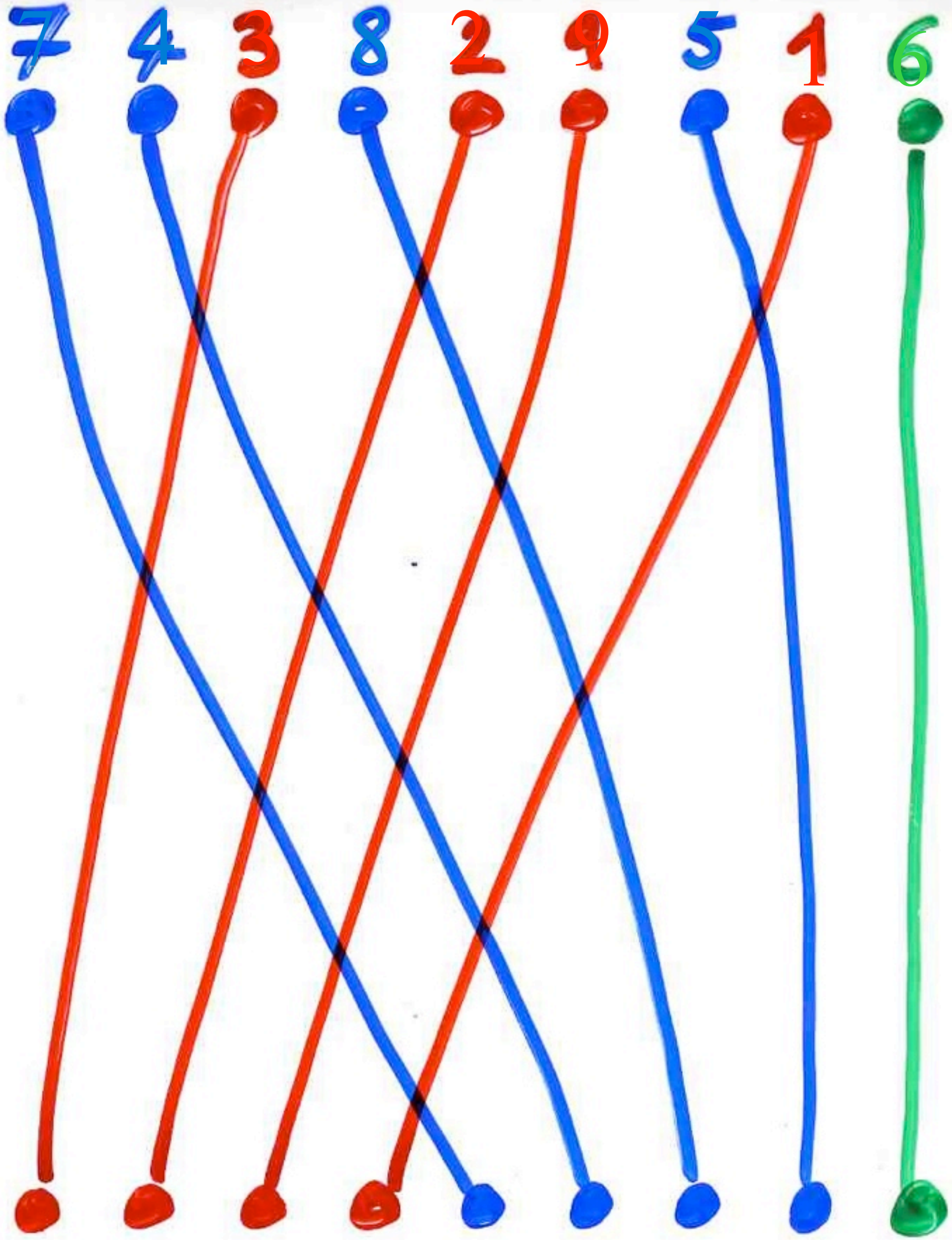
Chapter 4c

Alternative tableaux and the PASEP

complements

18 january 2011
Talca

the exchange deletion algorithm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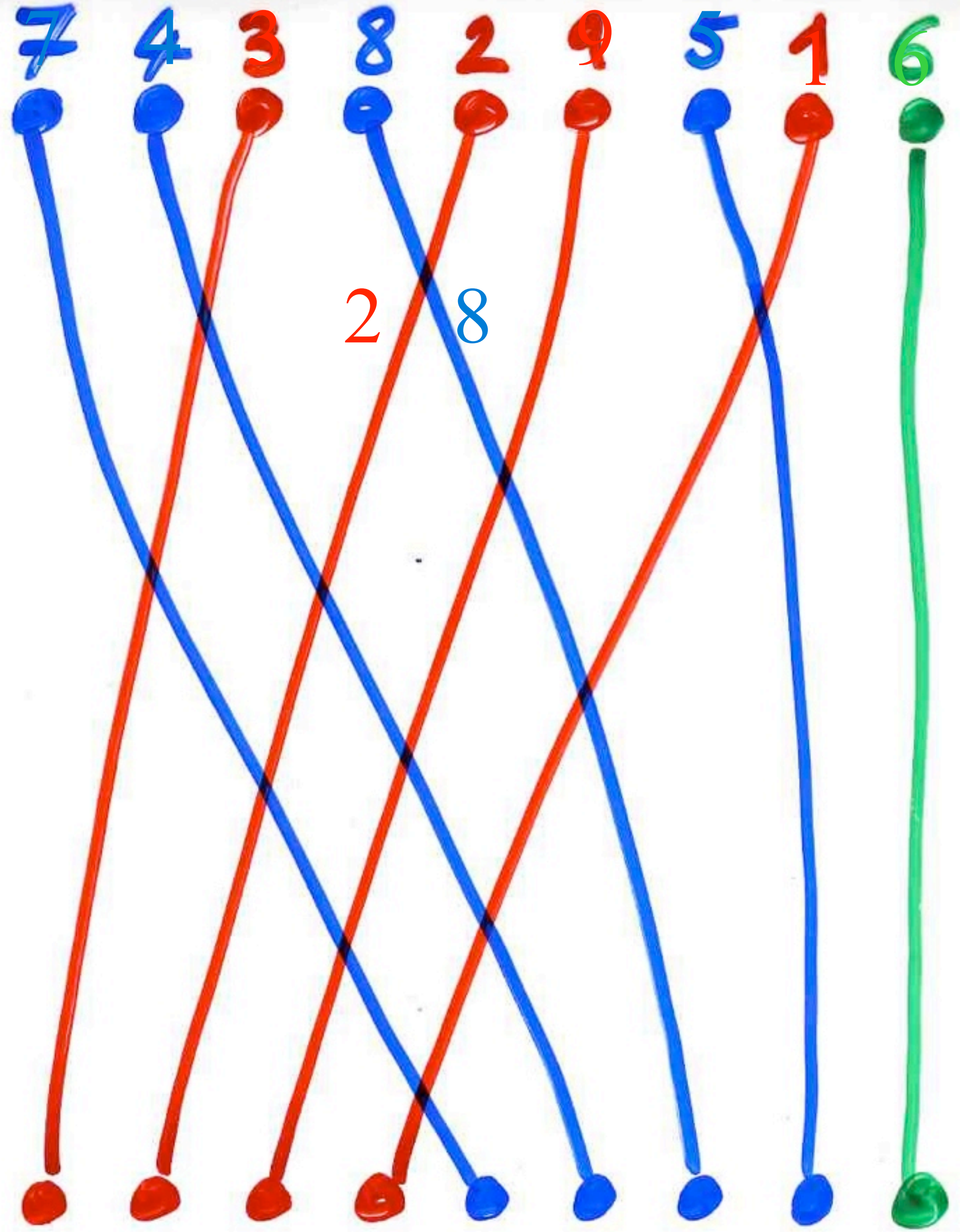
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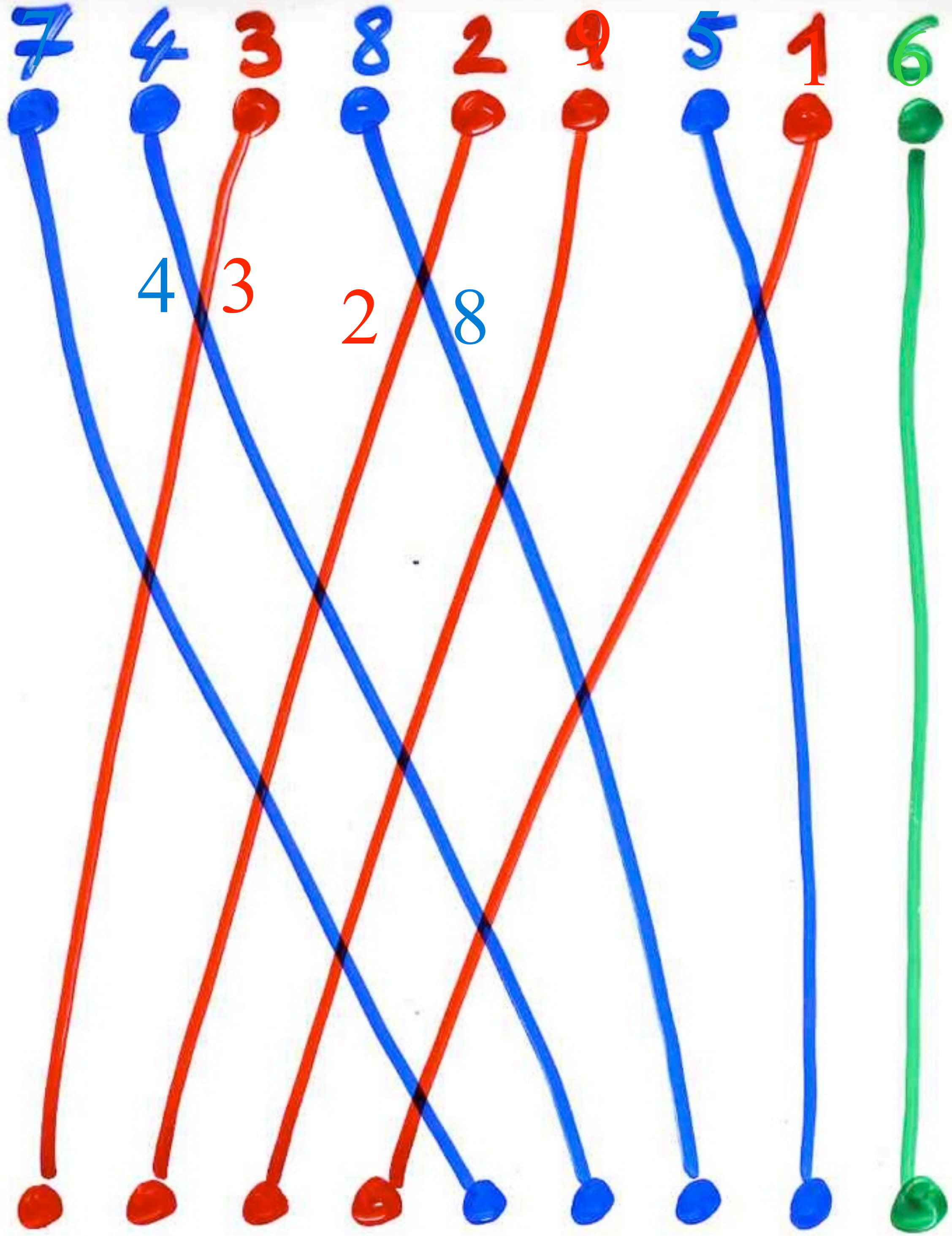
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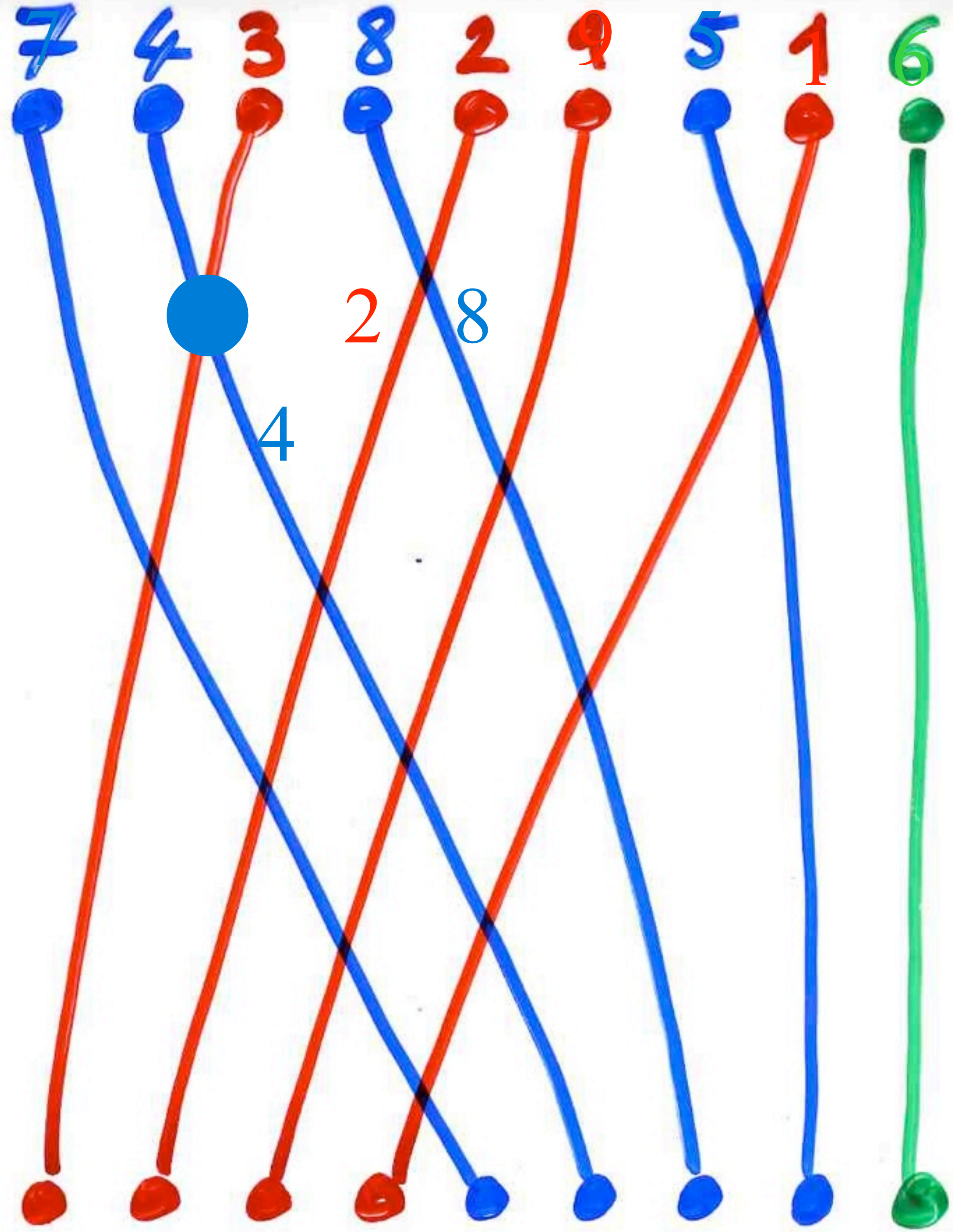
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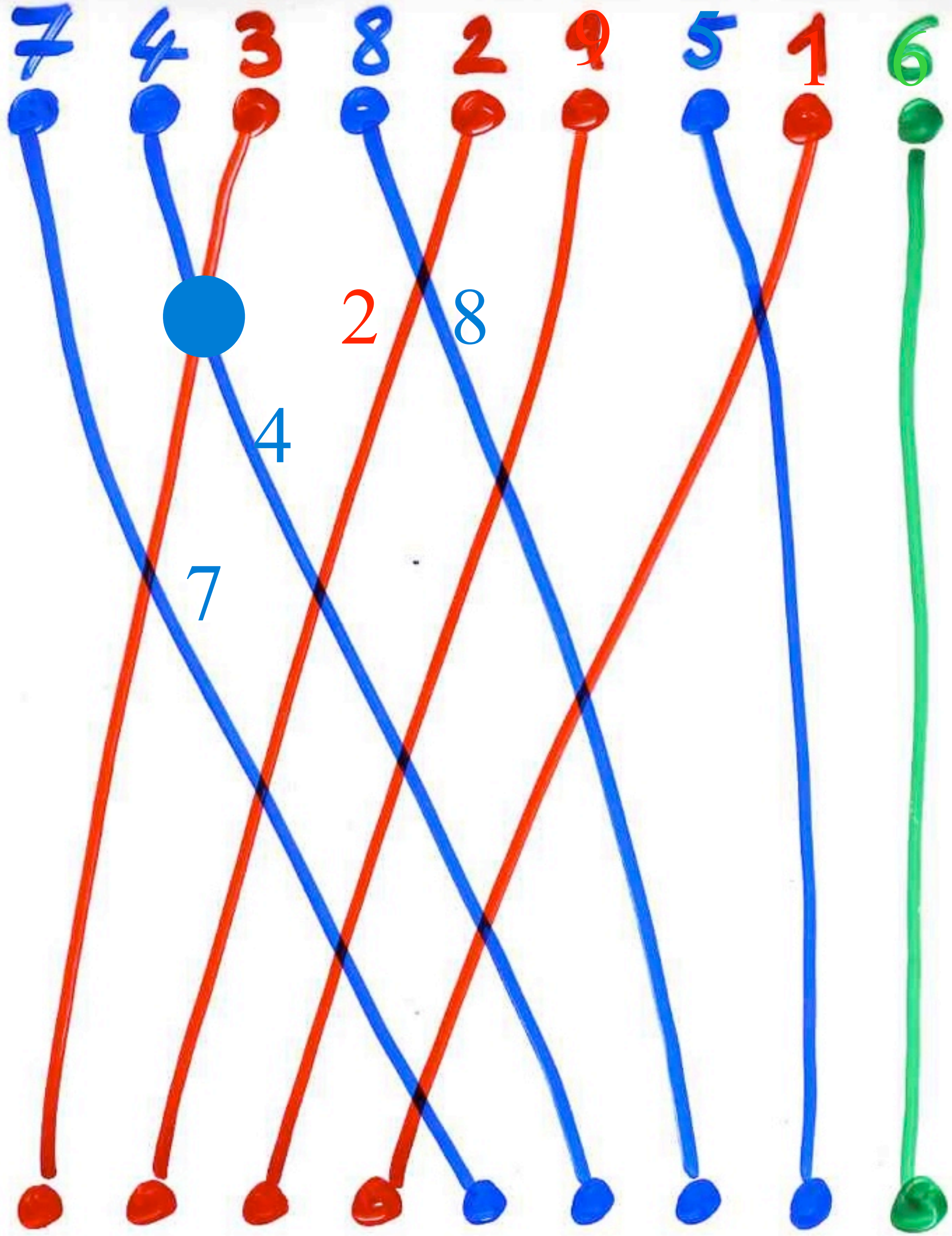

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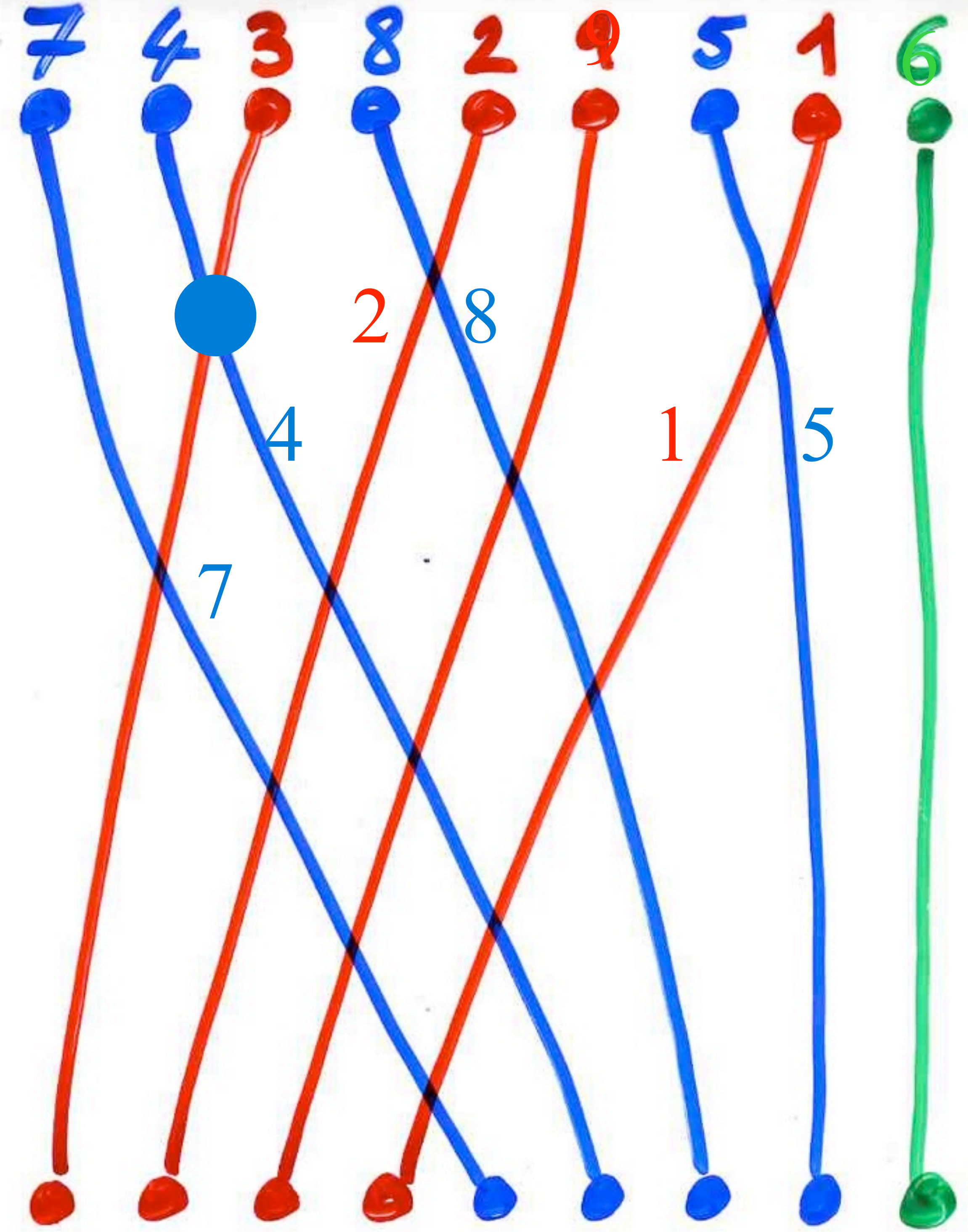
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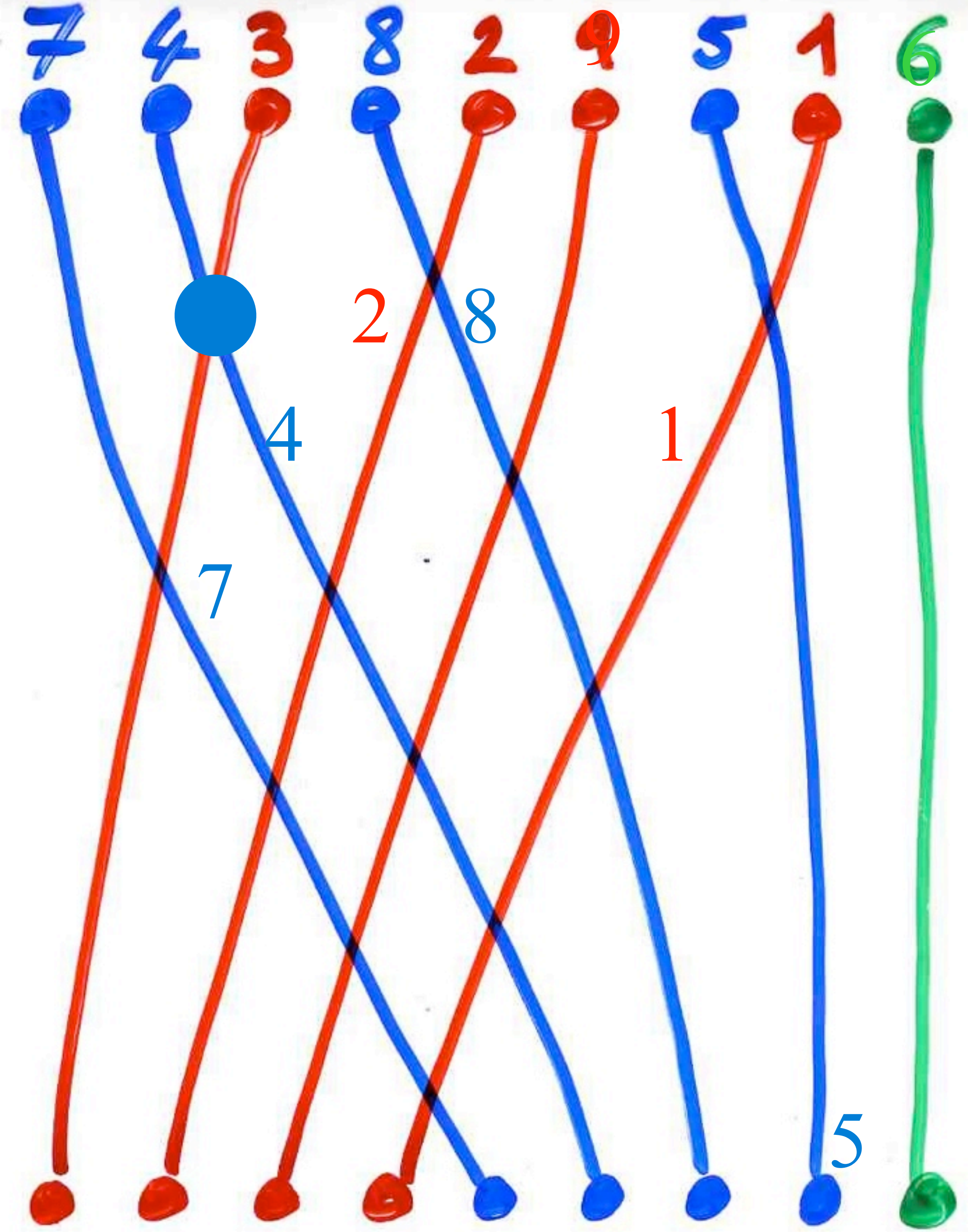

