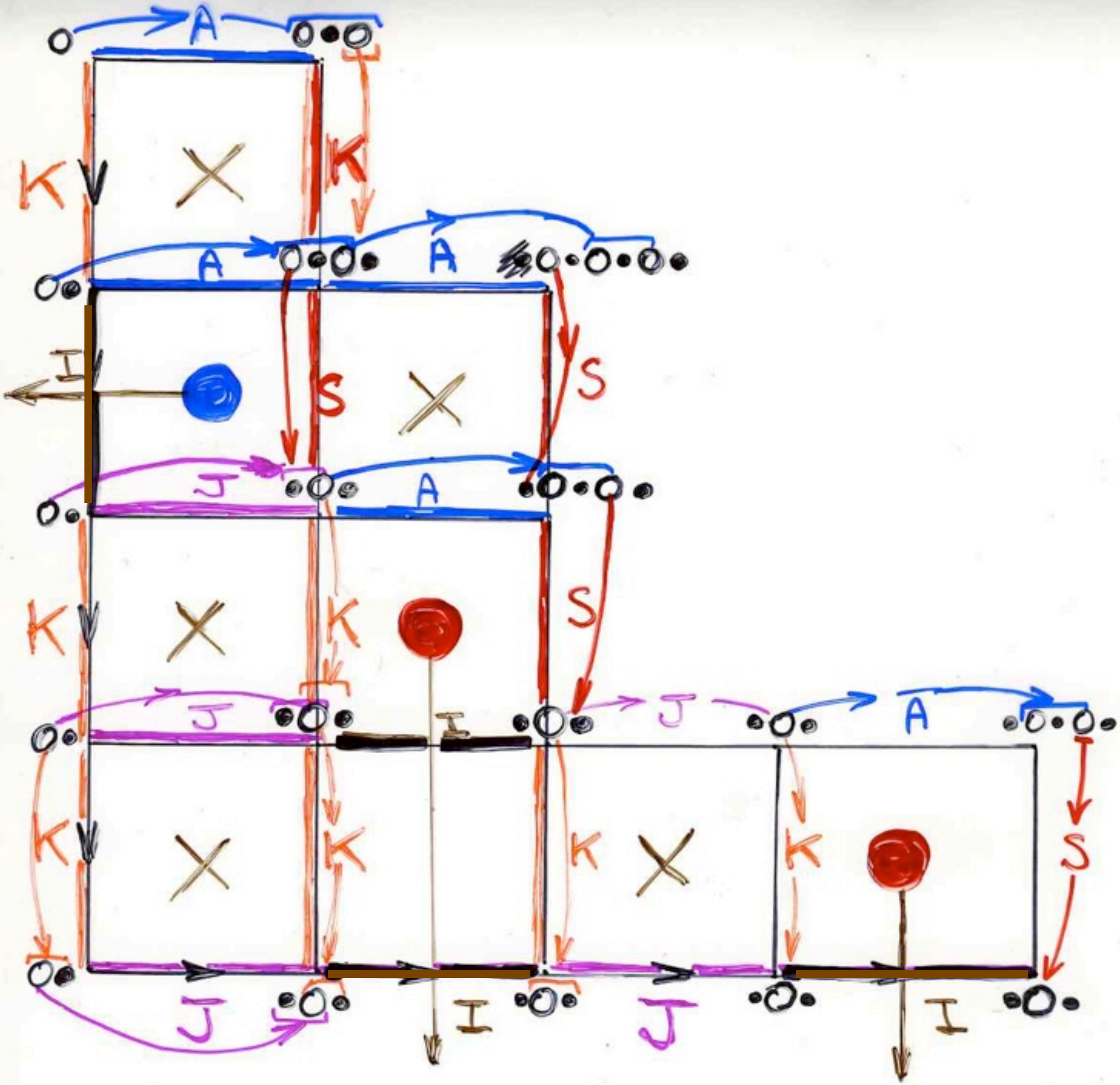