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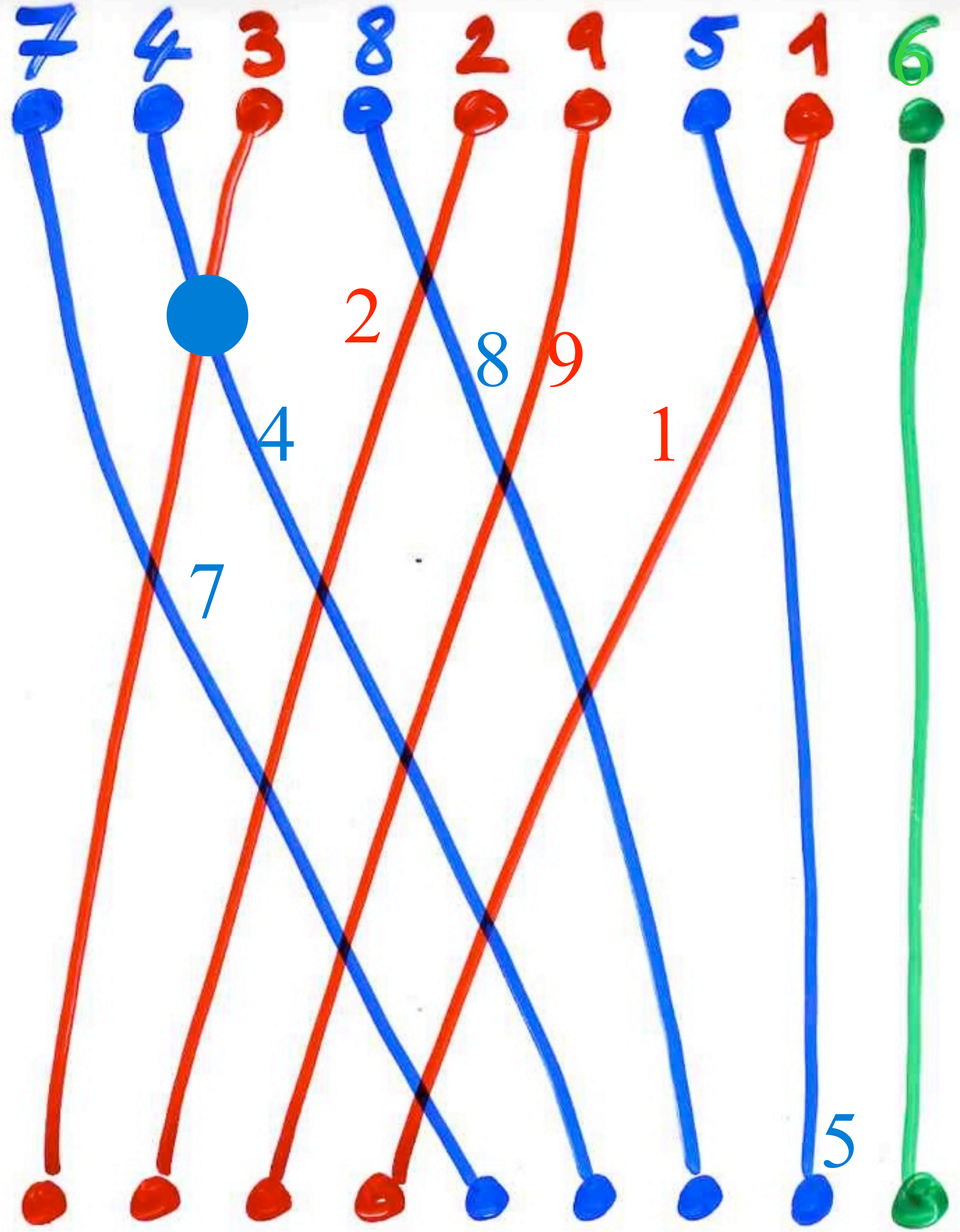

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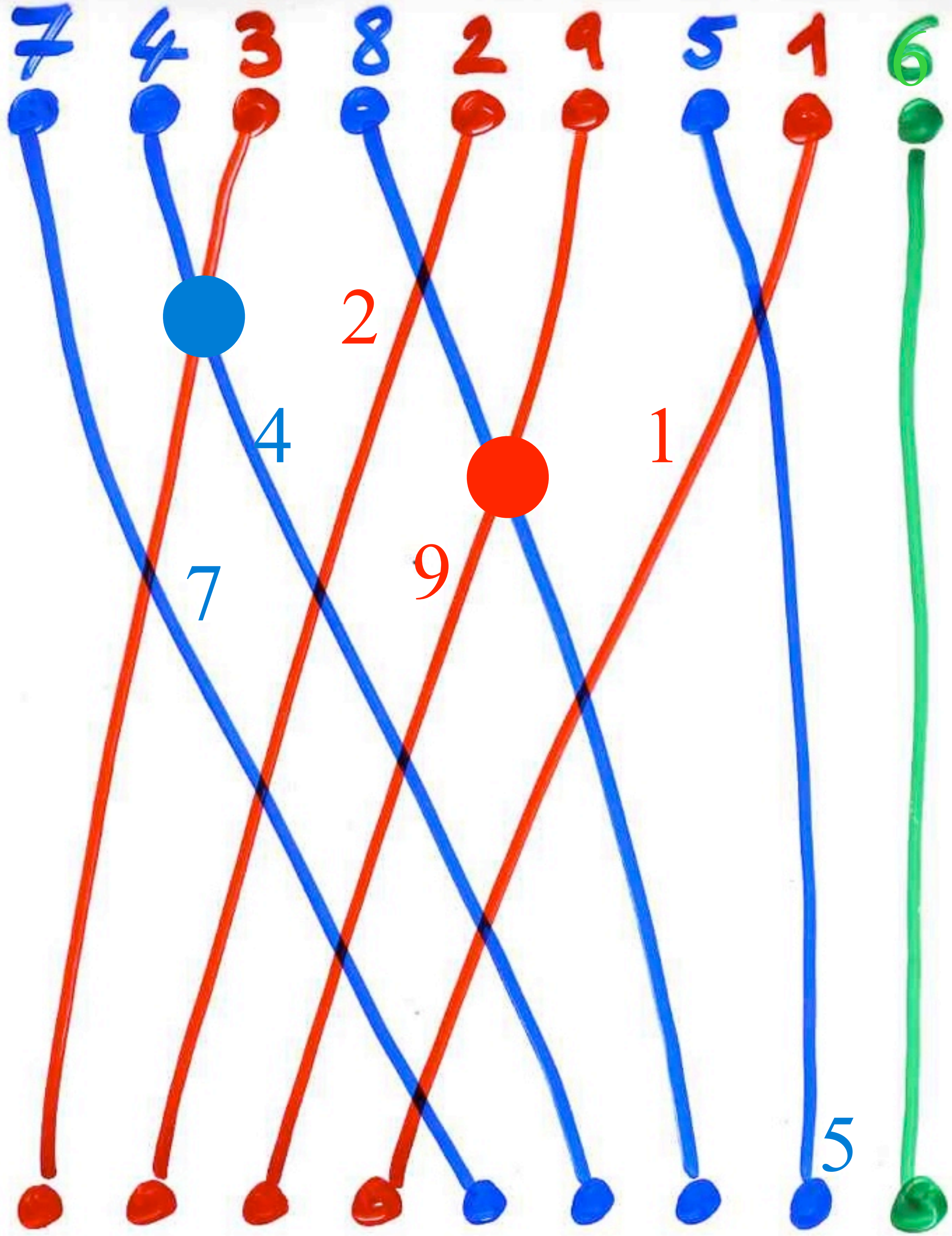

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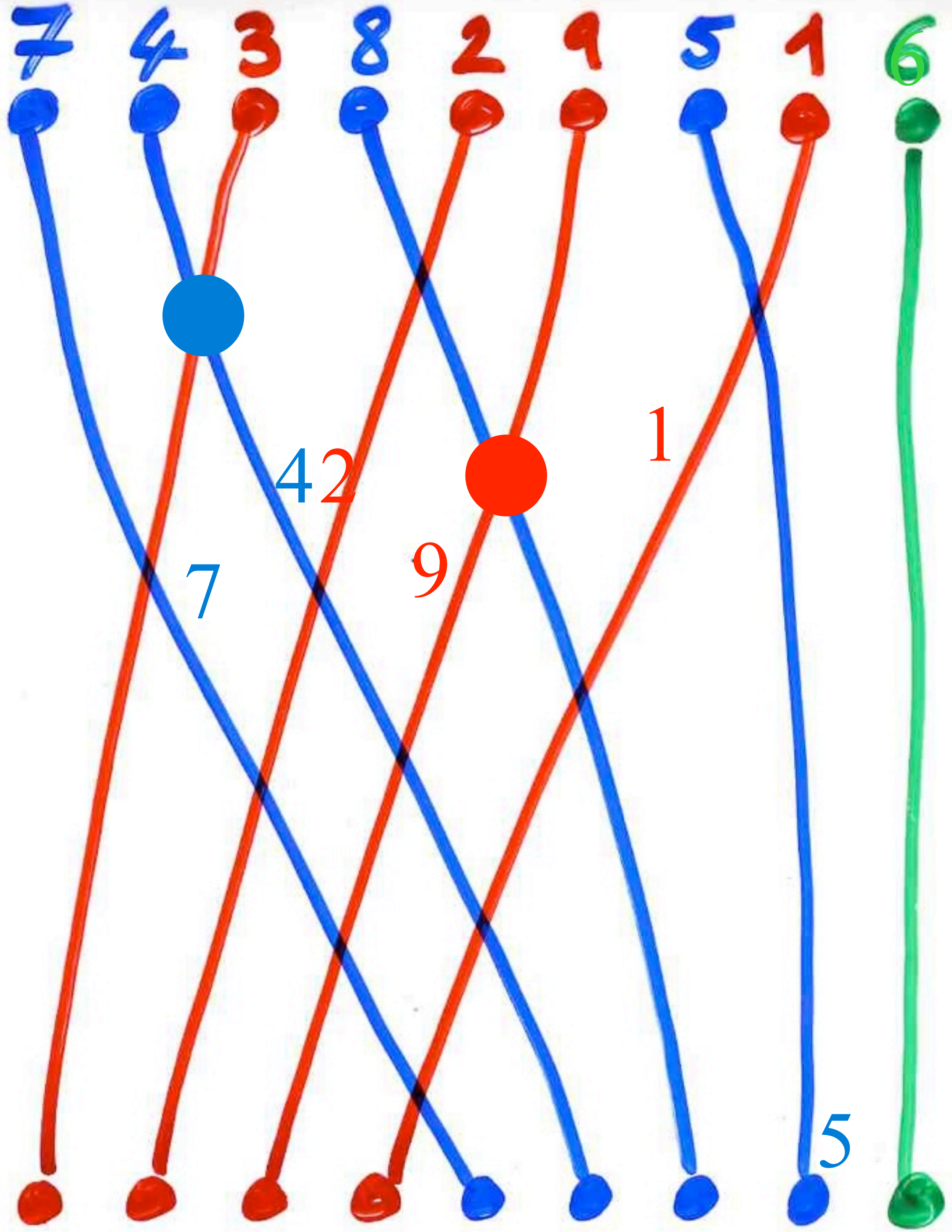
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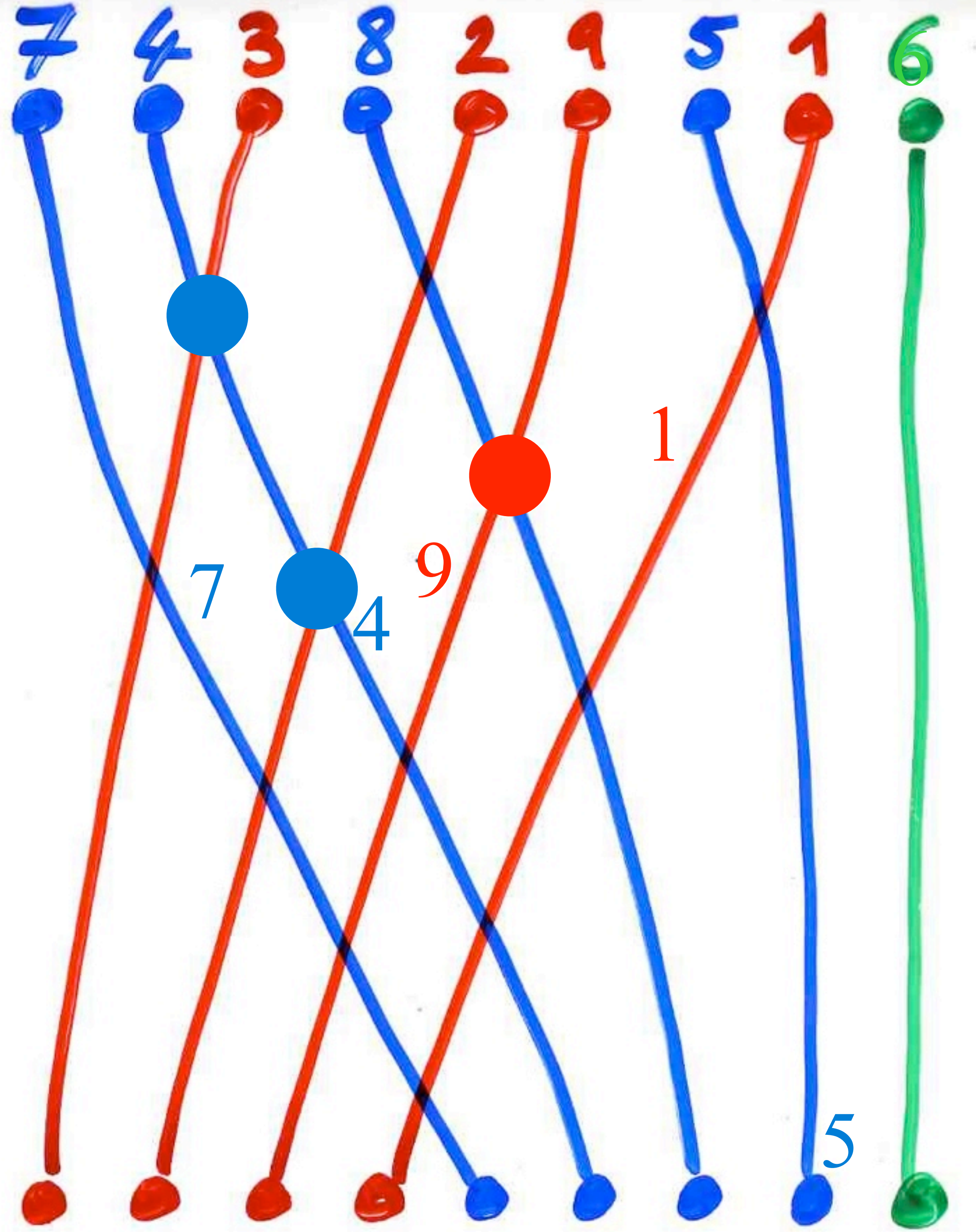

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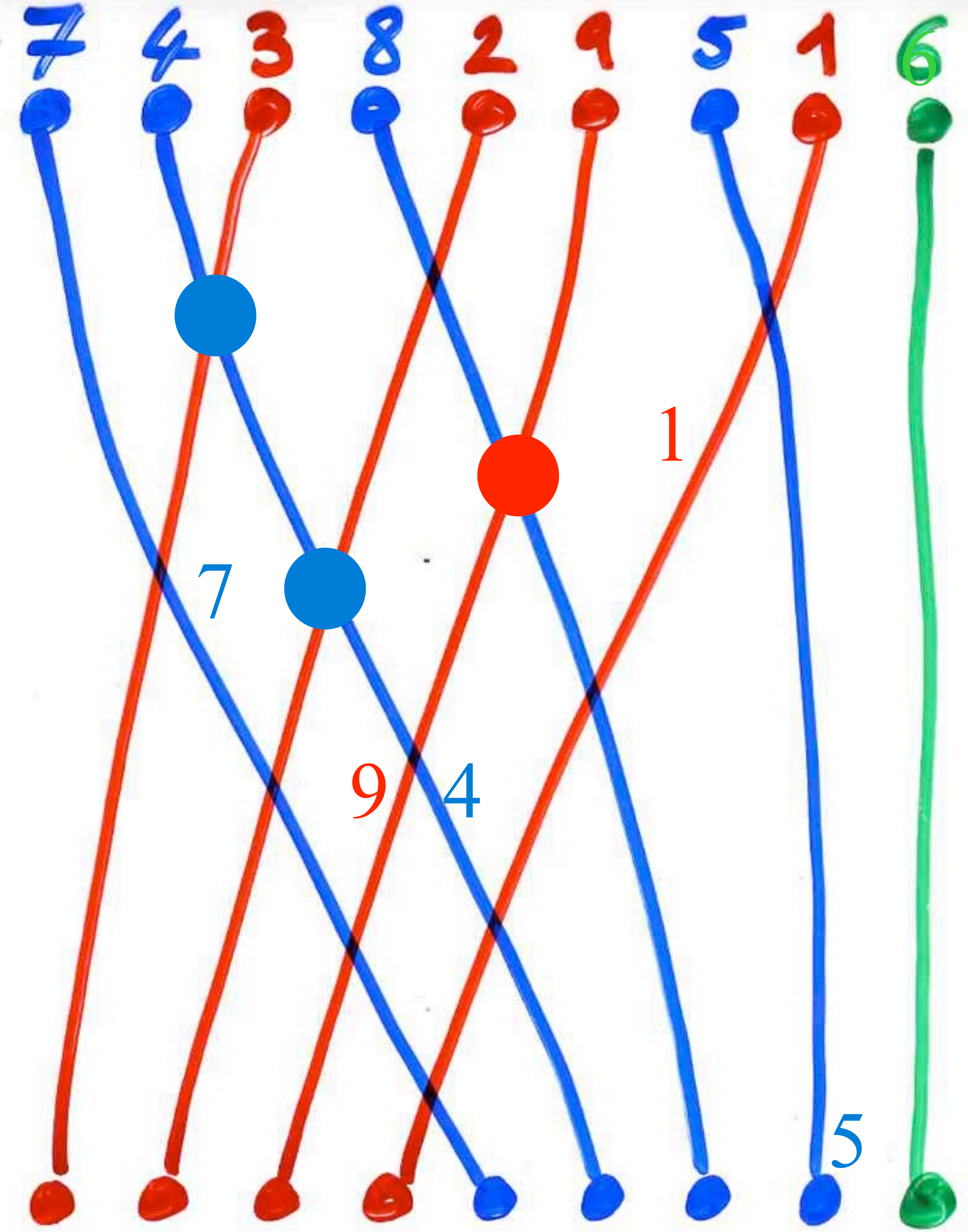

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7
9 4
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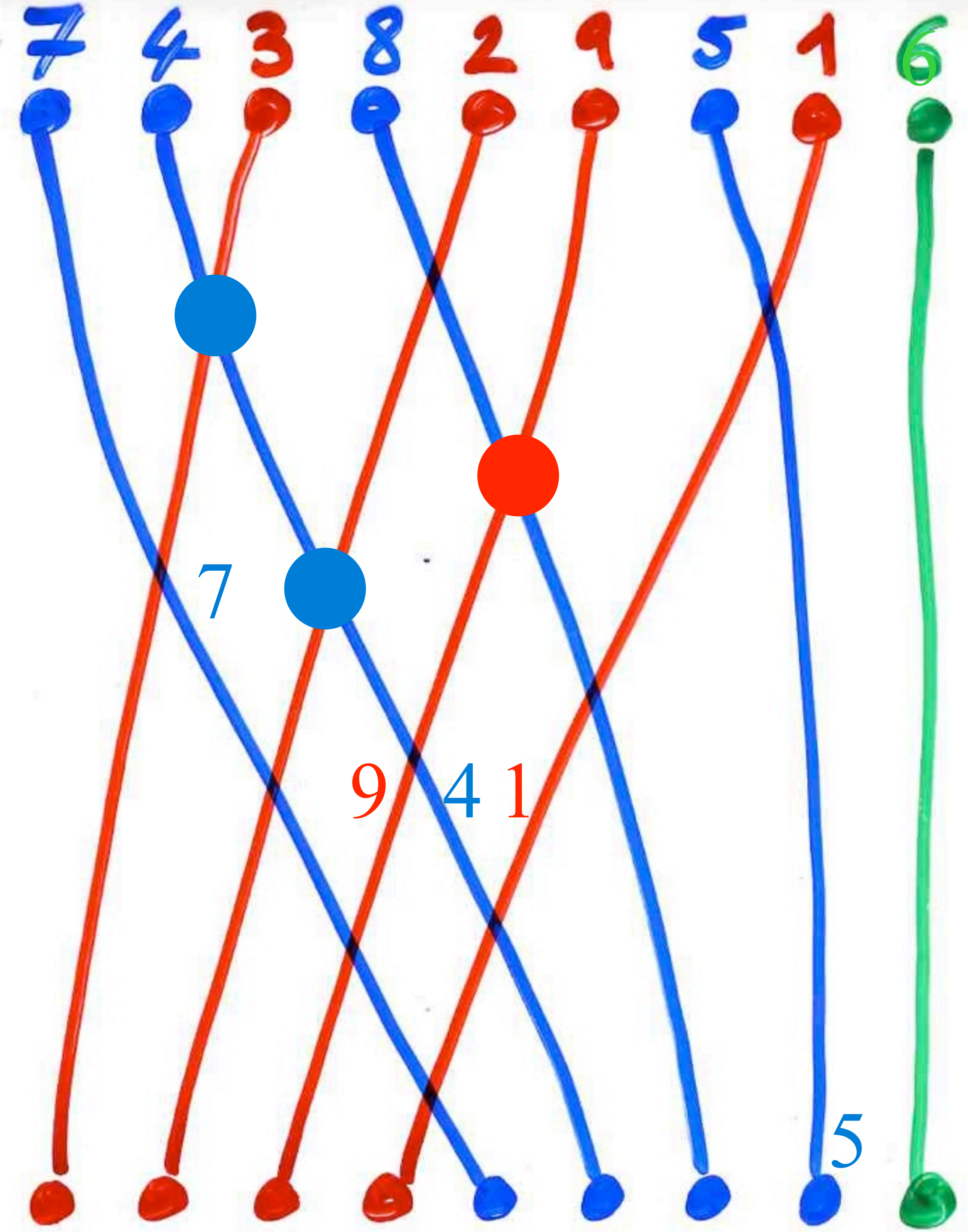
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9 4 1
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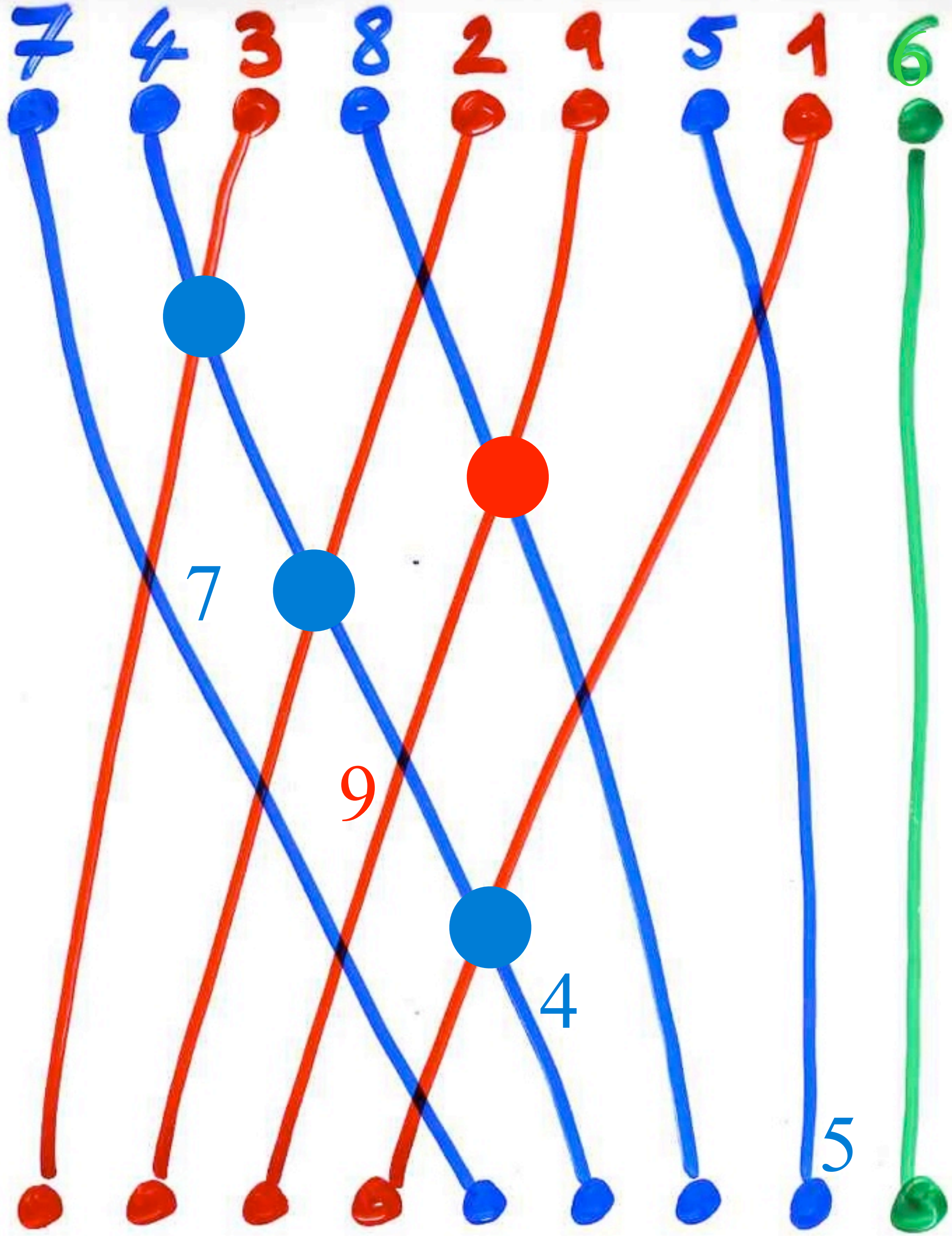