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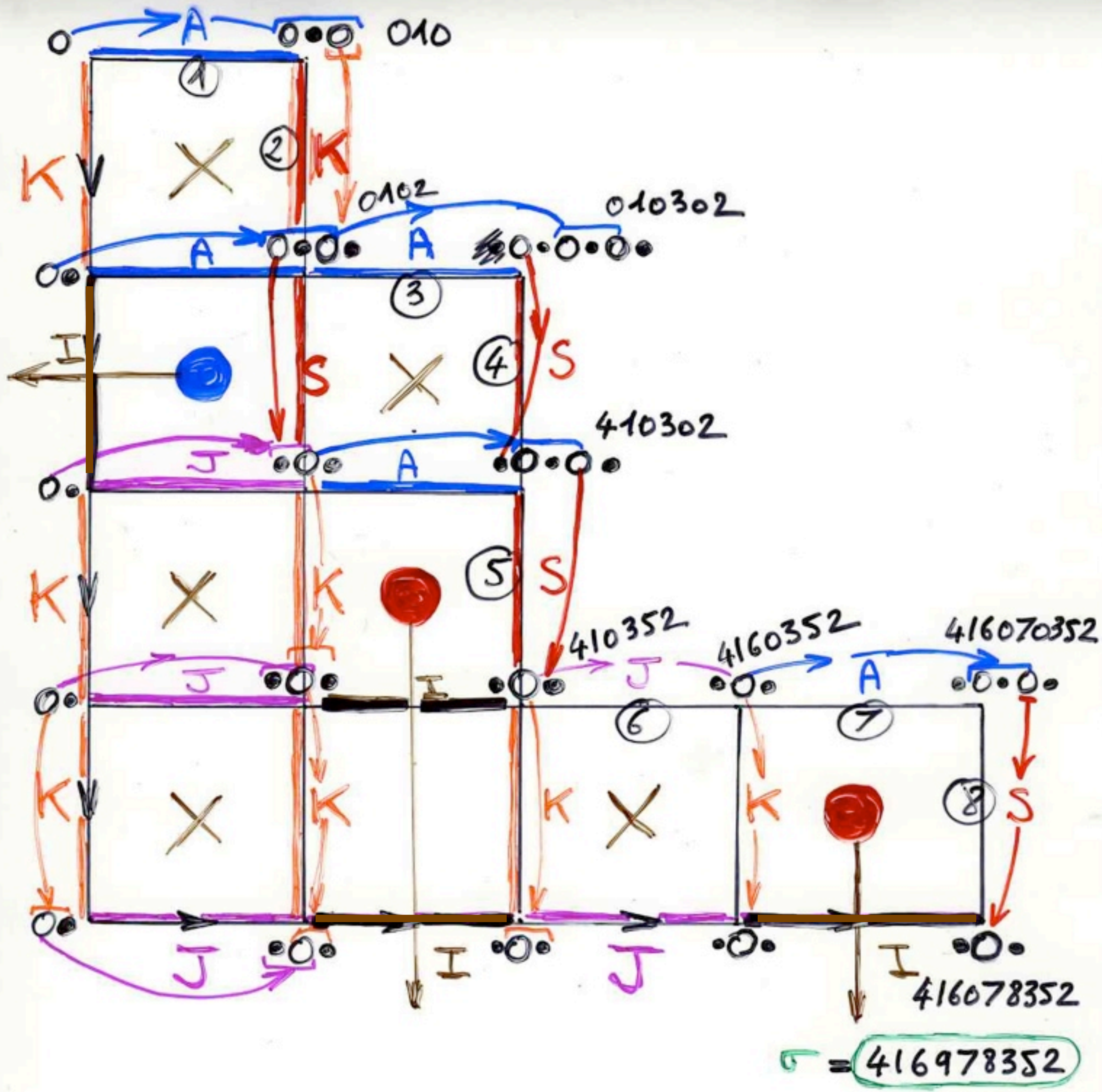