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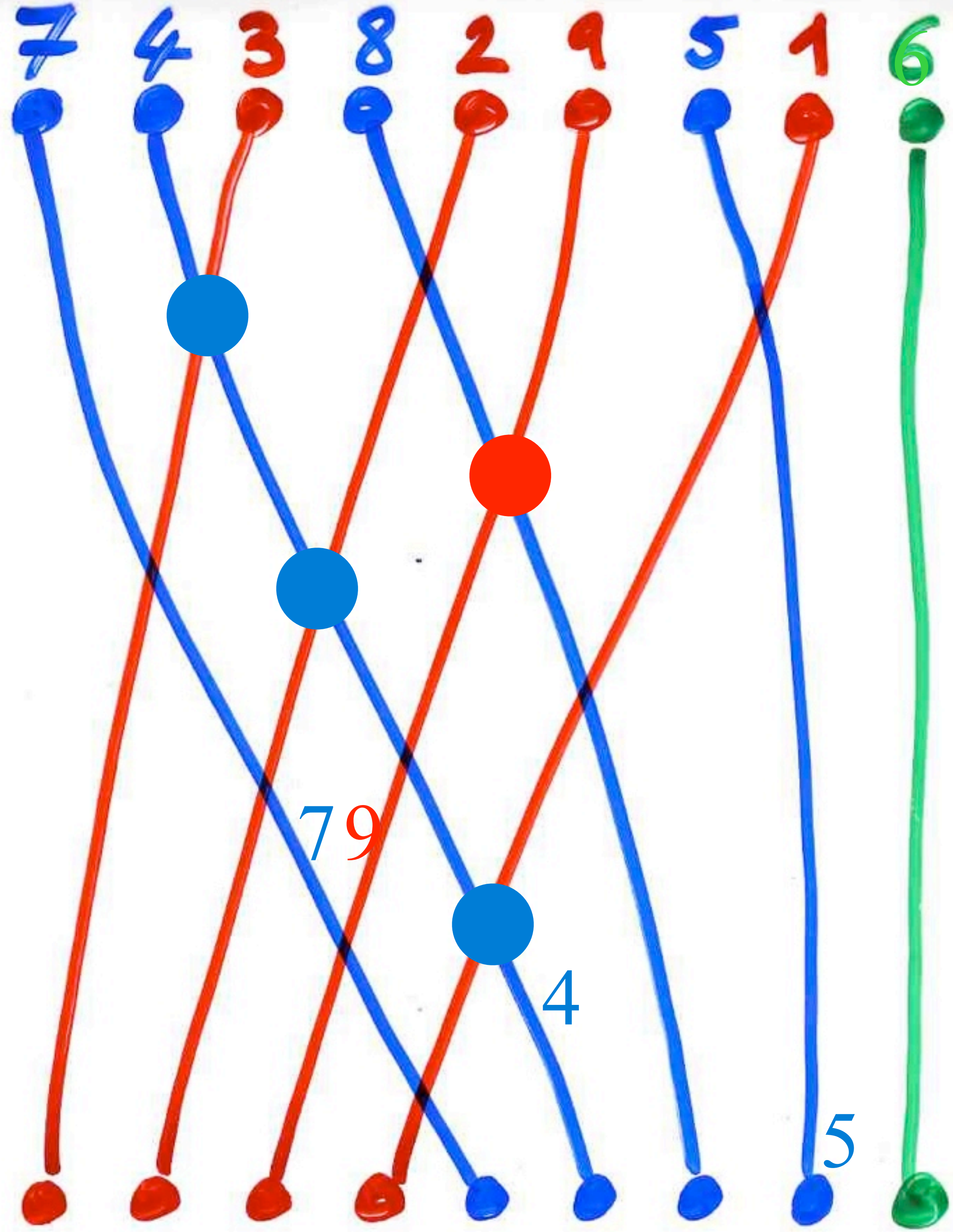
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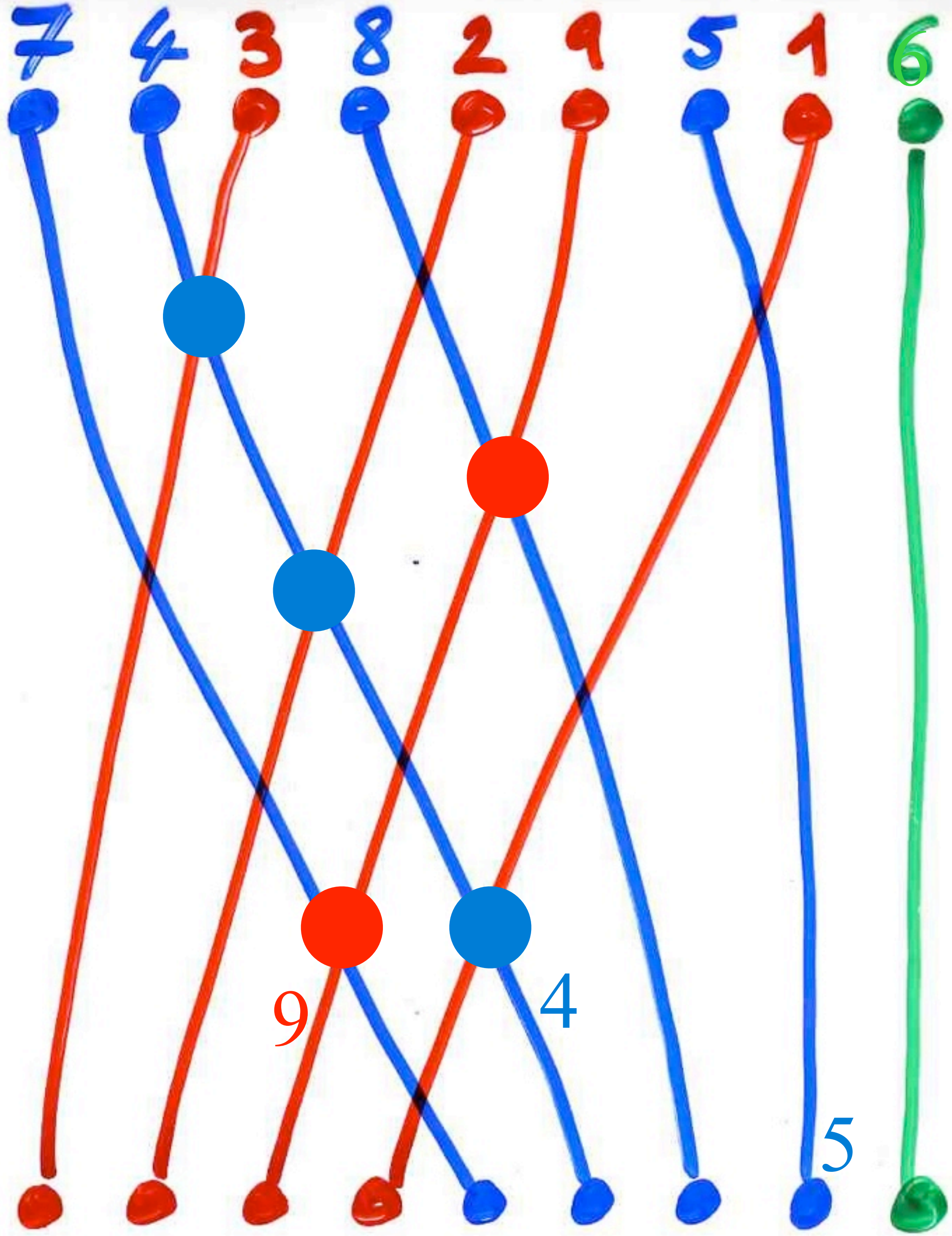
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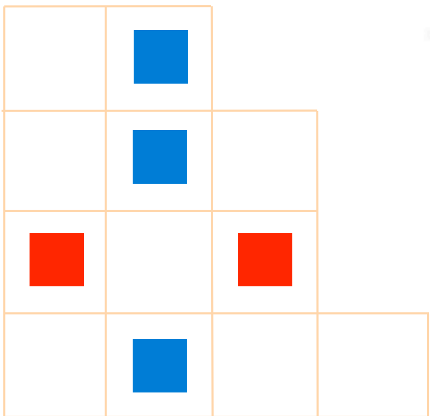
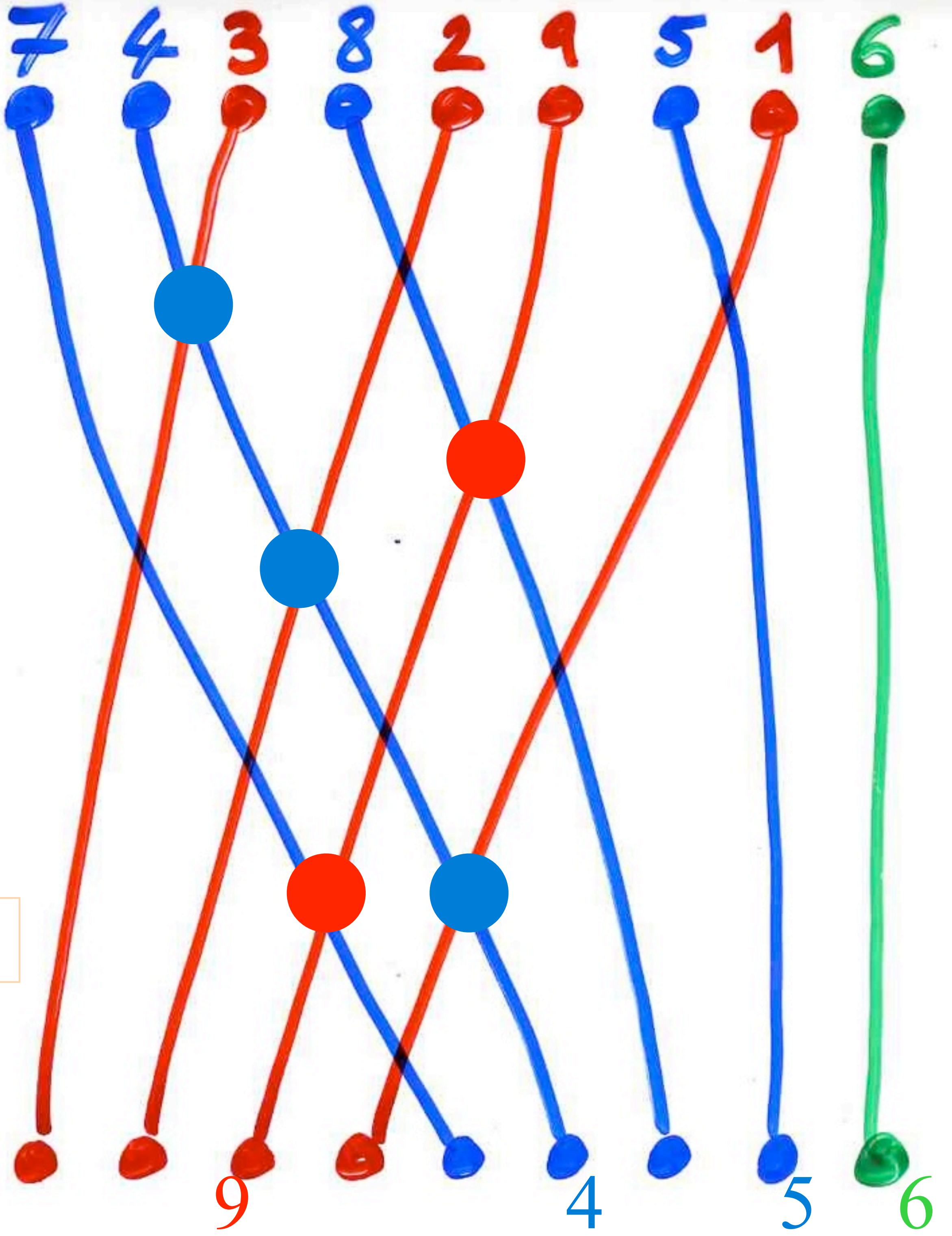
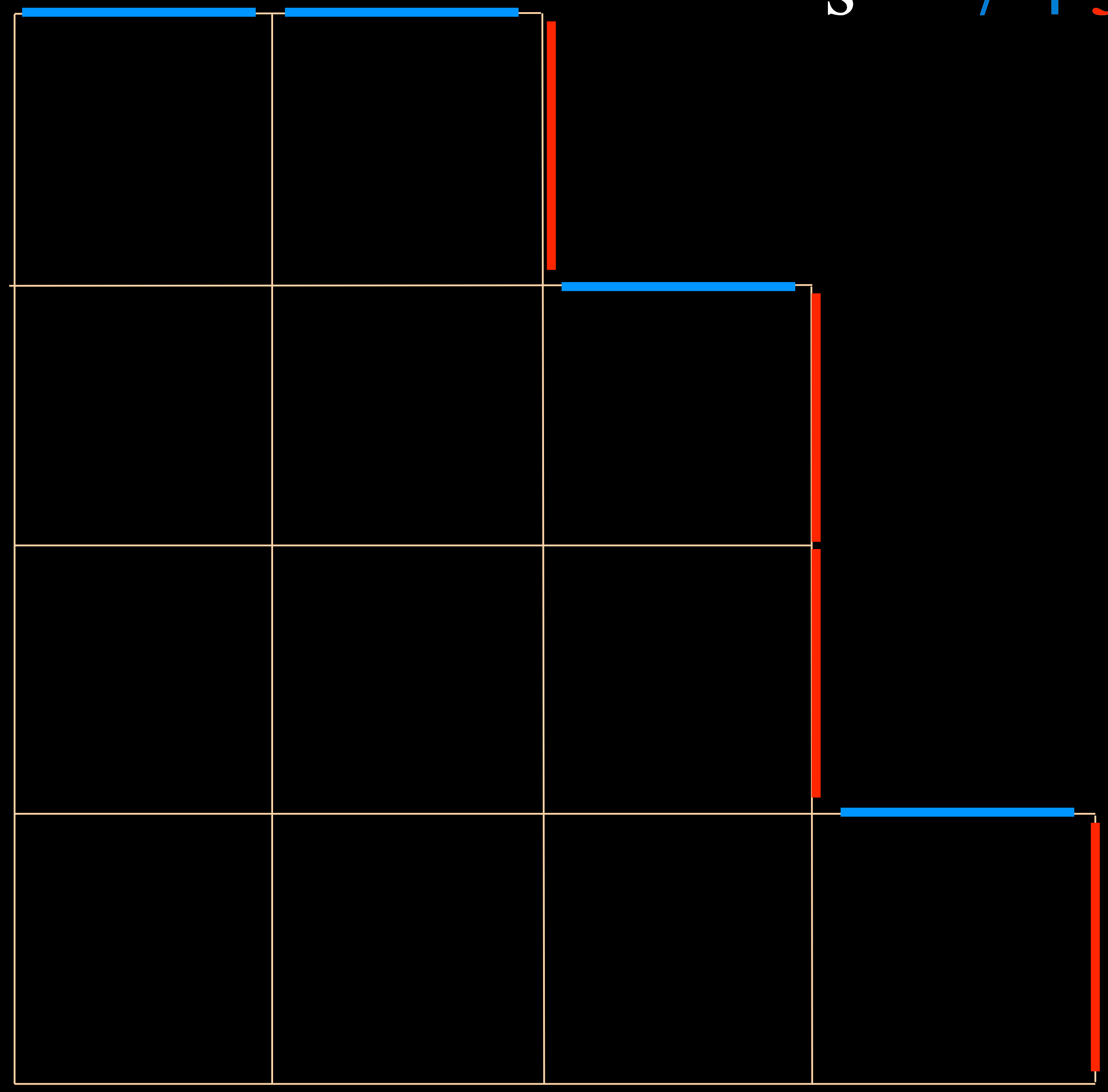
“exchange-deletion” algorithm



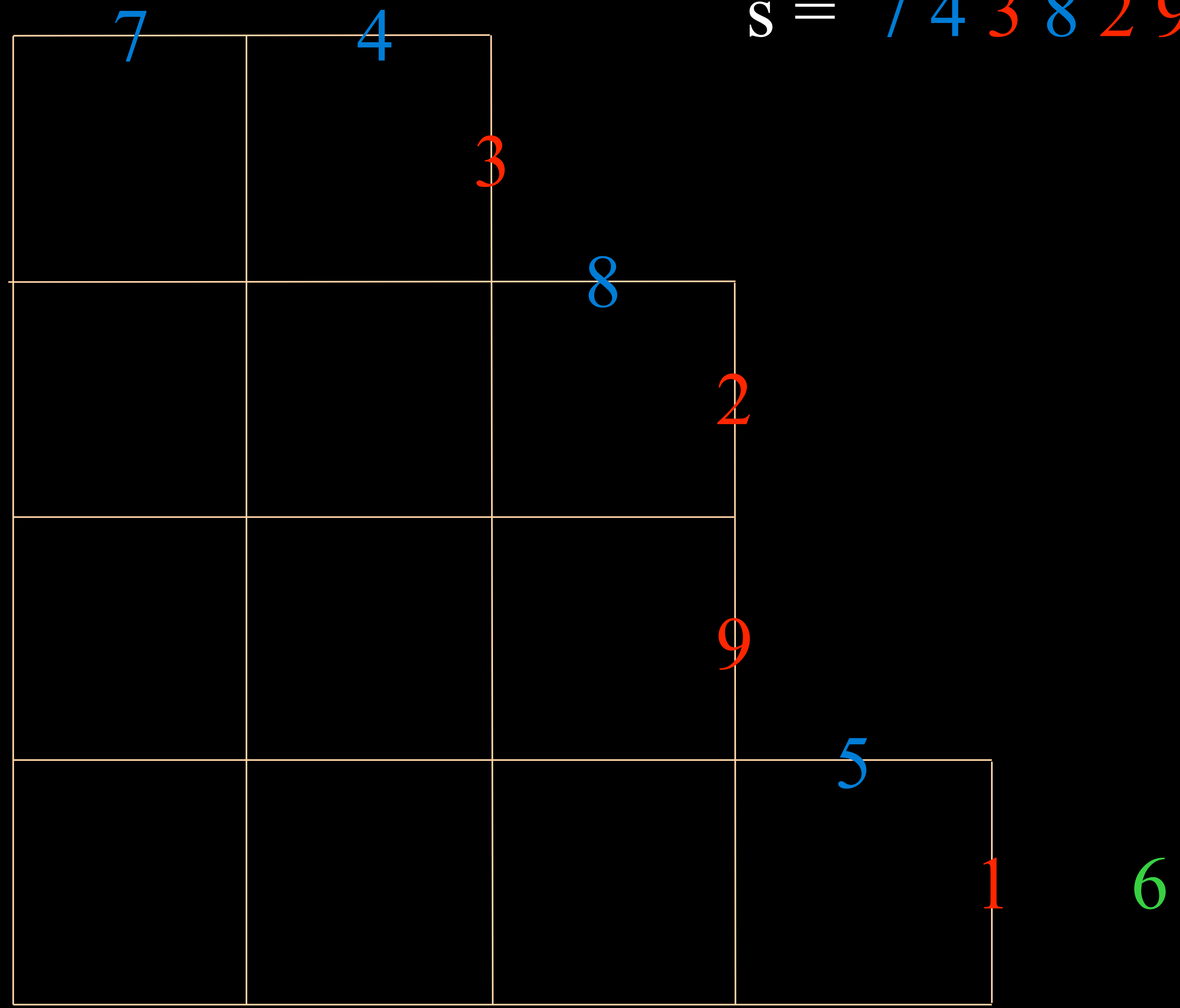


an alternative
“jeu de taquin”

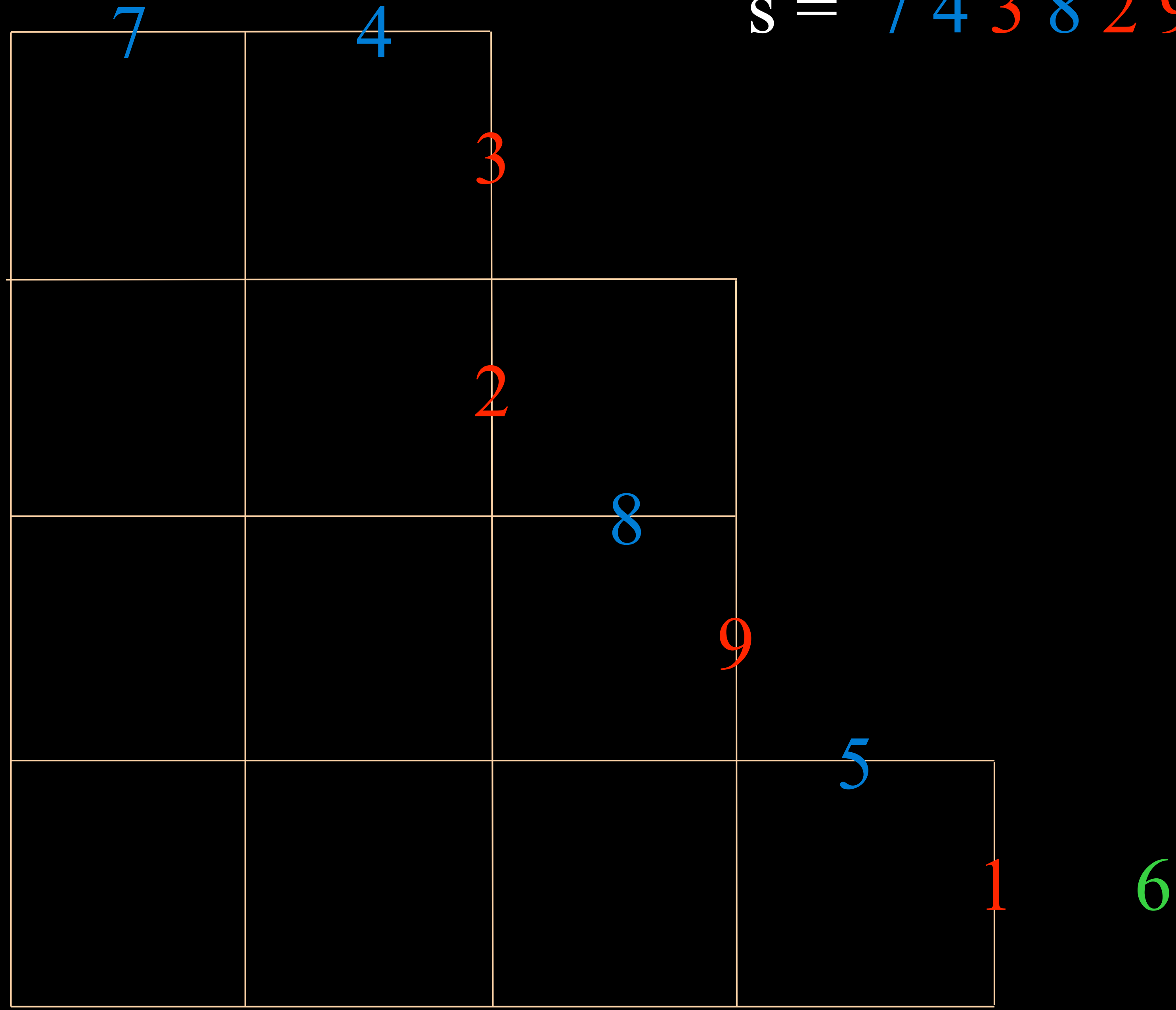
$s = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$