010
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416078352
σ = 416978352

two bijections
one theorem

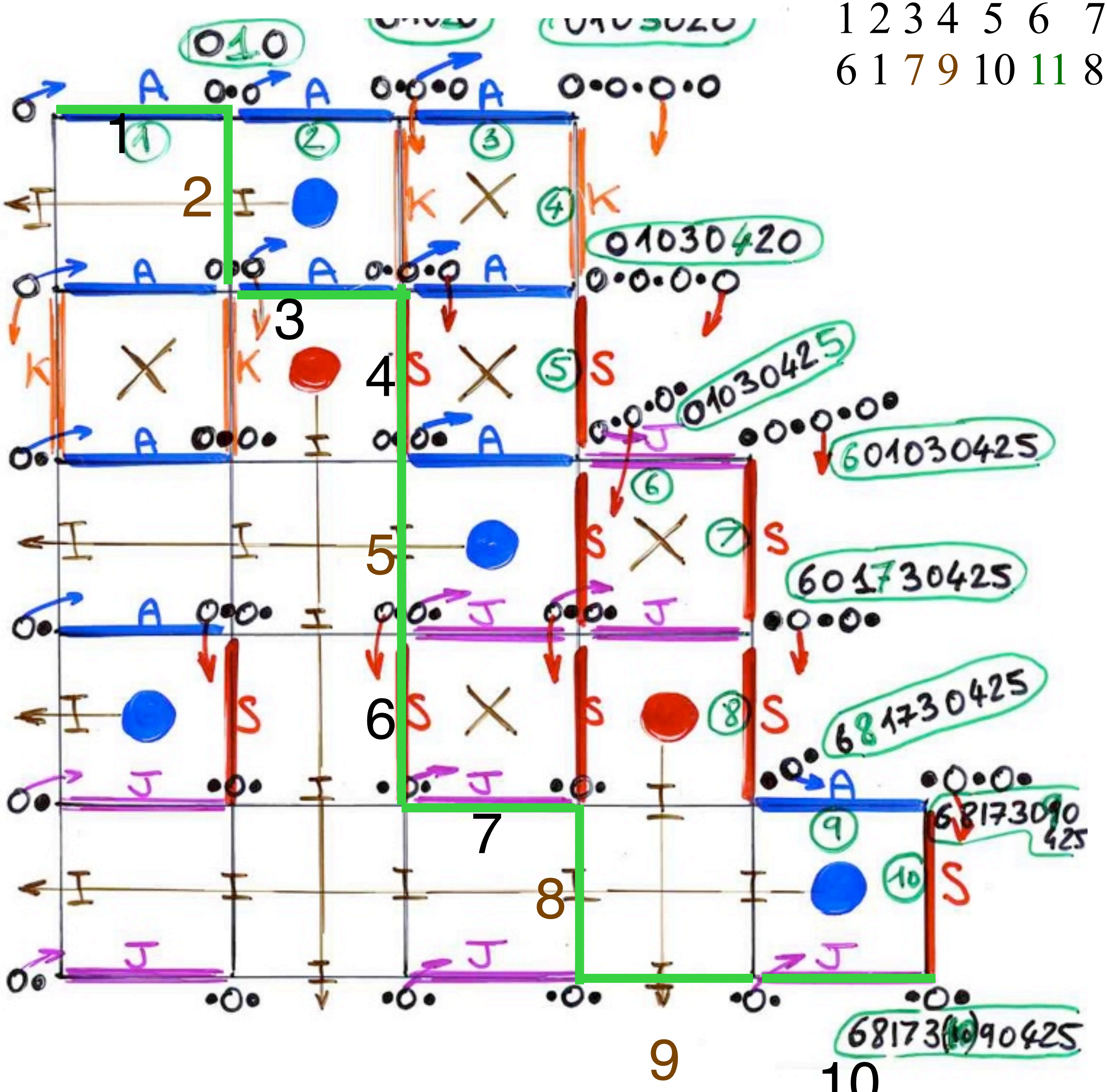
Prop.

$$\begin{tikzcd}[column sep=huge] & \sigma \\ T \text{ (alternative tableau)} \arrow[ur, "\text{inverse ``exchange-fusion'' algorithm}"] \arrow[dr, "\text{``local'' algorithm from } DE = ED + E + D"'] & \\ & \tau \end{tikzcd}$$

$$\sigma = \tau^{-1}$$

$\sigma = 6\ 8\ 1\ 7\ 3\ (10)\ 9\ (11)\ 4\ 2\ 5$

1	2	3	4	5	6	7	8	9	10	11
6	1	7	9	10	11	8	5	3	4	2 = S



Conclusion: In this talk I have presented a sort of

"cellular ansatz"

- Some (formal) operators satisfying some commutation relations are given and generate a certain quadratic algebra.

- The computations in this algebra are made by some (oriented) rewriting rules which are visualized in a planar way on a (square) elementary cell of a grid. May be the operator identity I has to be introduced as another formal operator.

- The rewriting of a word of the algebra is visualized by a kind of a 2D cellular oriented expansion. The edges of the grid are labeled by the operators, the cells are labeled by each of the possible rewriting rules.

- The grid with the final labeling of the cells is in bijection with a class P of combinatorial objects (Permutations, Alernative tableaux, ASM, FPL, Tilings, etc ...).

- If the operators can be represented as combinatorial operators acting on a certain class F of combinatorial objects, then a simple combinatorial explanation of the commutation rules can be "attached" to each labeled cell of the grid. The vertices of the grid becomes labeled by the objects of F and "local rules" should be defined. In the case (as in the two examples of RSK and Alternating tableaux) when only the labels of the cells, and not those of the edges, are needed for defining the local rules, then from the cellular propagation of these local rules across the grid, one obtain a bijection between the objects of P and some other objects coded by the sequence of the F-labels on the border of the grid.

The "exchange-fusion" algorithm

The inverse "exchange-fusion" algorithm

Genocchi sequence of a permutation

some parameters

Laguerre histories

The cellular Ansatz $\qquad DE = ED + E + D$

representation of the operators E and D

Cellular Ansatz: bijection permutations - alternative tableaux

the inverse bijection permutations - alternative tableaux

two bijections, one theorem

complements

the exchange deletion algorithm

Postnikov bijection