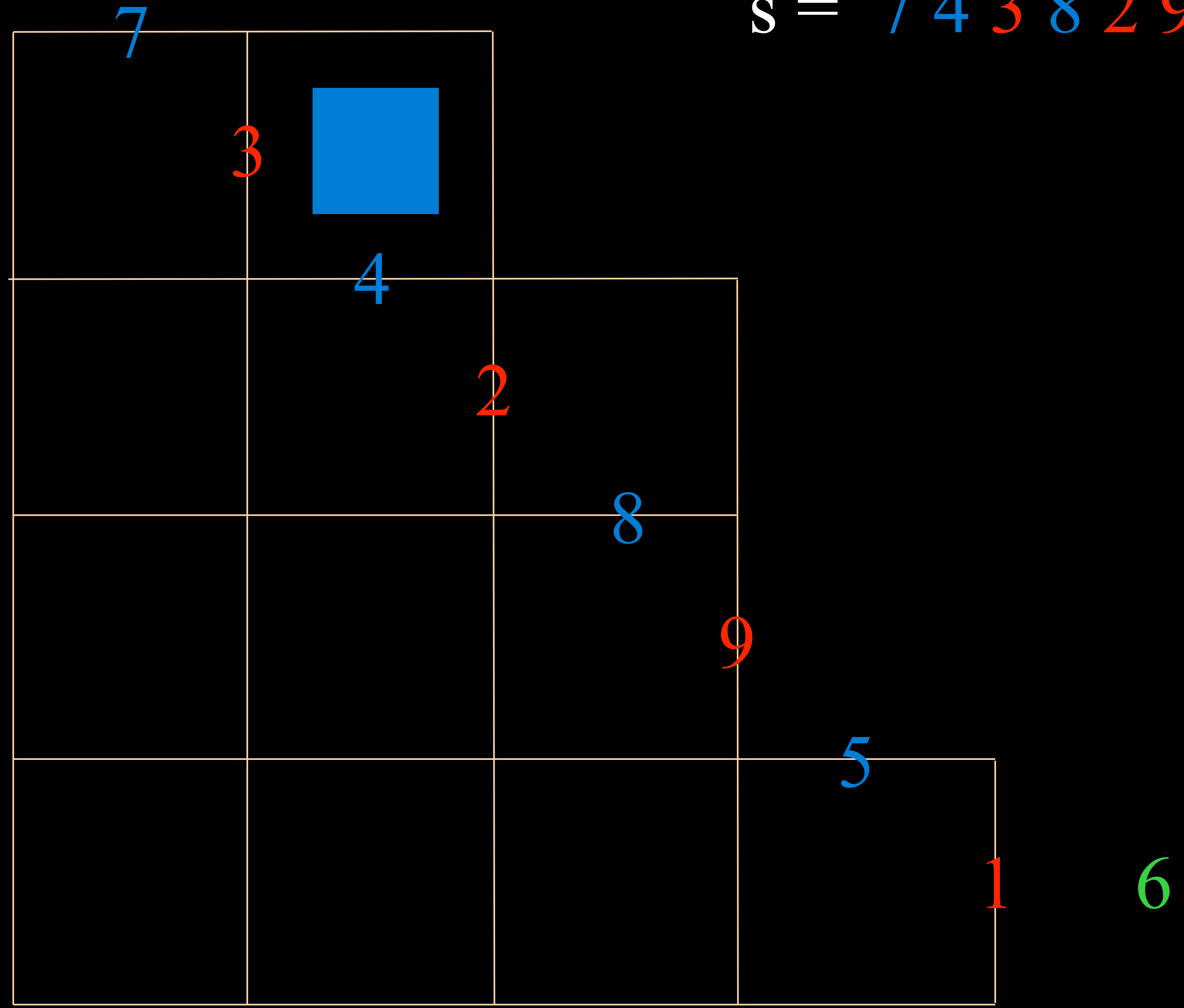
$s = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$



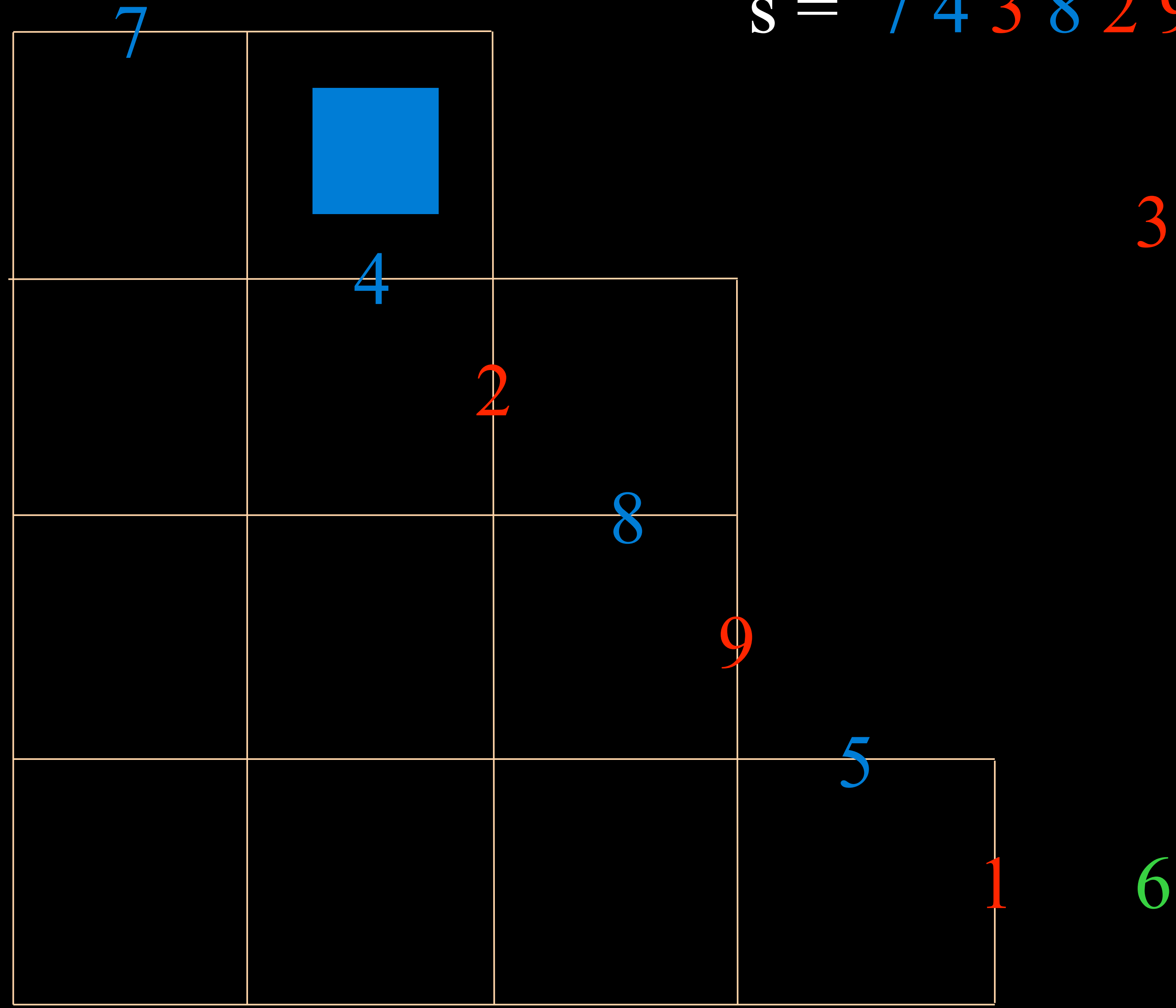
$s = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$



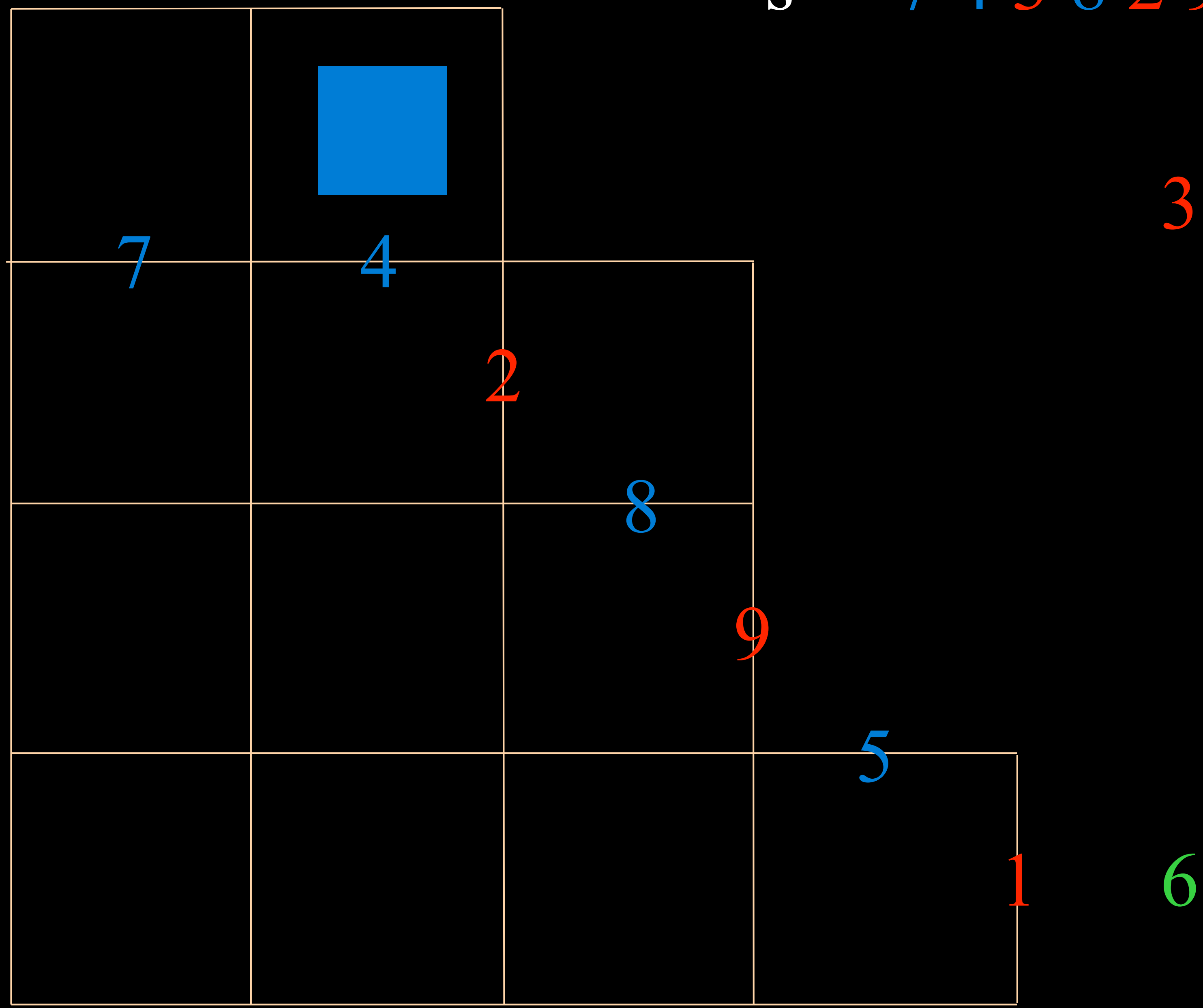
s = 7 4 3 8 2 9 5 1 6



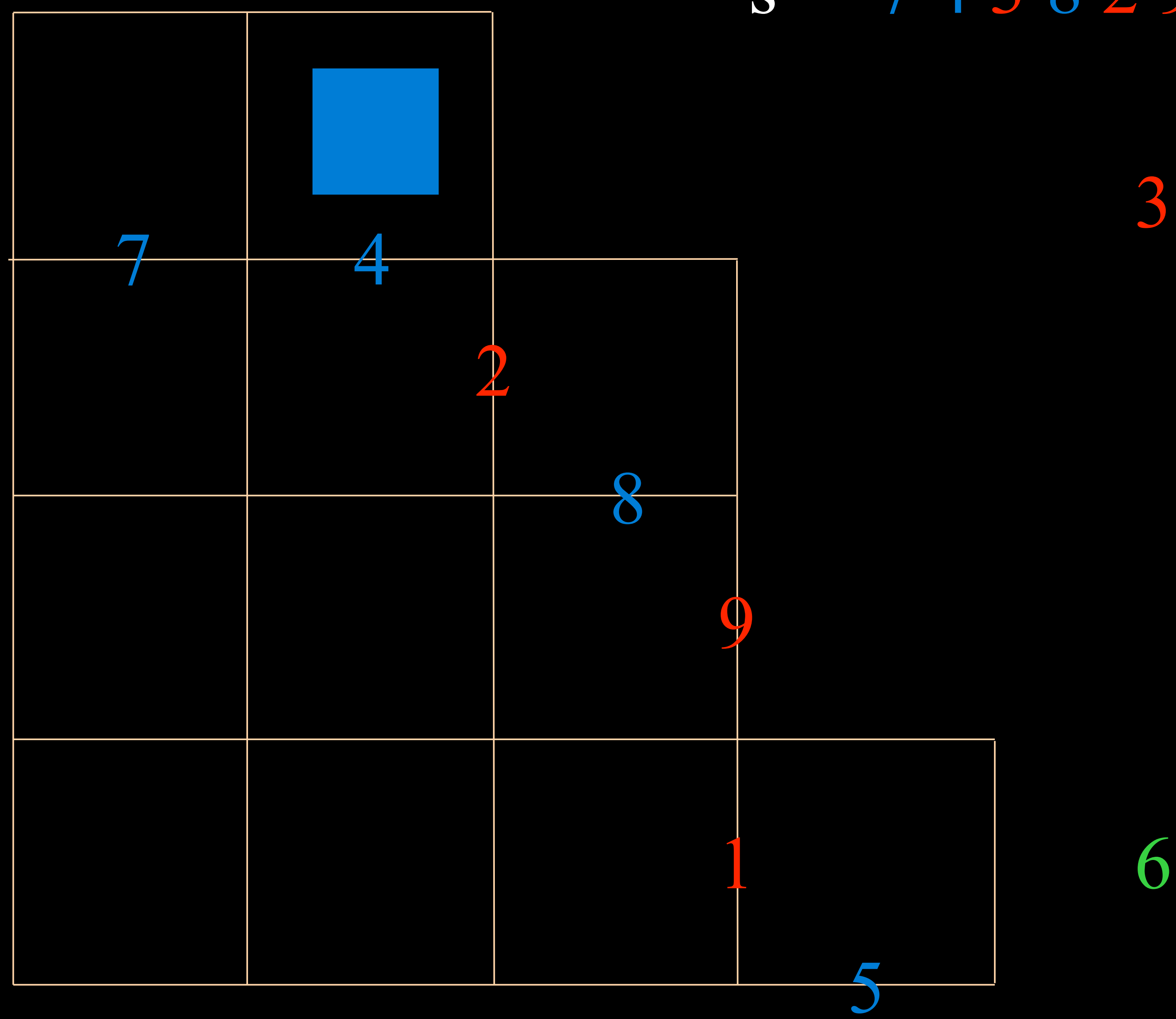
$s = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$



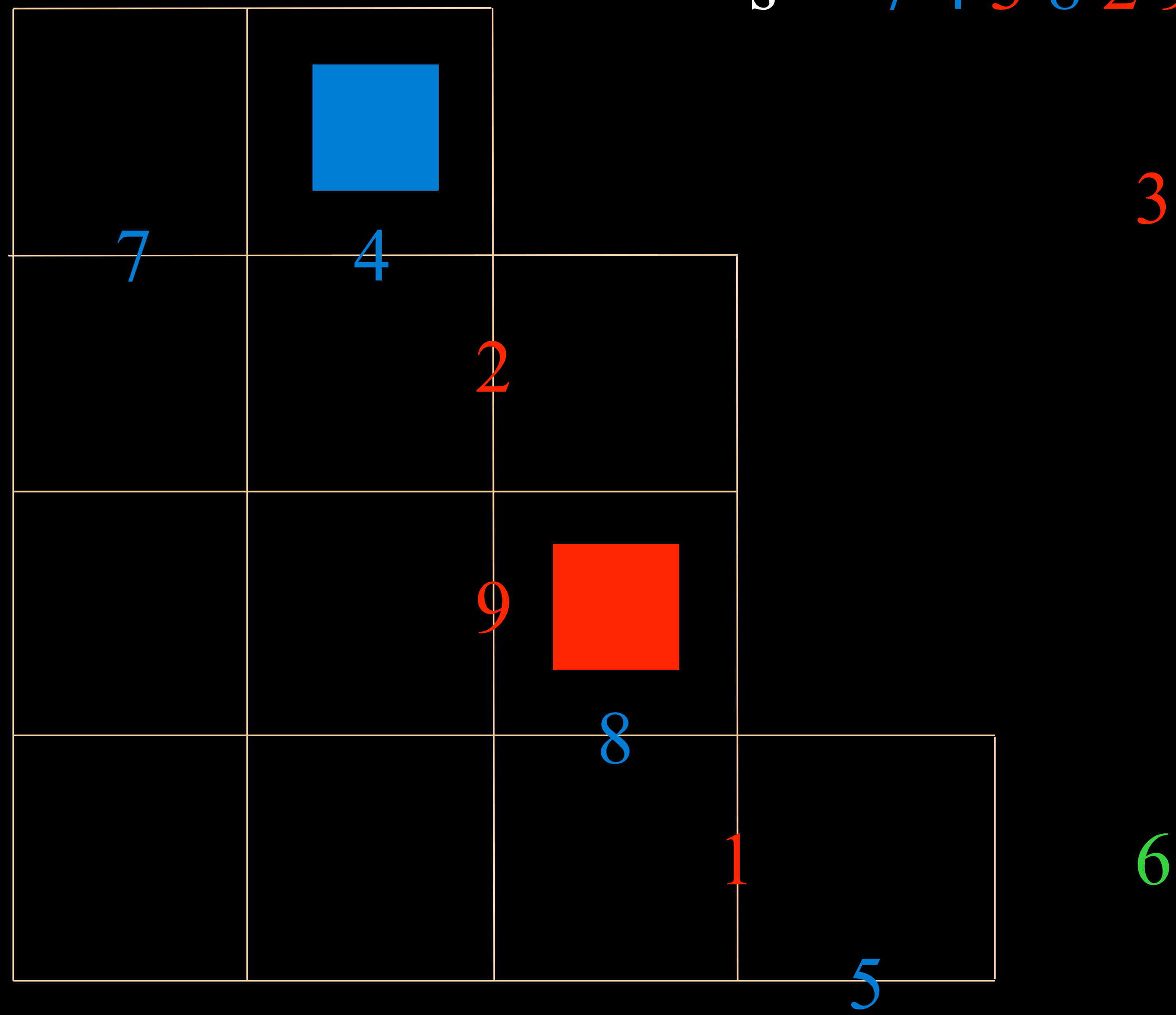
s = 7 4 3 8 2 9 5 1 6



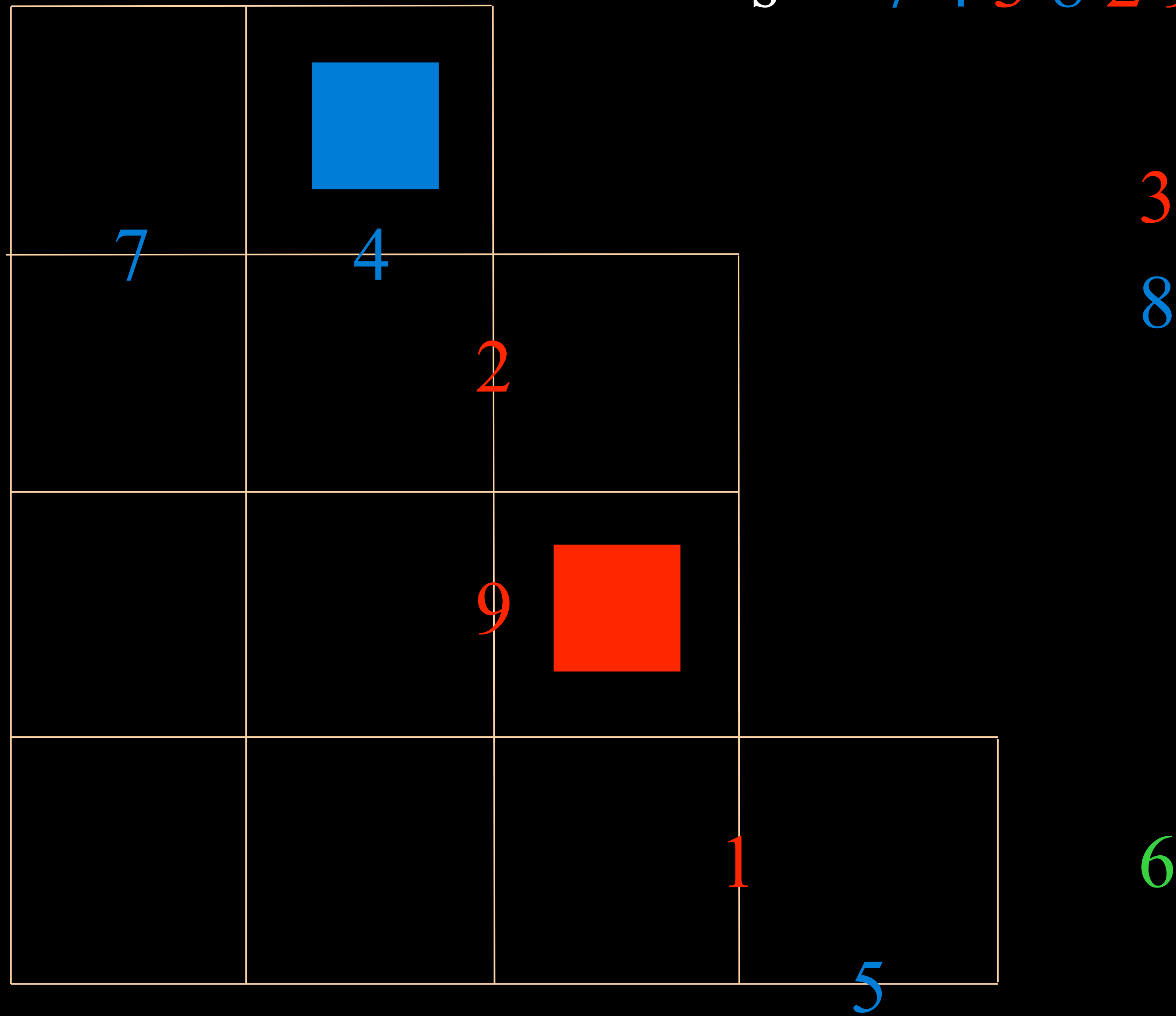
s = 7 4 3 8 2 9 5 1 6



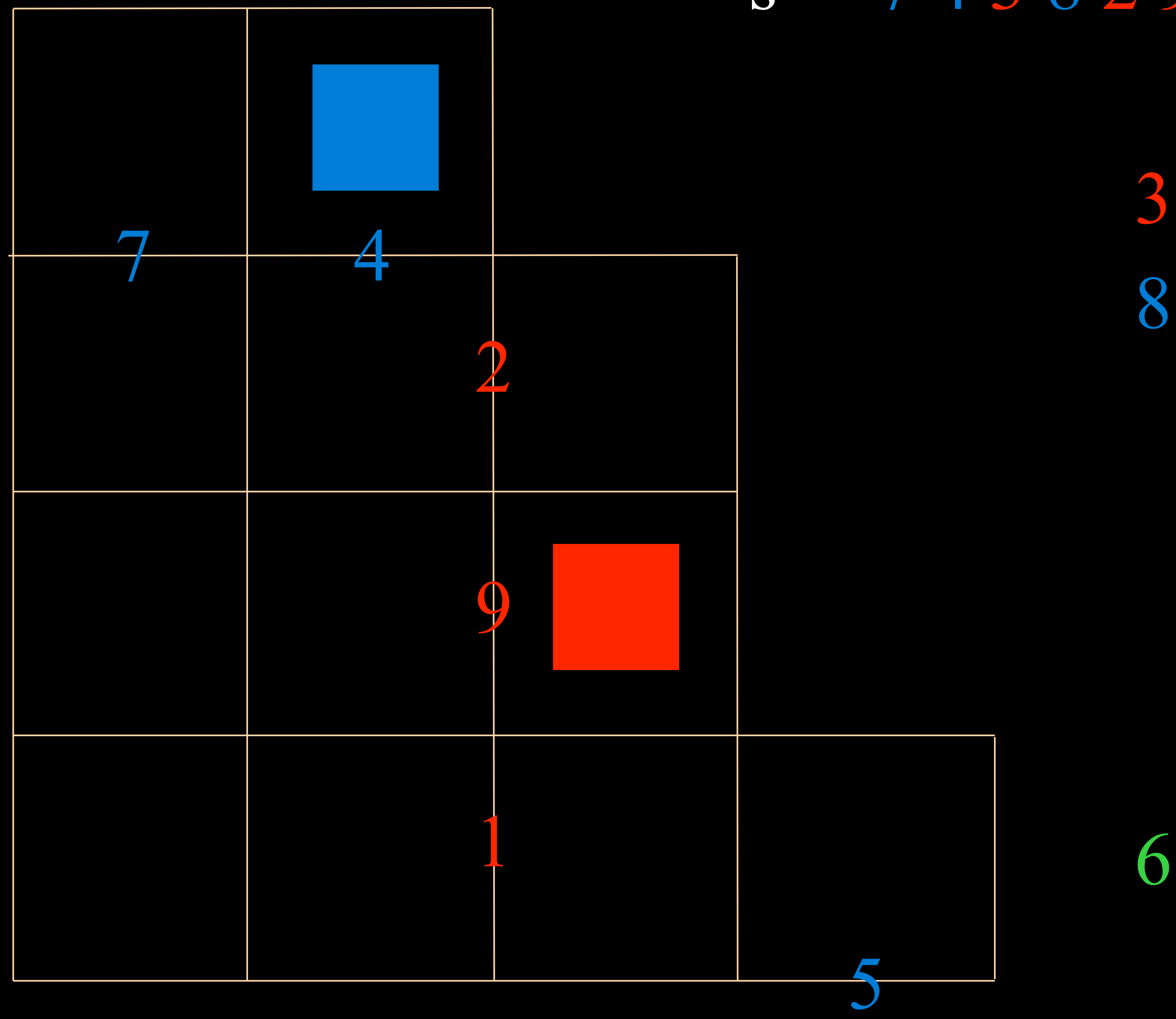
$s = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$



$s = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$



$s = 7\ 4\ 3\ 8\ 2\ 9\ 5\ 1\ 6$



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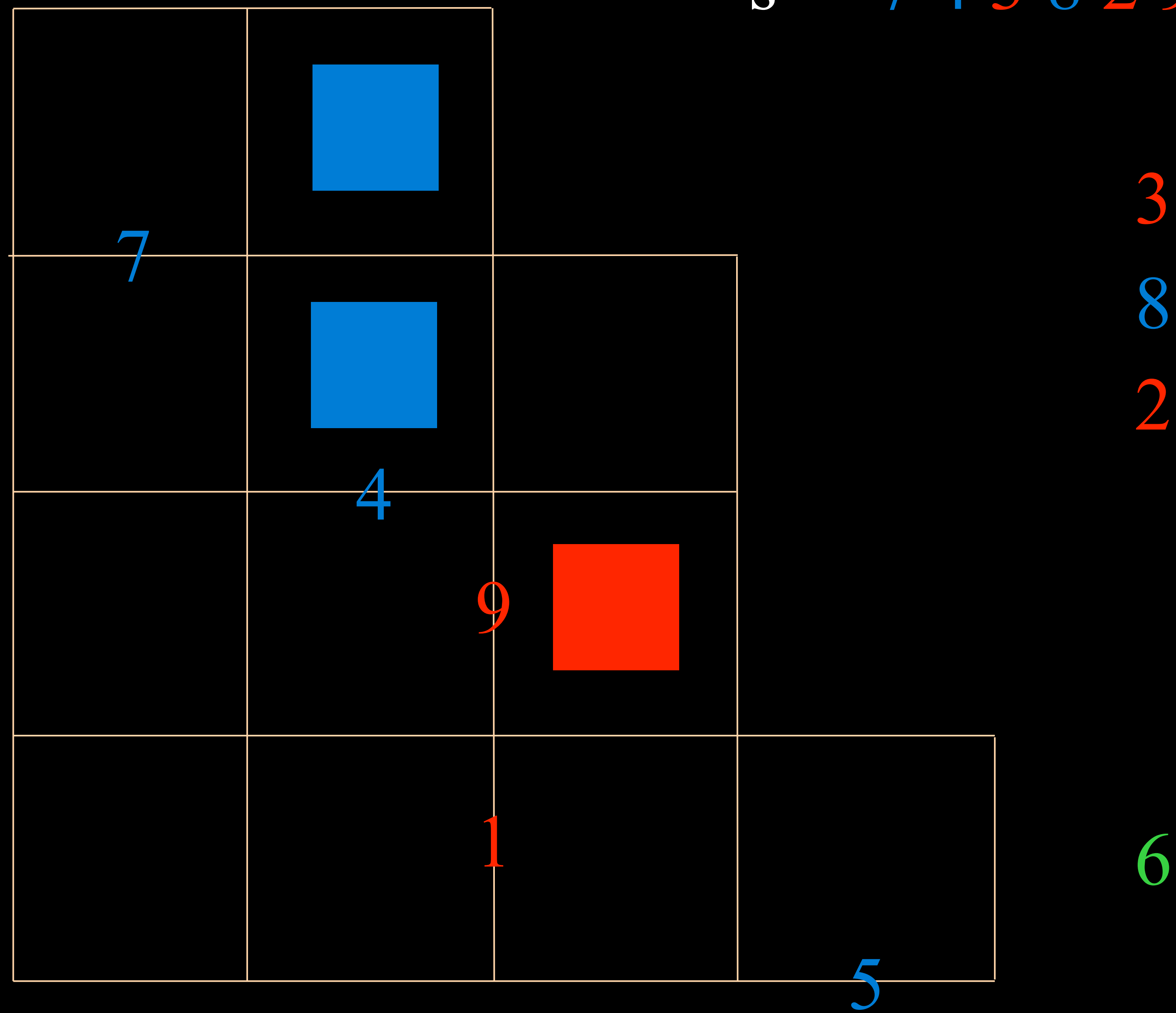

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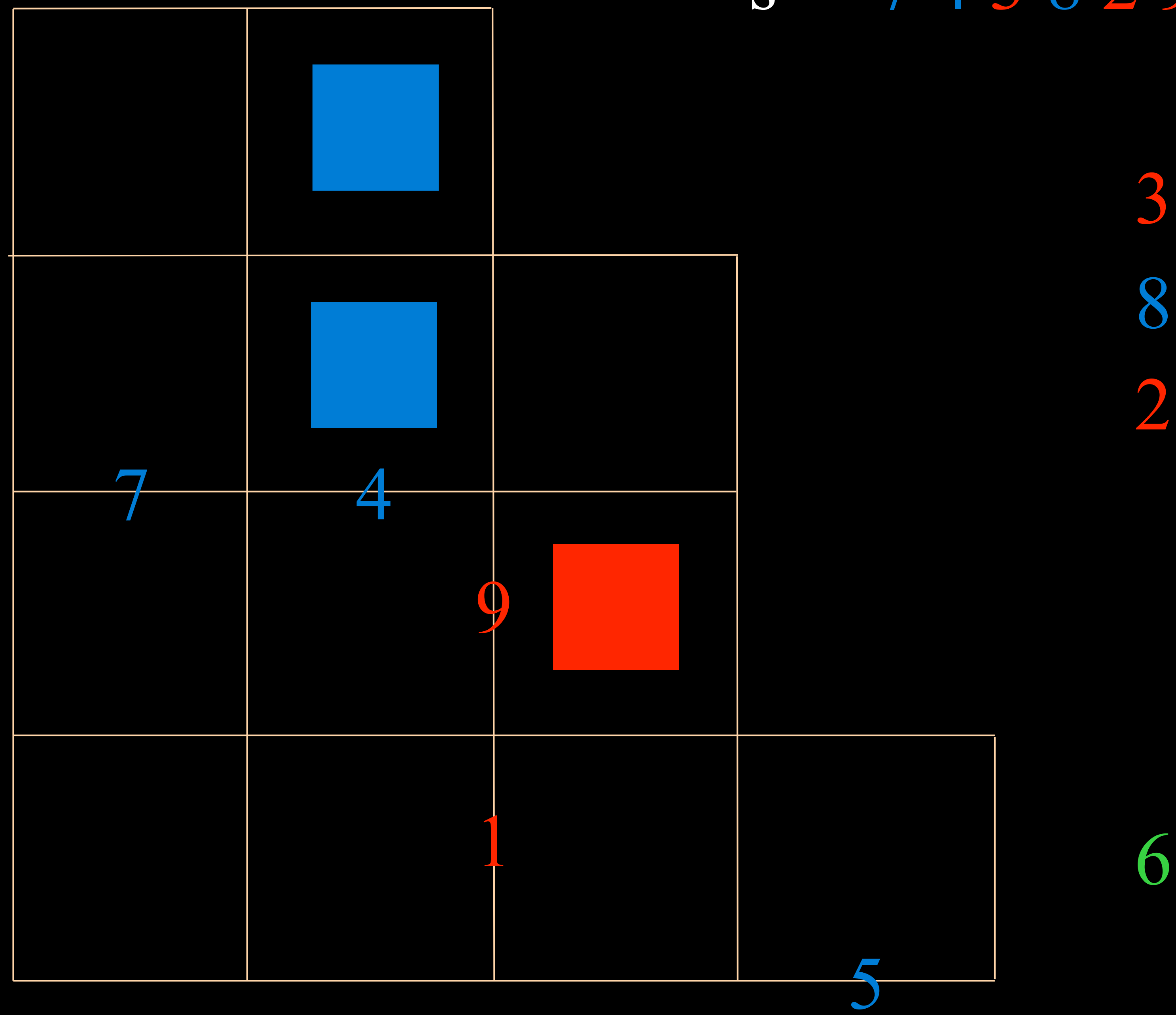

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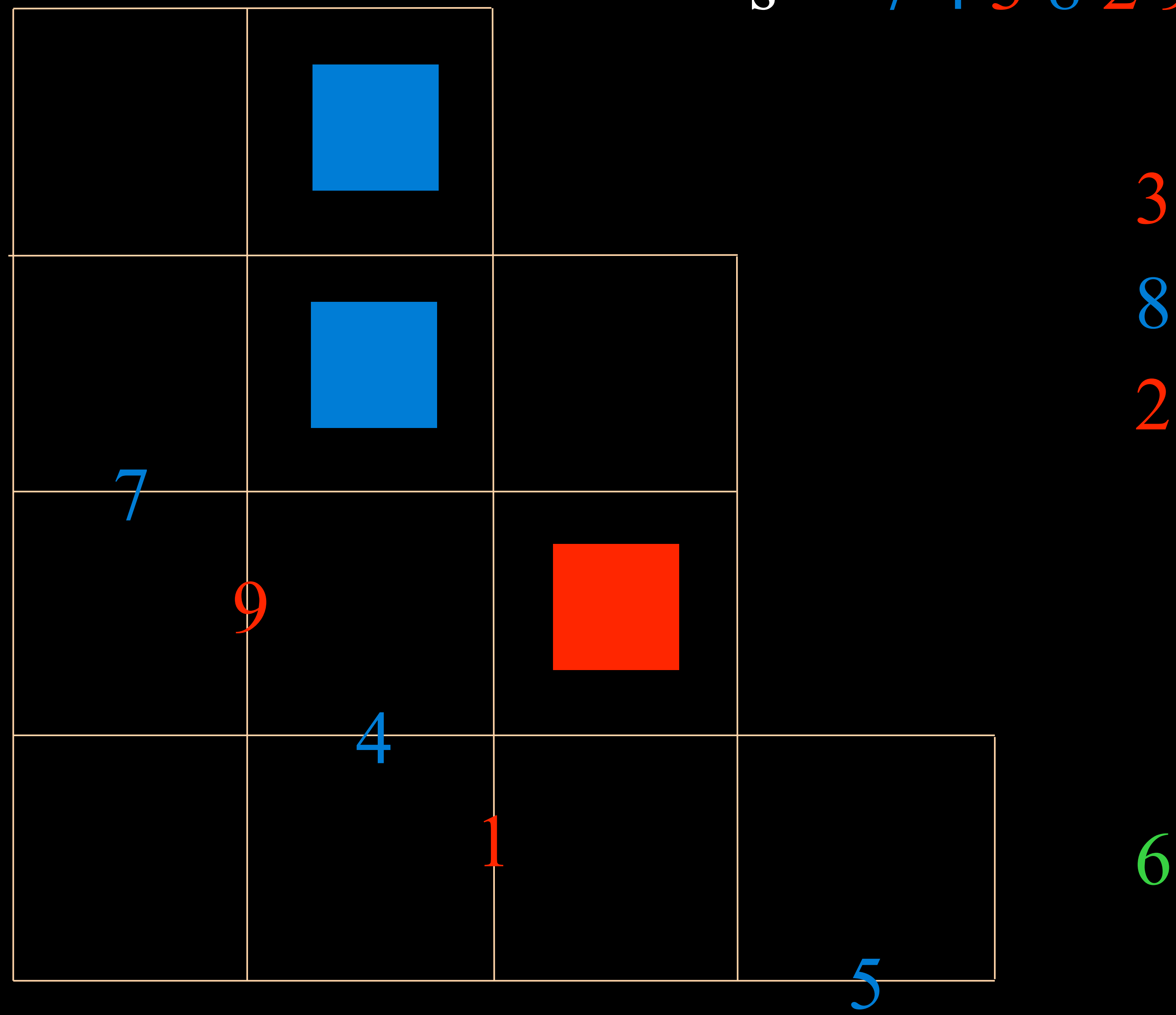

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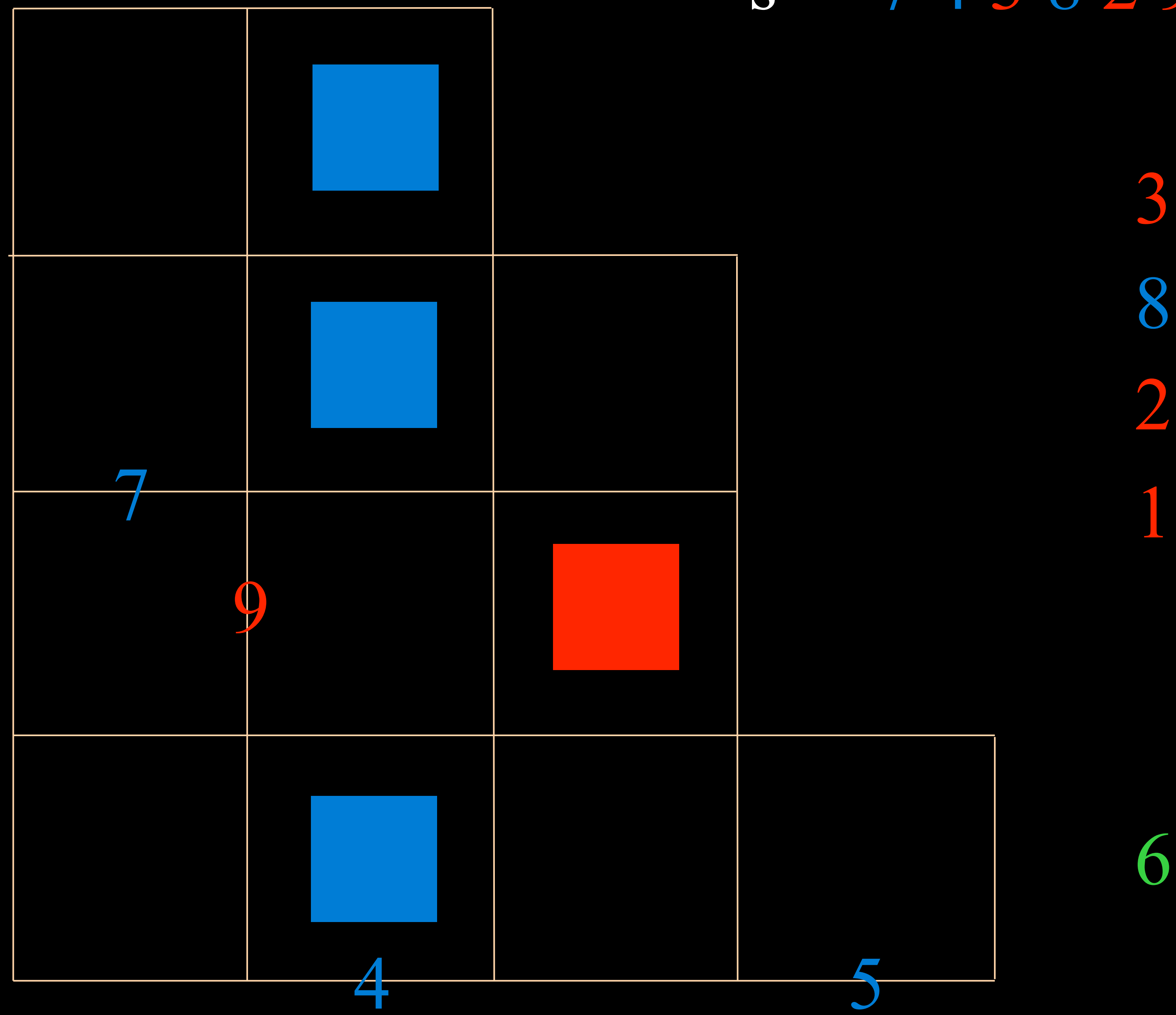

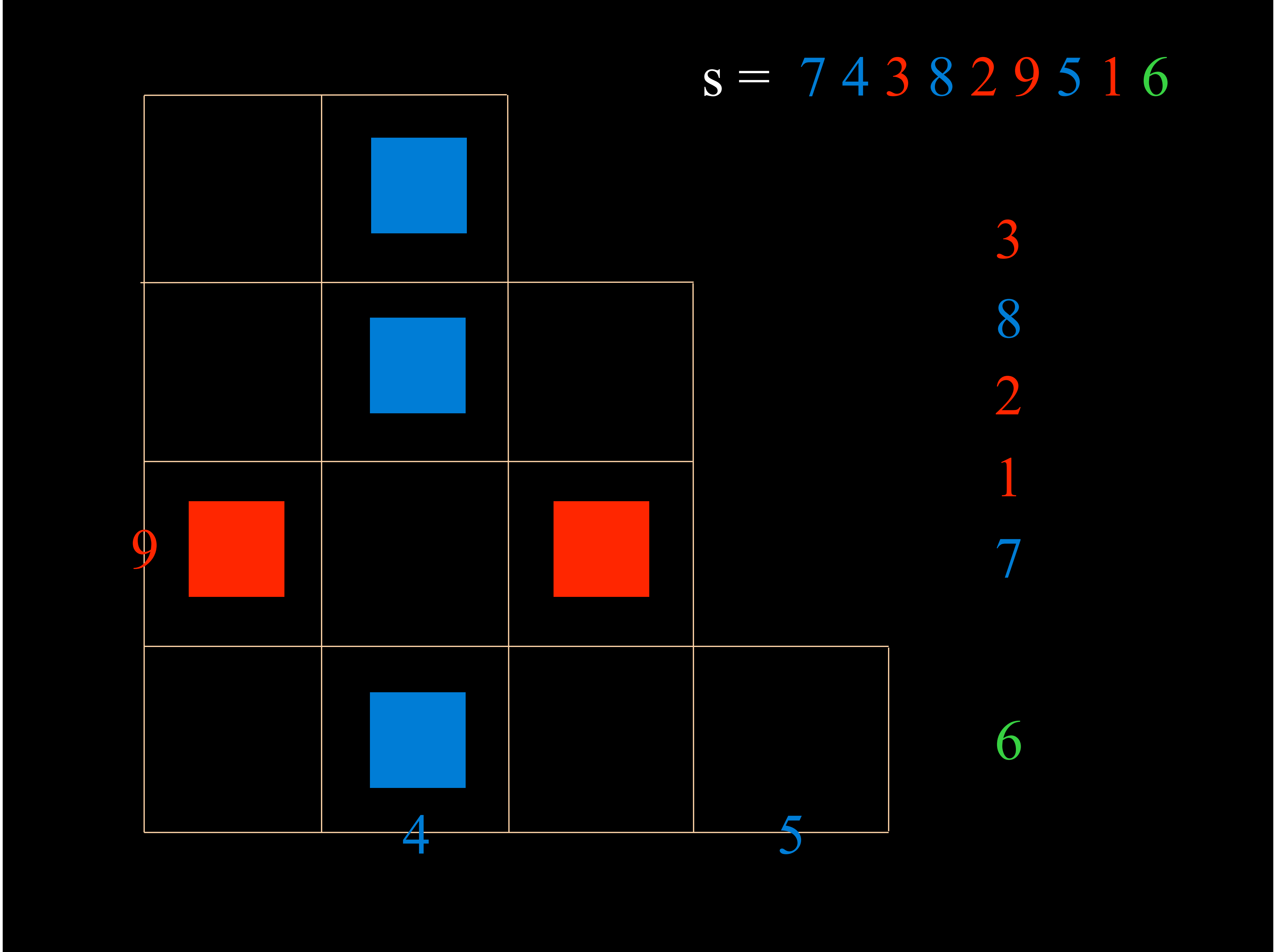
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3
8
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7
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4
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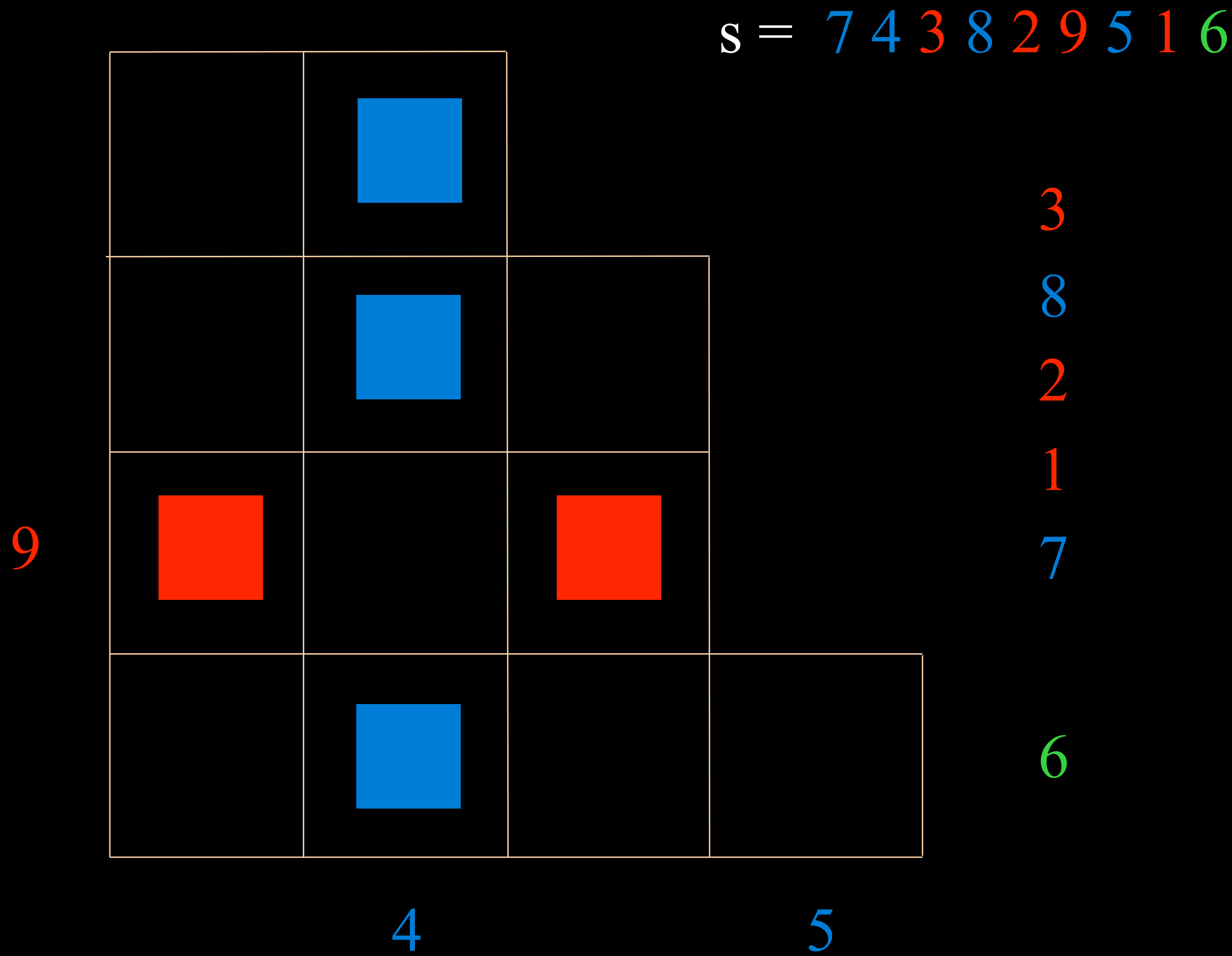
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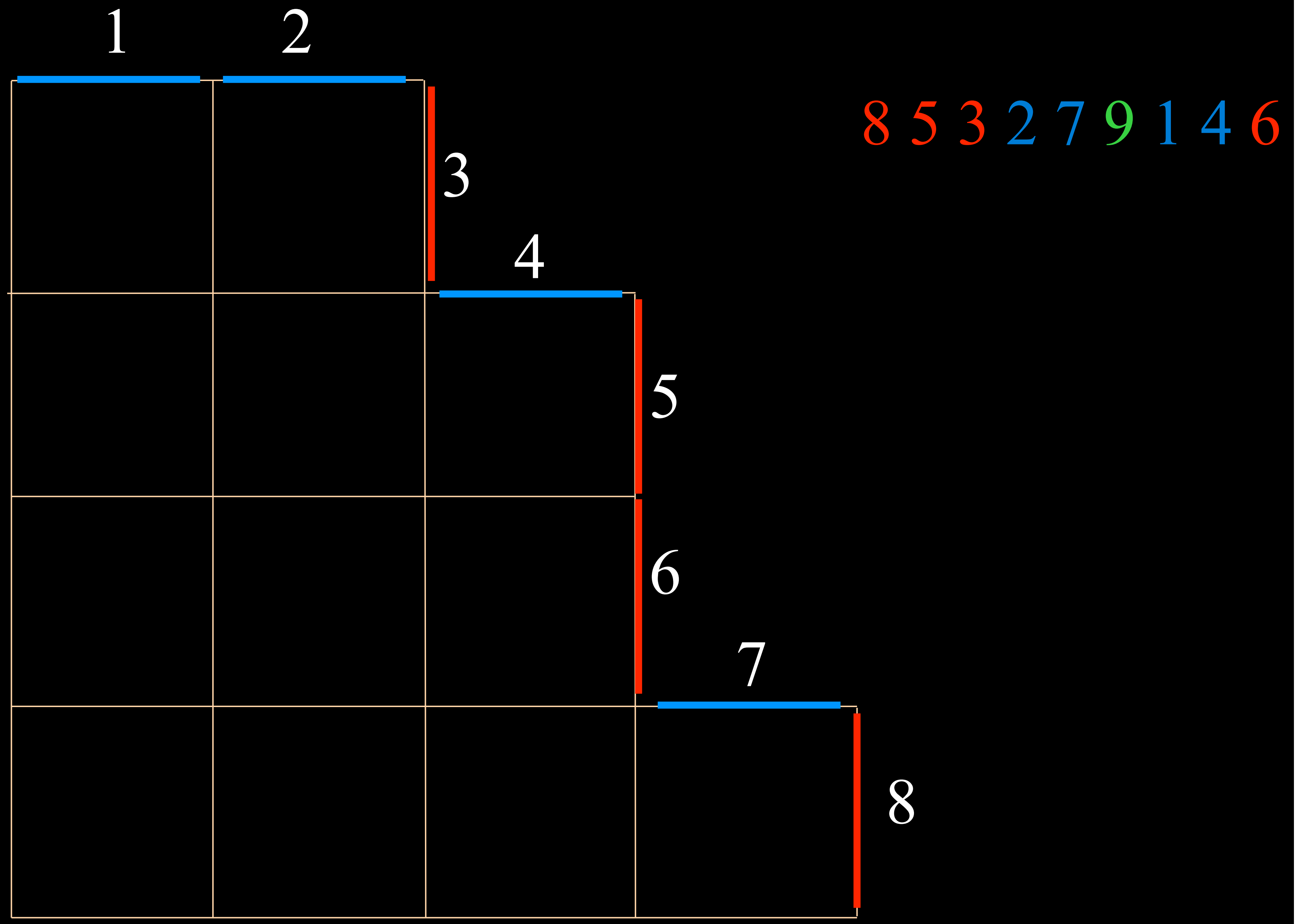
5

same example denoted with
the inverse permutation



1
2
3
4
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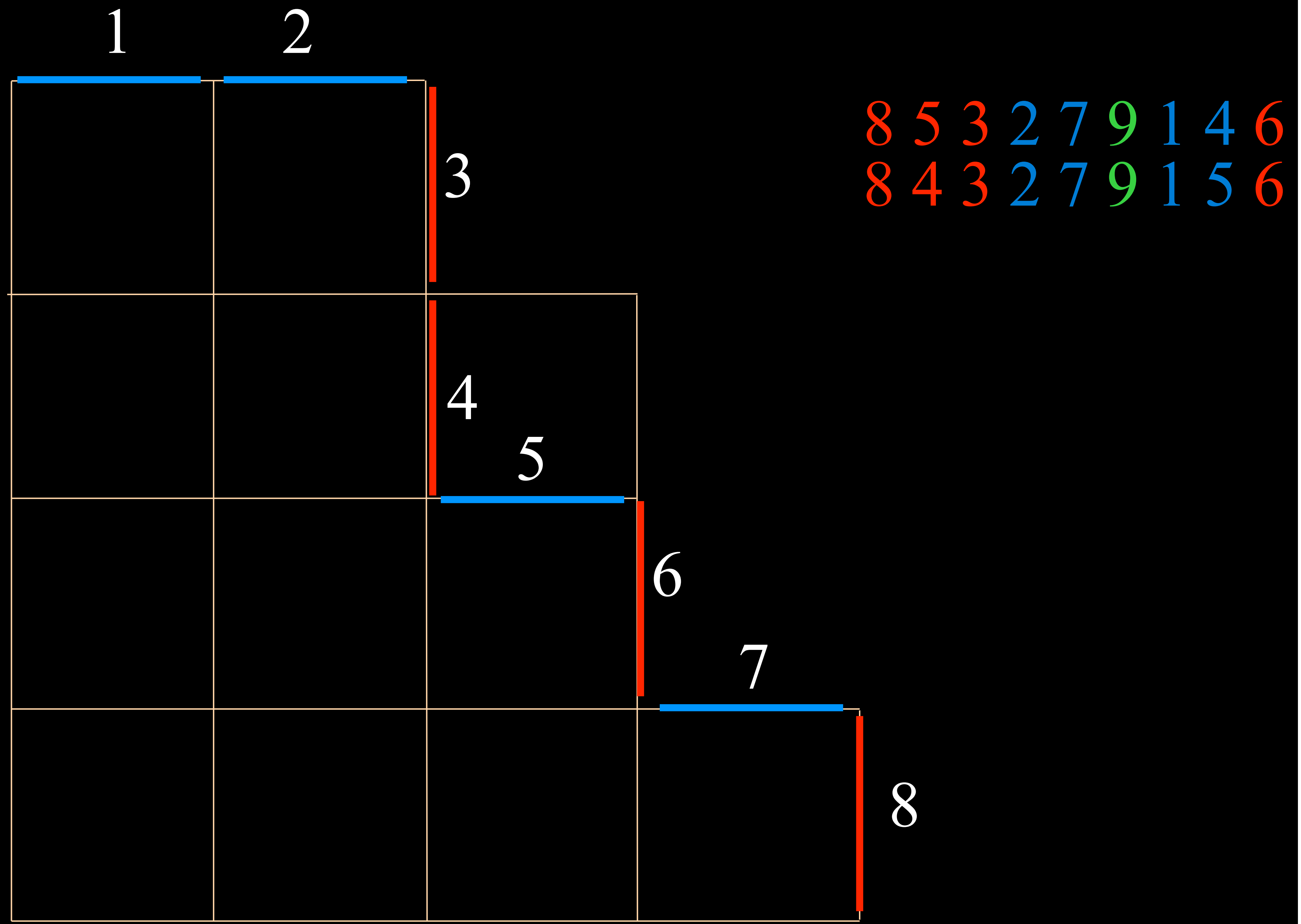
7
8
8 5 3 2 7 9 1 4 6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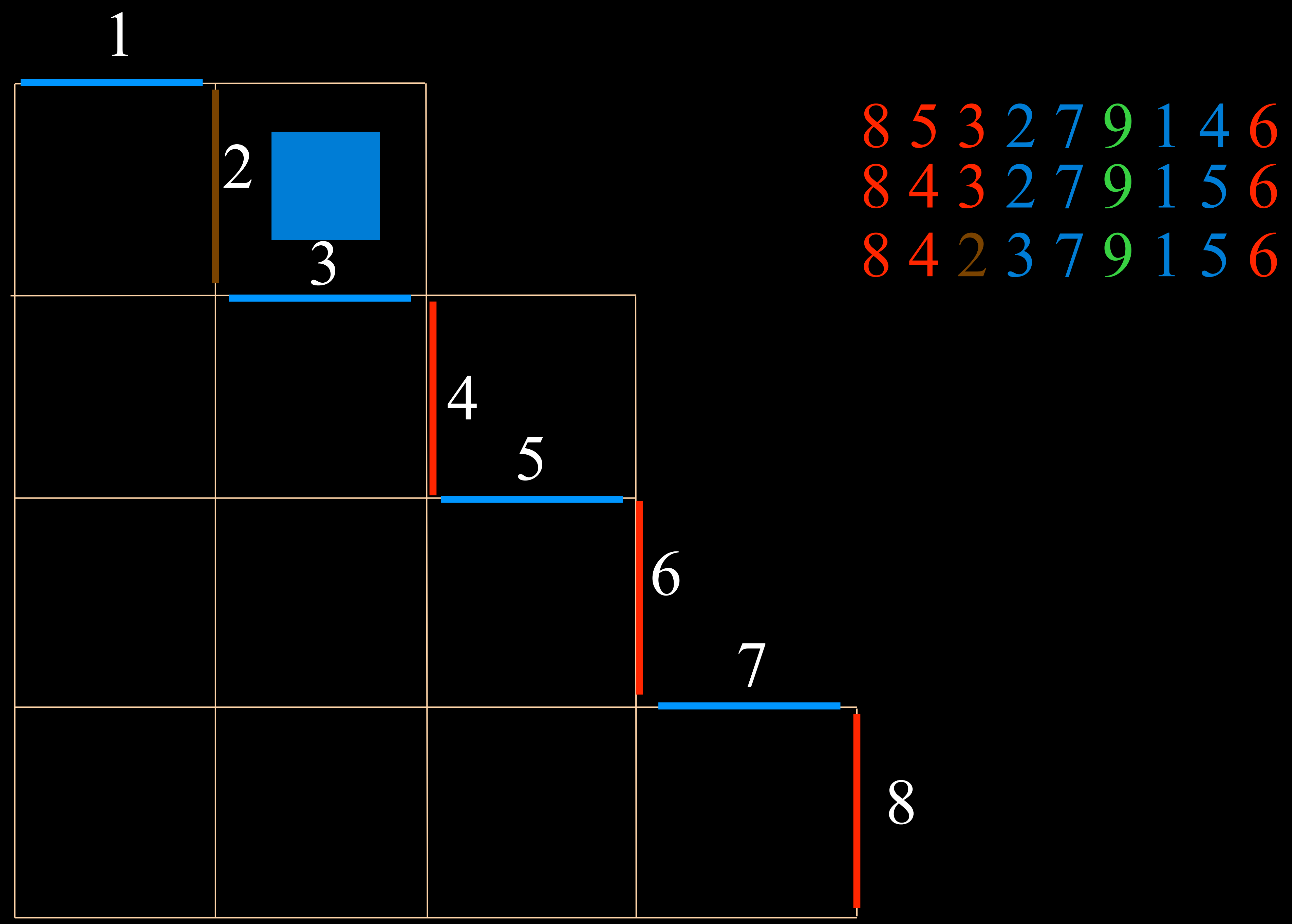
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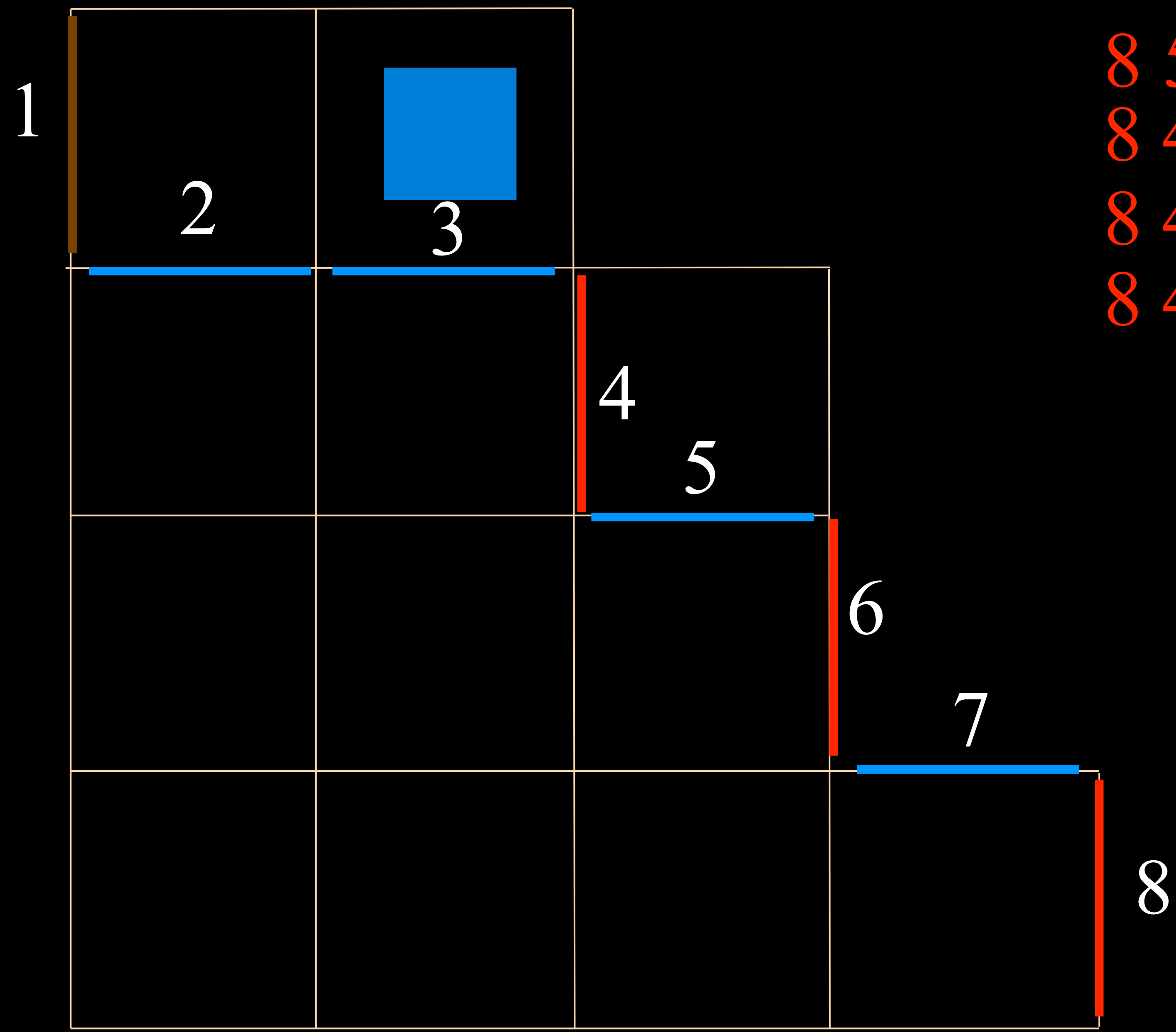

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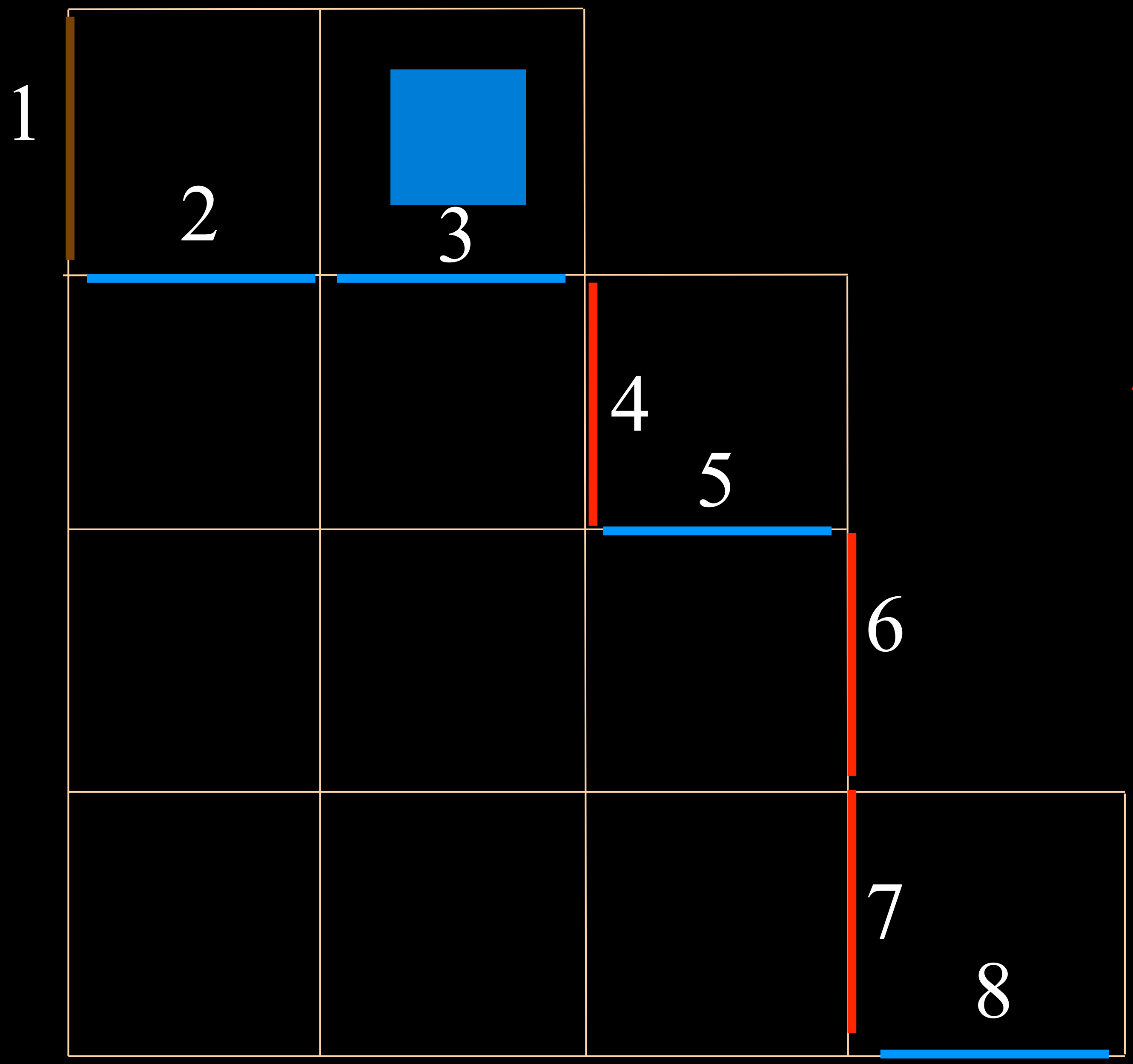

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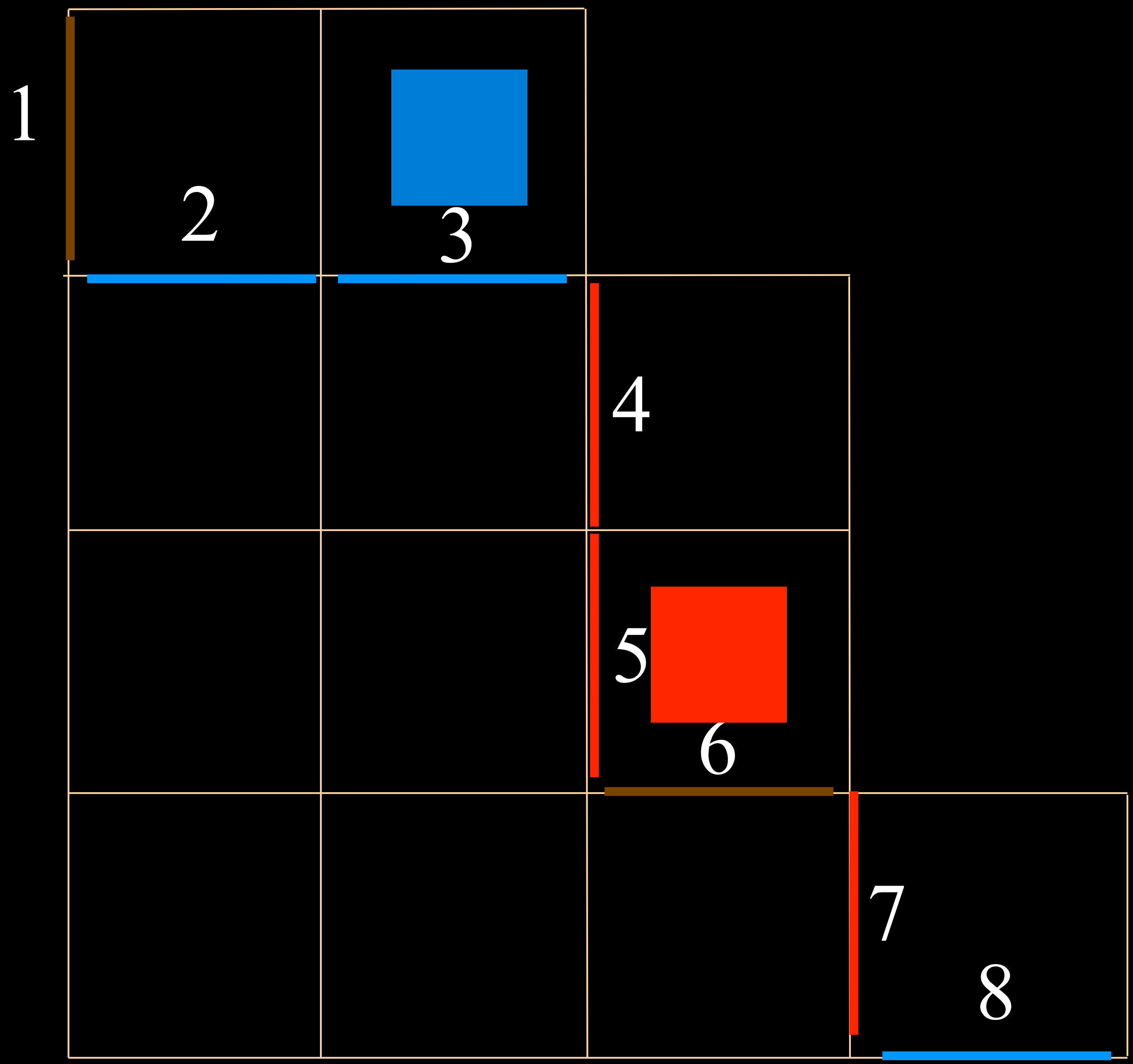
7 4 1 3 8 9 2 5 6



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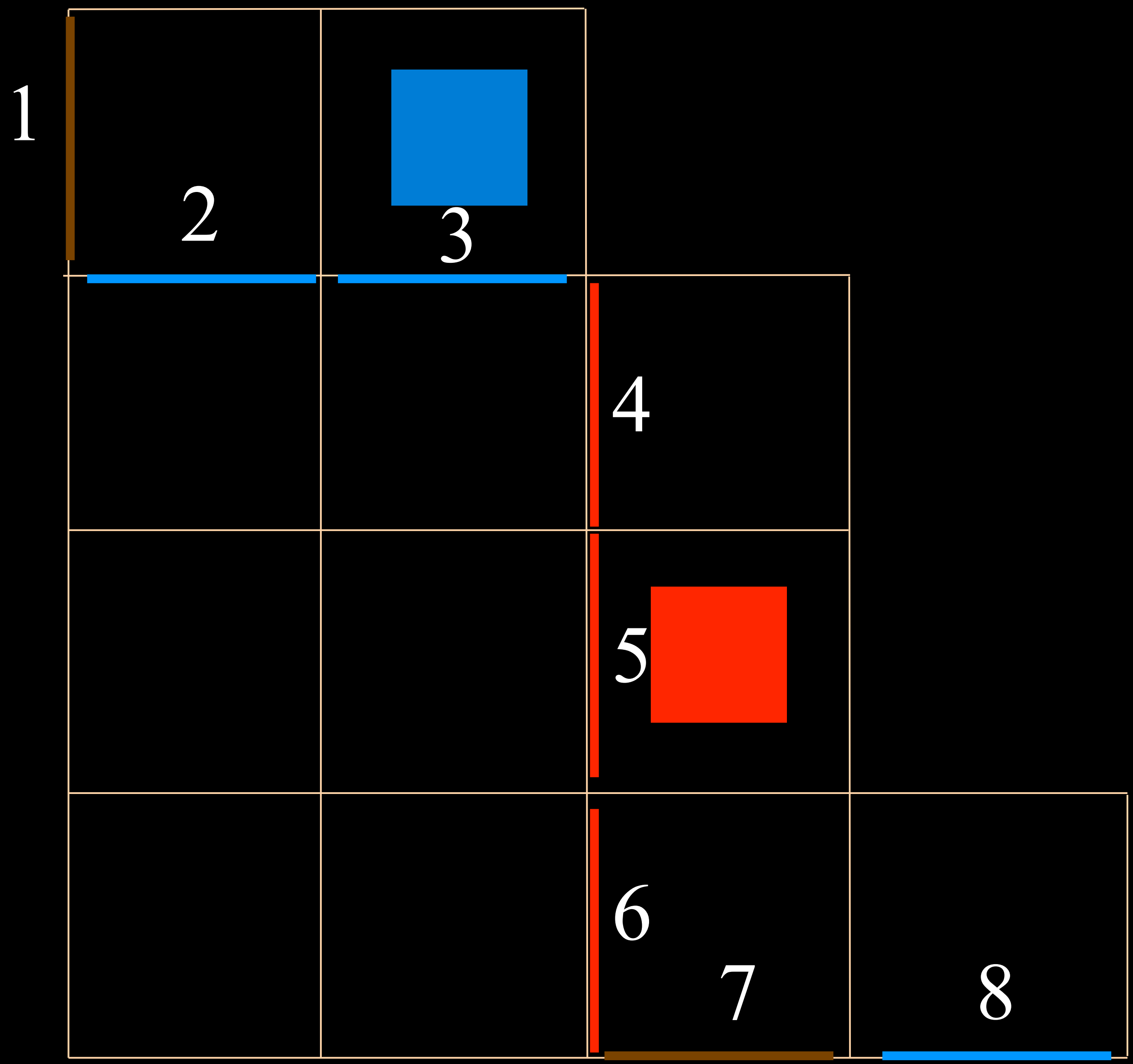

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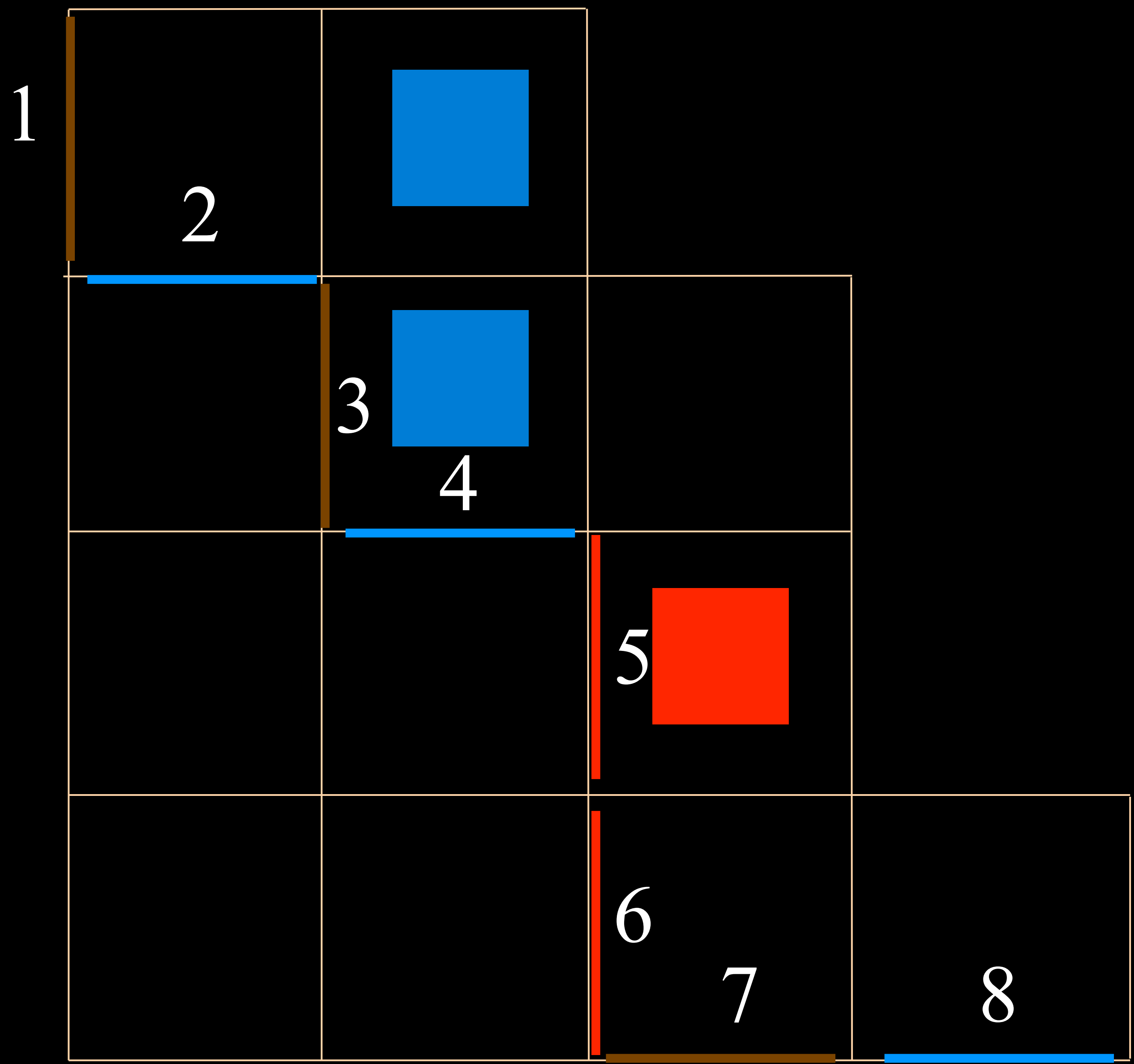

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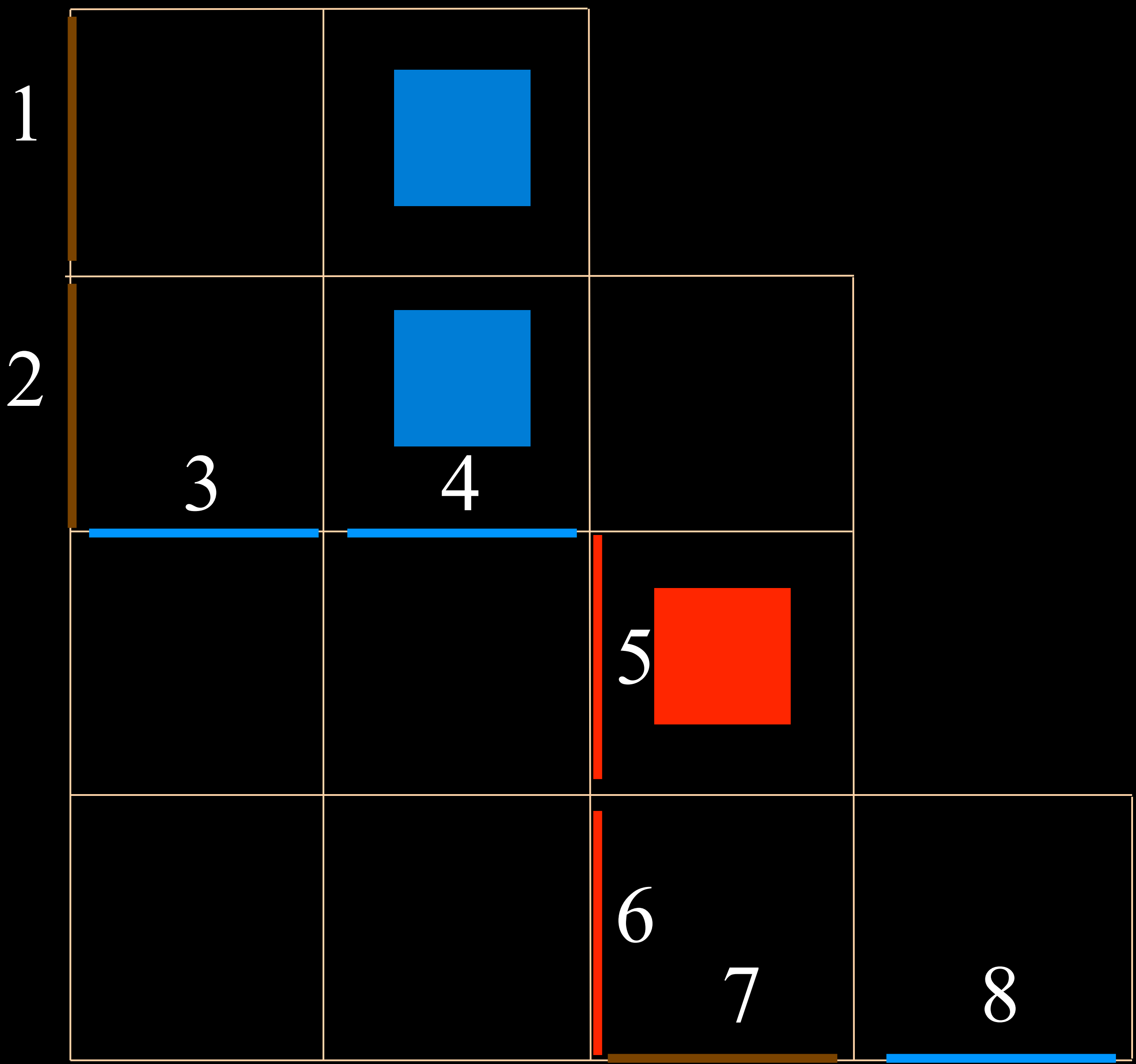

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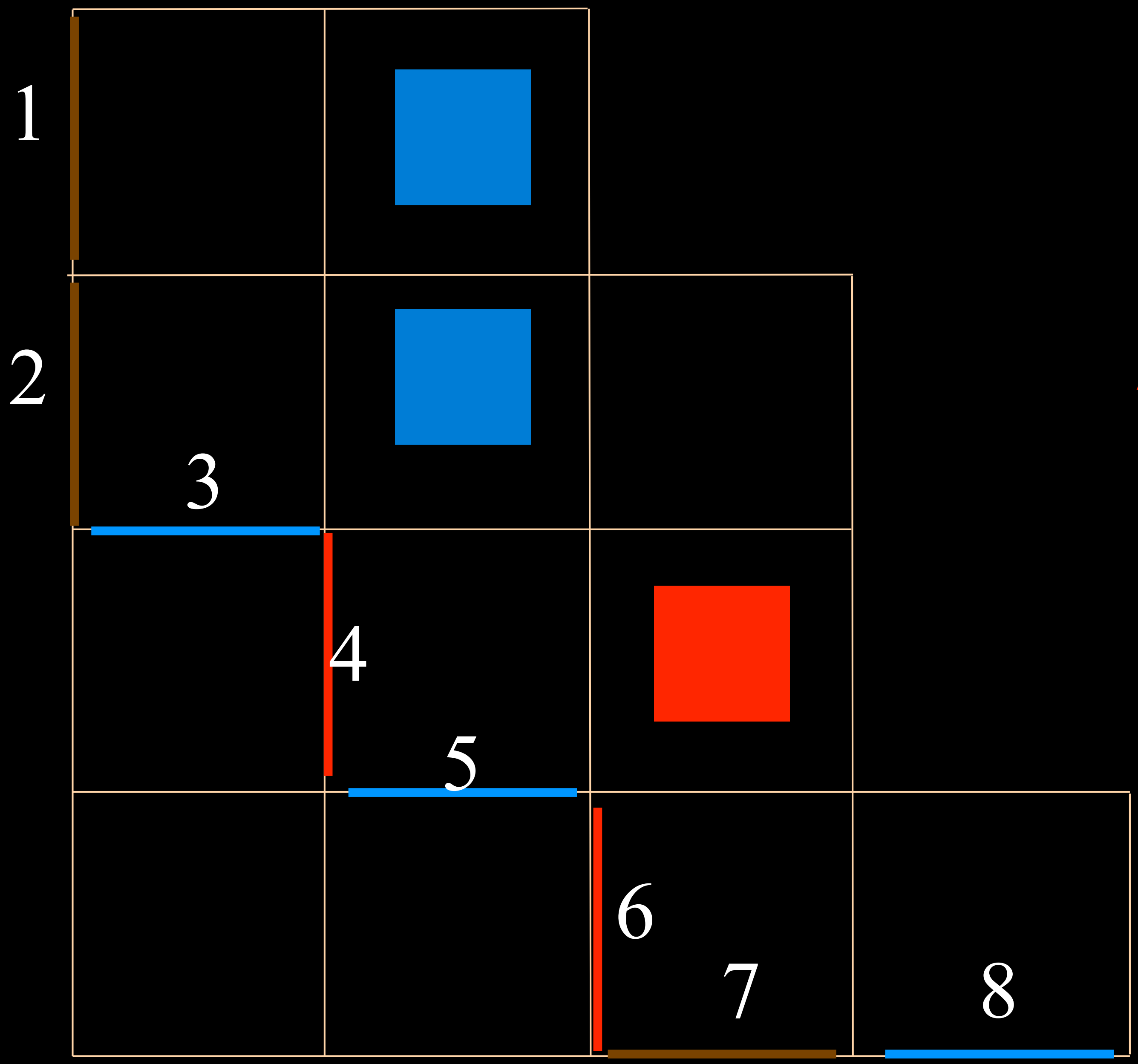

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8 4 2 3 7 9 1 5 6

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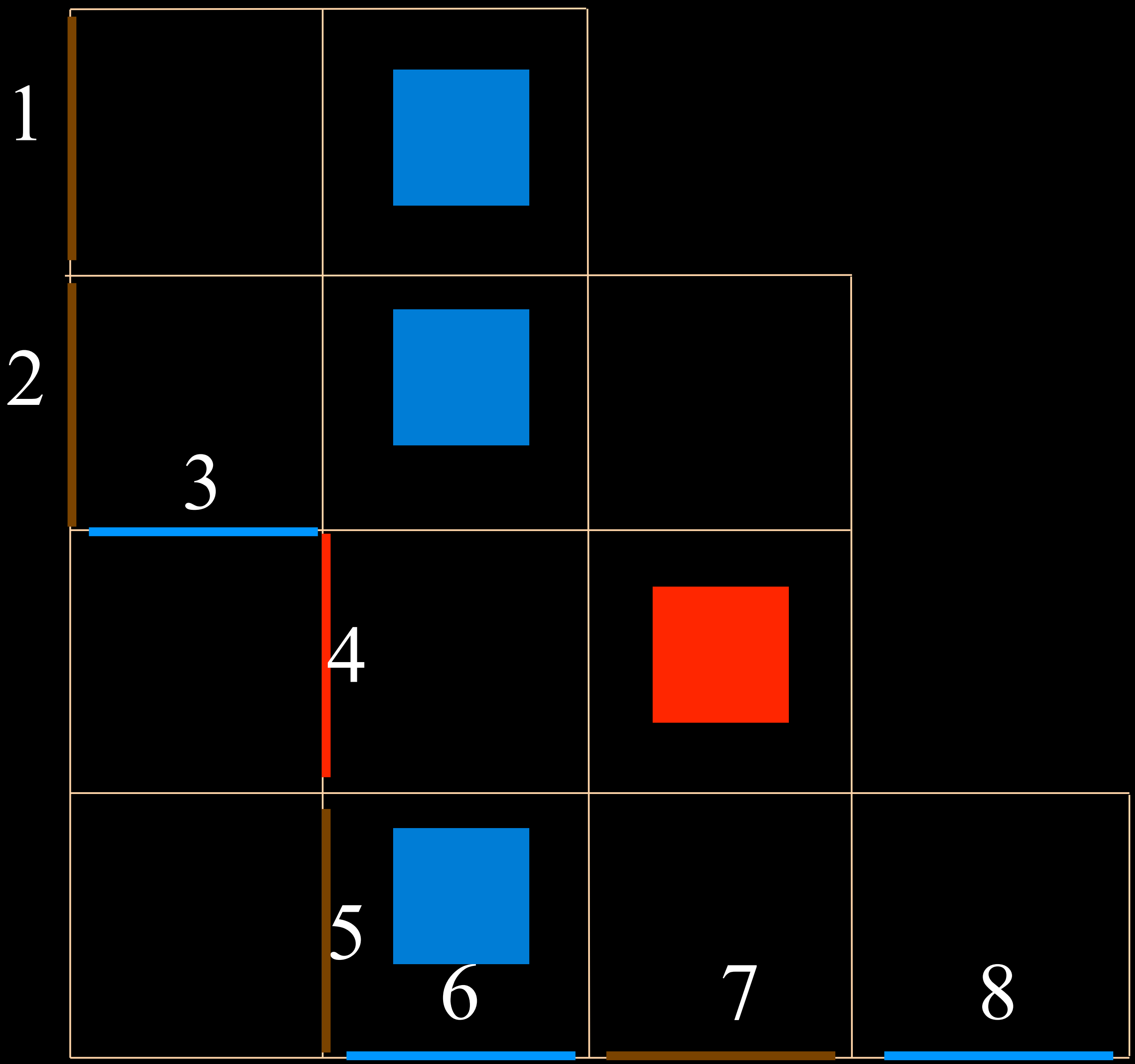

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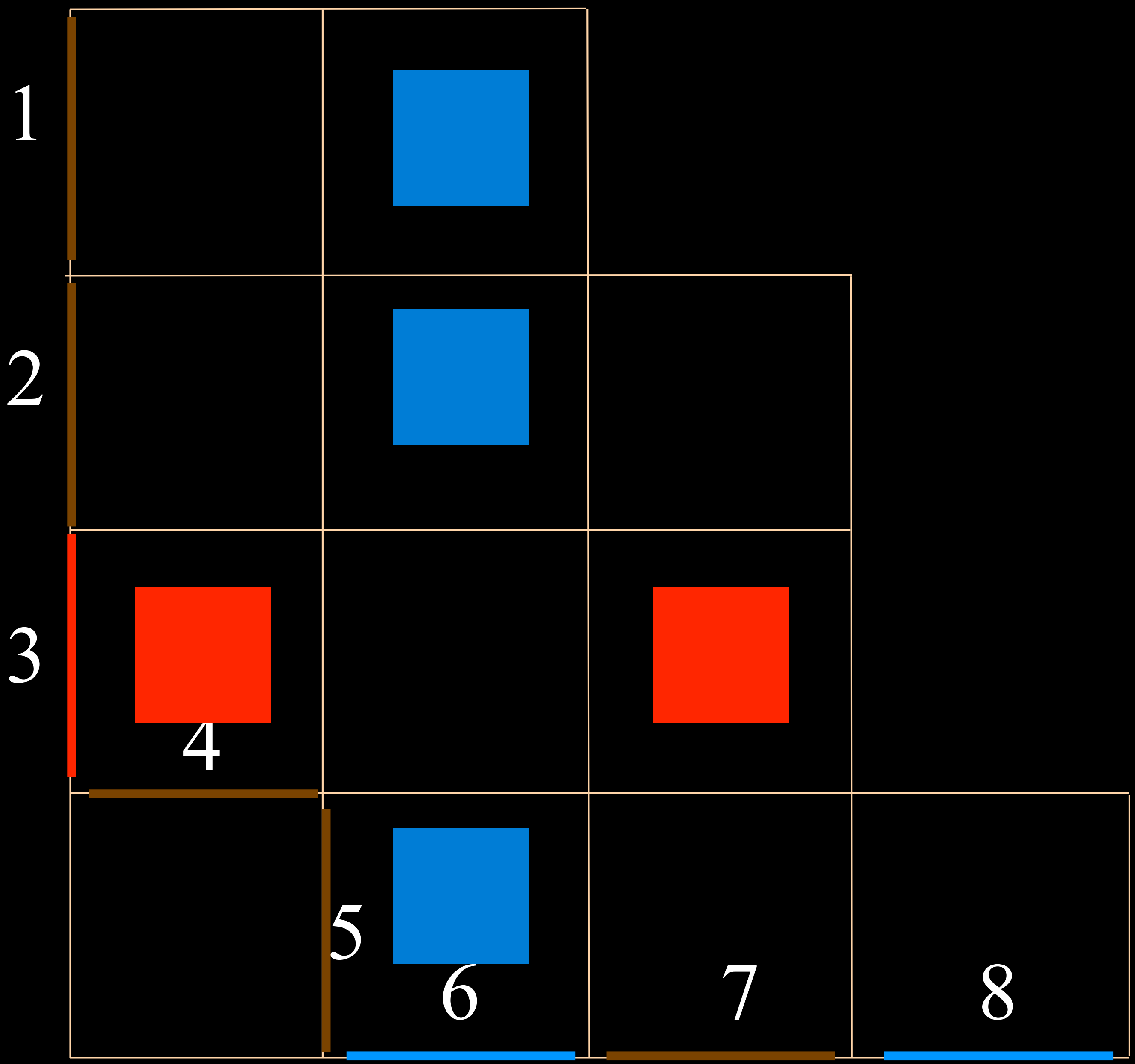

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8 4 3 2 7 9 1 5 6
8 4 2 3 7 9 1 5 6
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7 4 1 3 8 9 2 5 6
7 4 1 3 8 9 2 6 5
6 4 1 3 8 9 2 7 5
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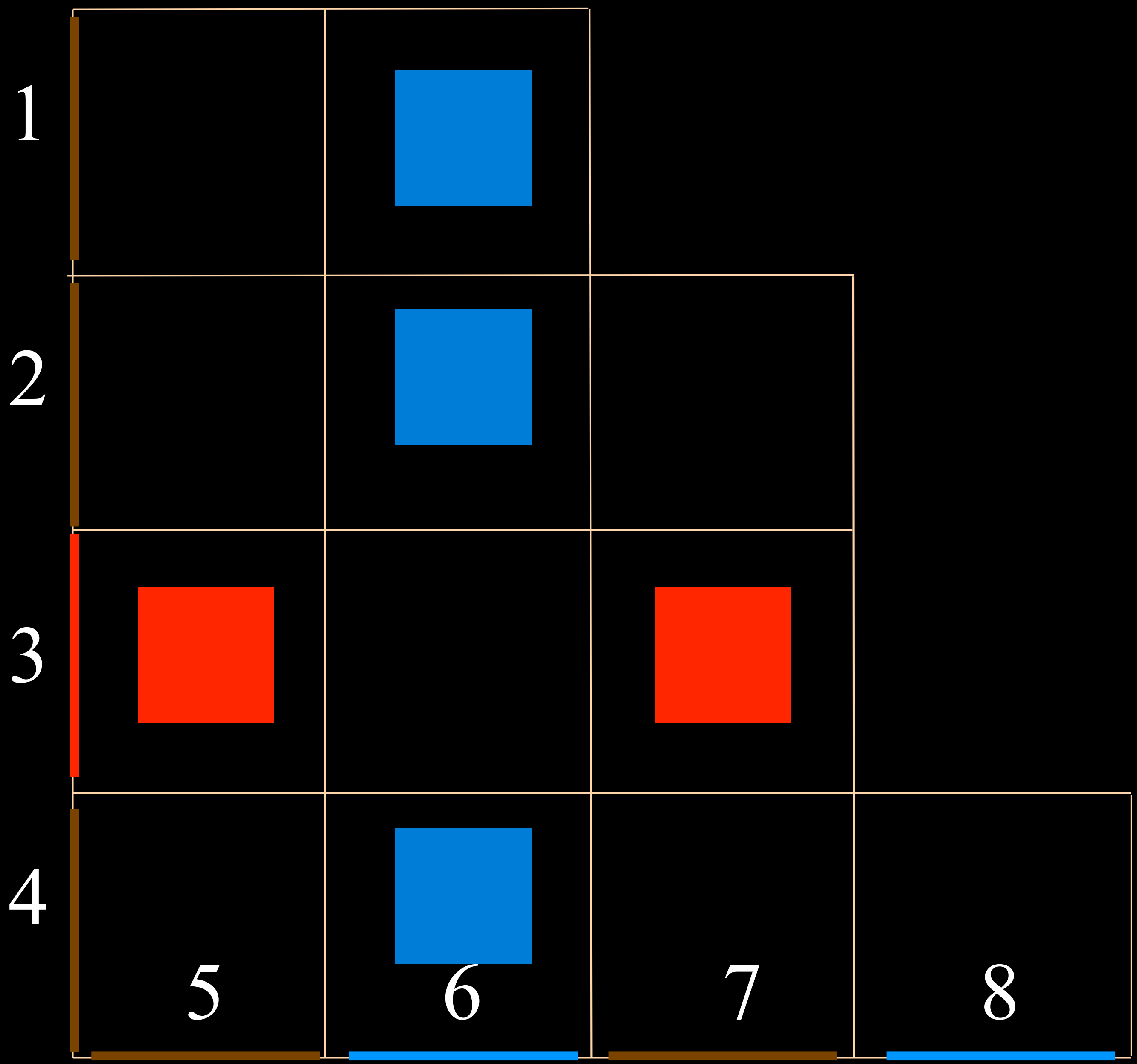

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8 4 1 3 7 9 2 5 6
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6 4 1 3 8 9 2 7 5
6 3 1 4 8 9 2 7 5
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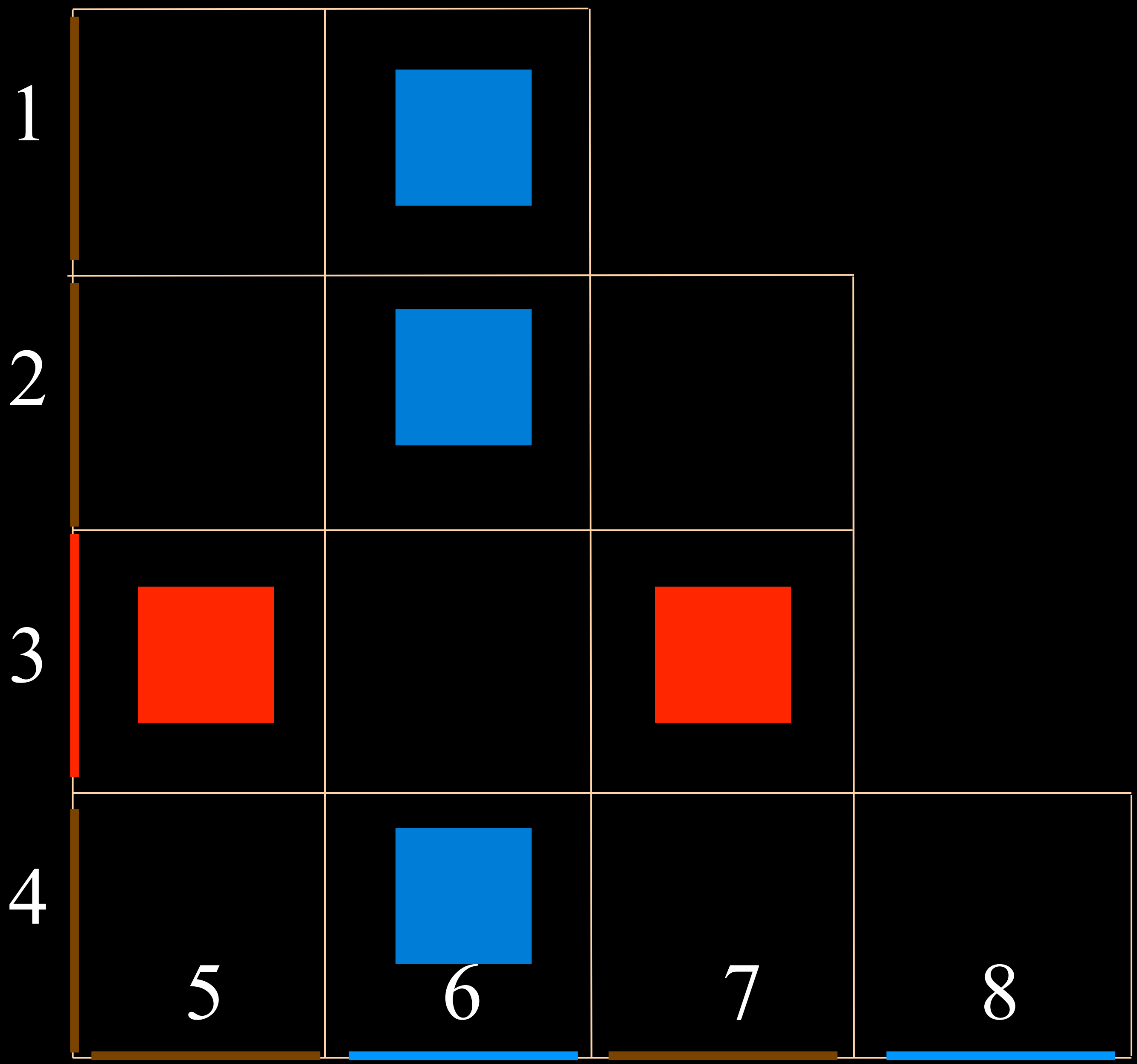
4 2 1 6 8 9 5 7 3



s = 7 4 3 8 2 9 5 1 6

1 2 3 4 5 6 7 8 9

8 5 3 2 7 9 1 4 6

8 4 3 2 7 9 1 5 6

8 4 2 3 7 9 1 5 6

8 4 1 3 7 9 2 5 6

7 4 1 3 8 9 2 5 6

7 4 1 3 8 9 2 6 5

6 4 1 3 8 9 2 7 5

6 3 1 4 8 9 2 7 5

6 2 1 4 8 9 3 7 5

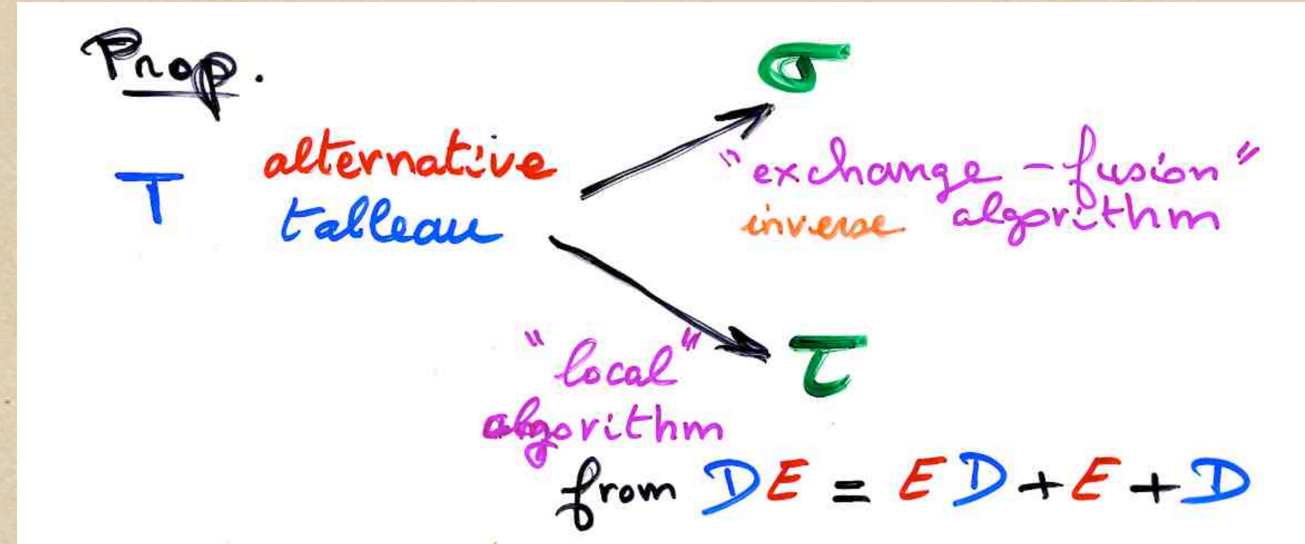
6 2 1 5 8 9 3 7 4

5 2 1 6 8 9 3 7 4

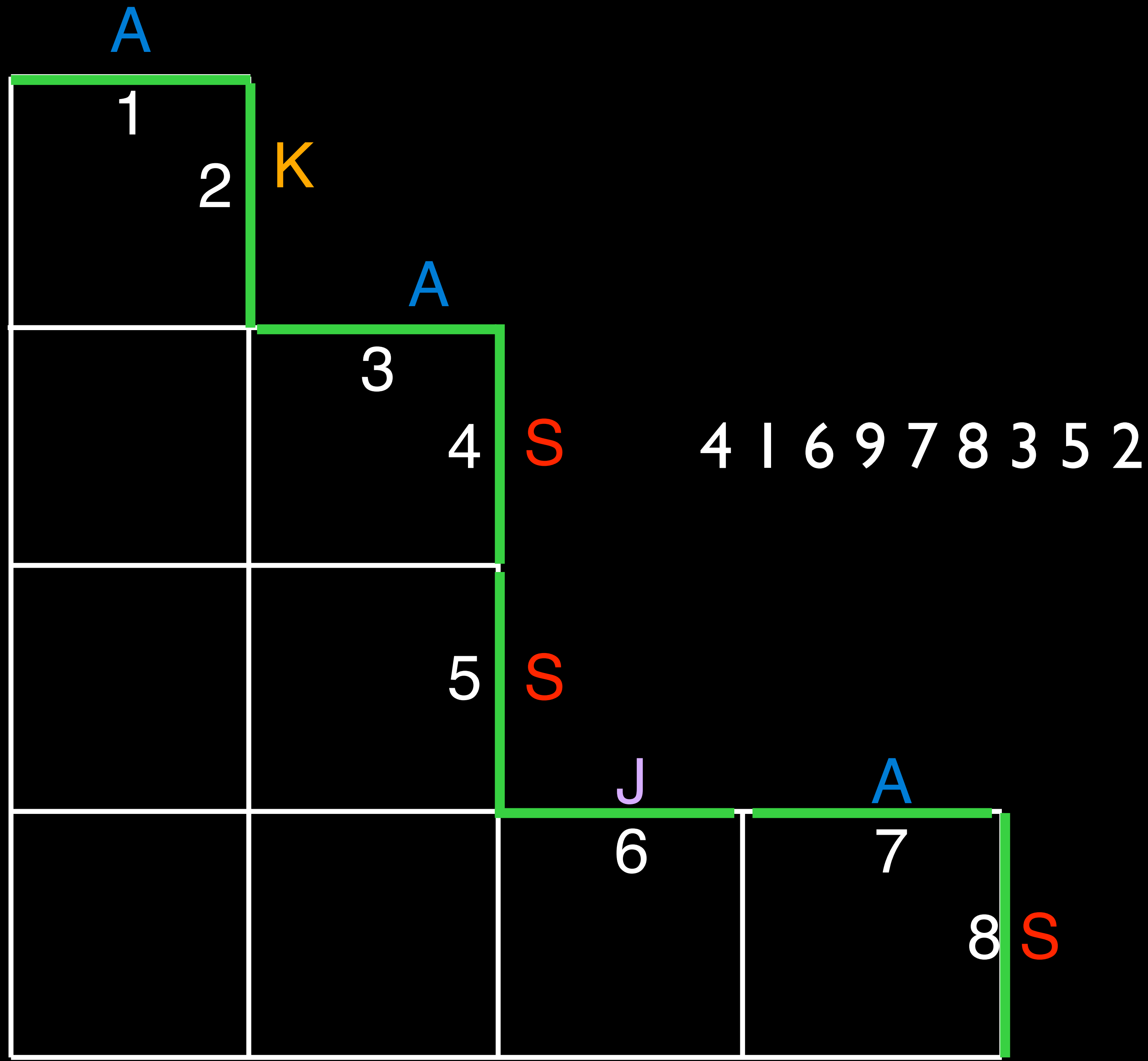
5 2 1 6 8 9 4 7 3

4 2 1 6 8 9 5 7 3

proof of the main theorem



$$\sigma = \tau^{-1}$$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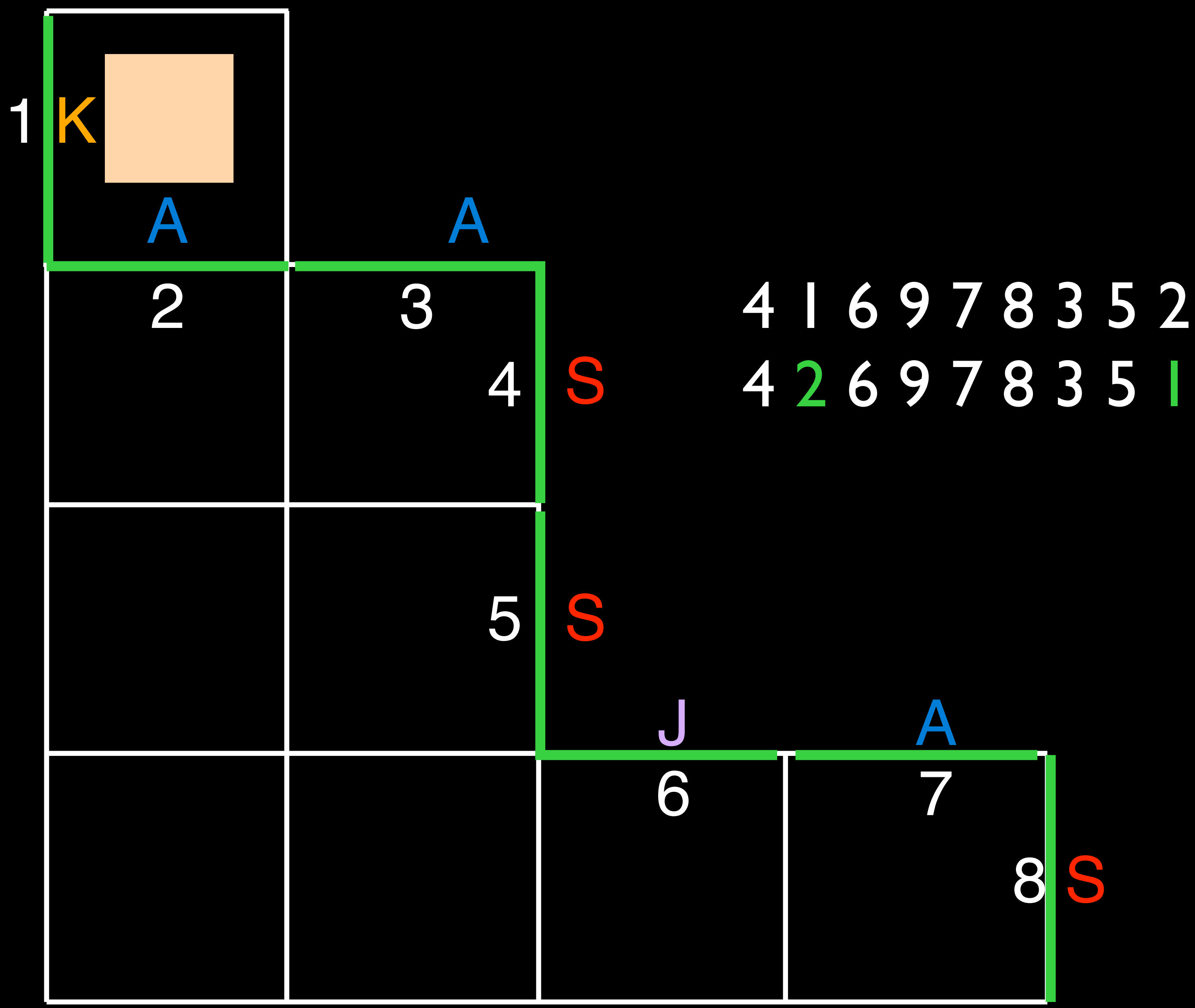
K
A
3
4
S
4 1 6 9 7 8 3 5 2
5
S
J
A
6
7
8
S


1
K
A
A
2
3
4
S
5
S
J
A
6
7
8
S
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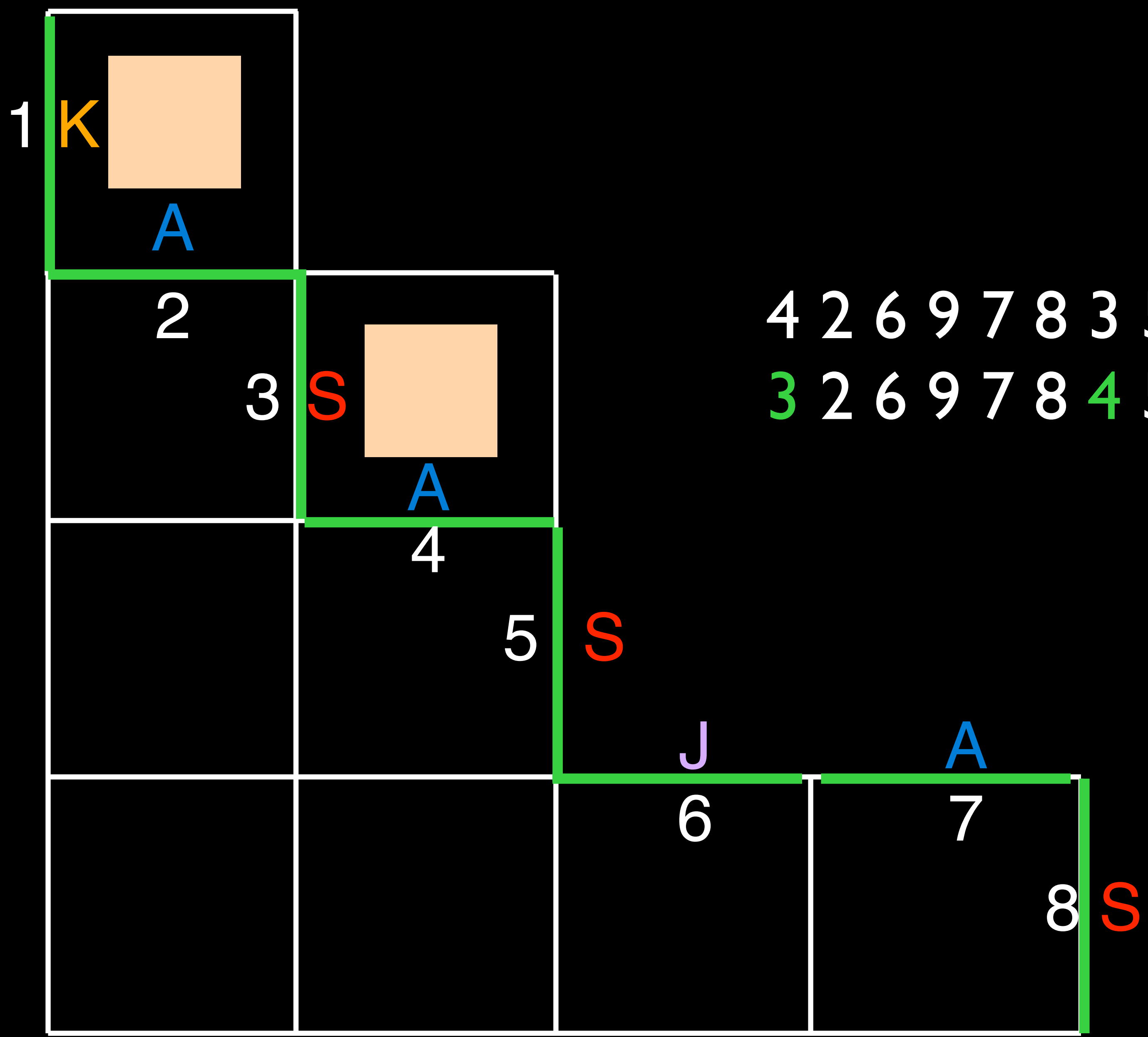
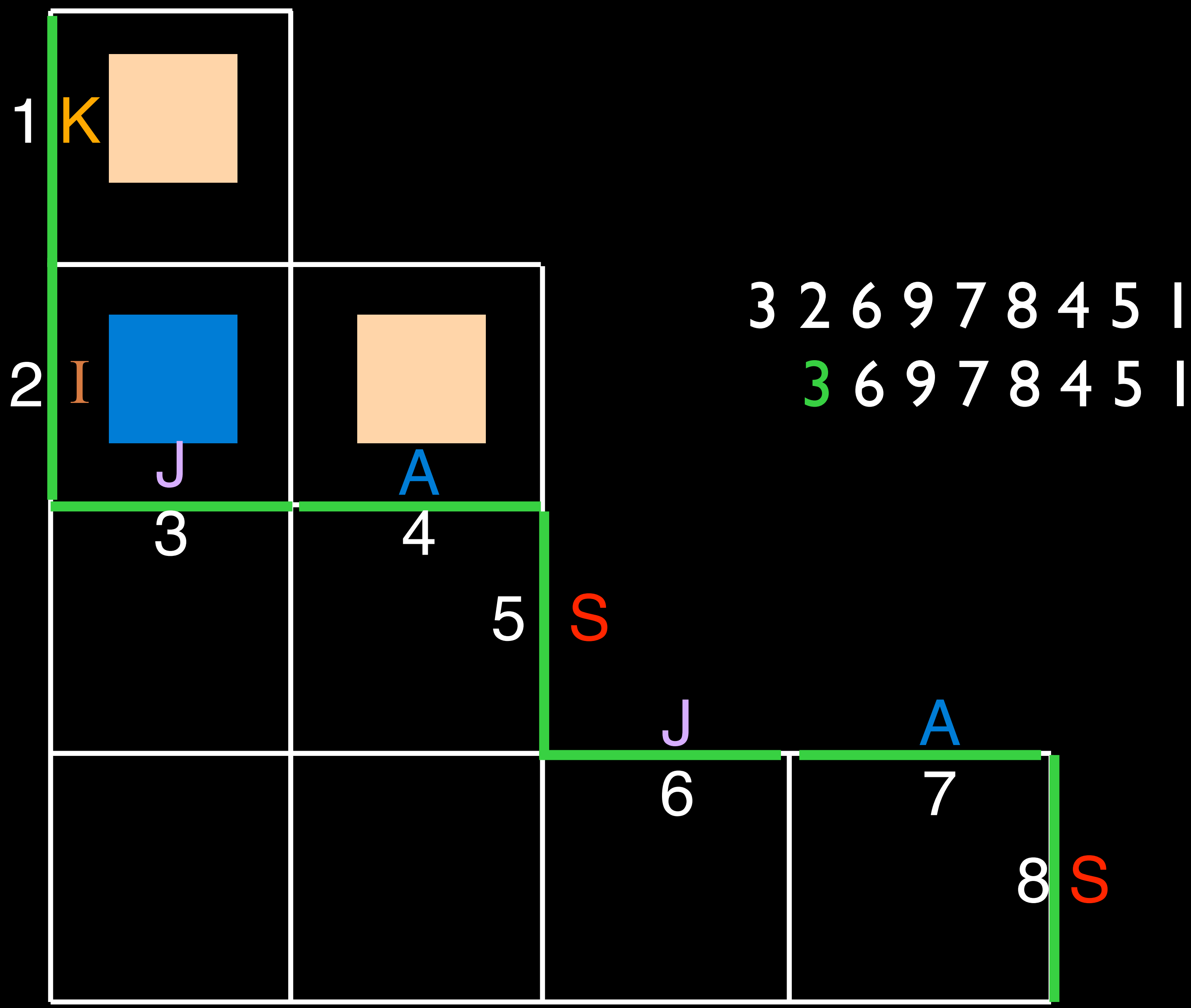
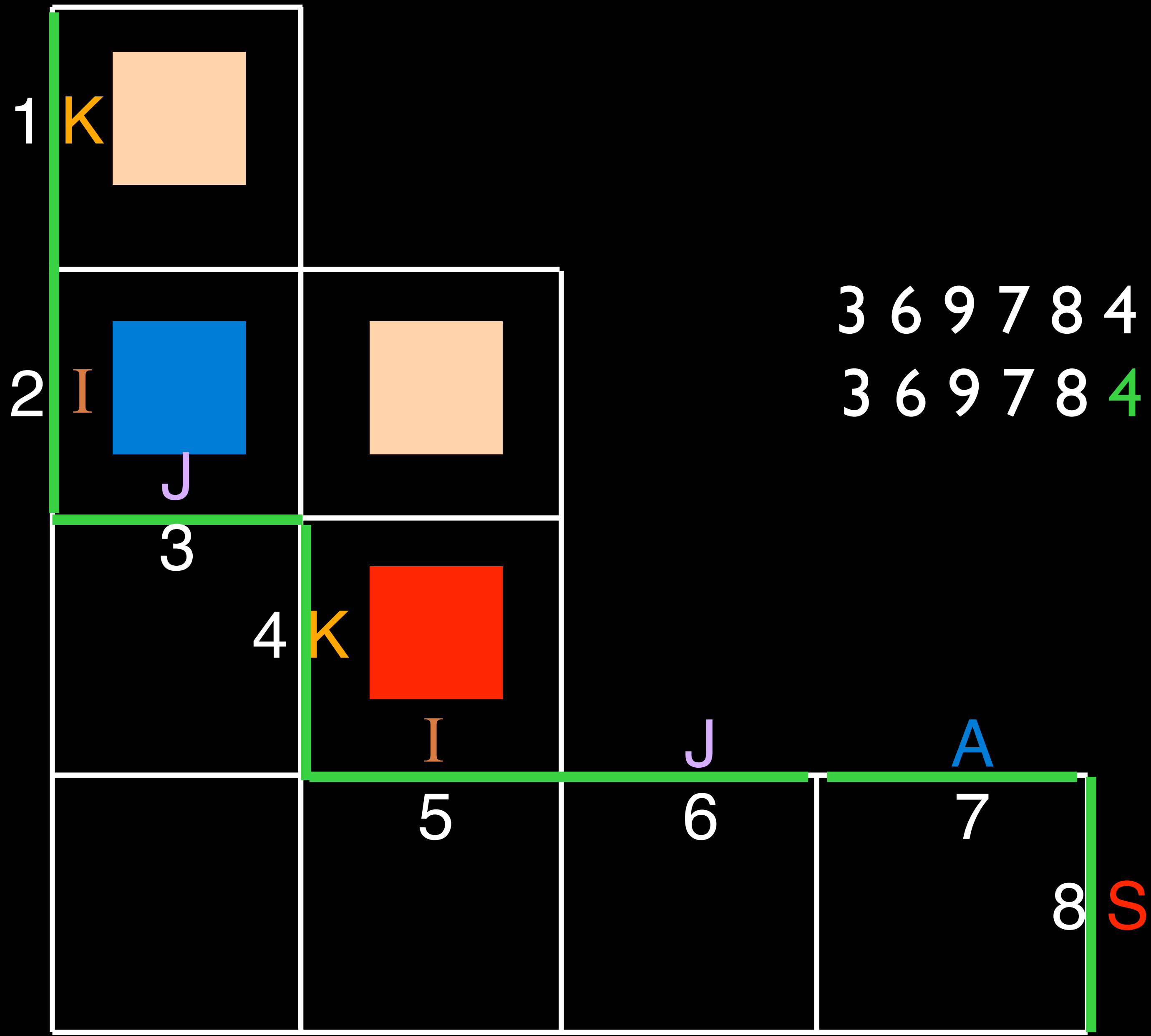
4 2 6 9 7 8 3 5 1



4 2 6 9 7 8 3 5 1

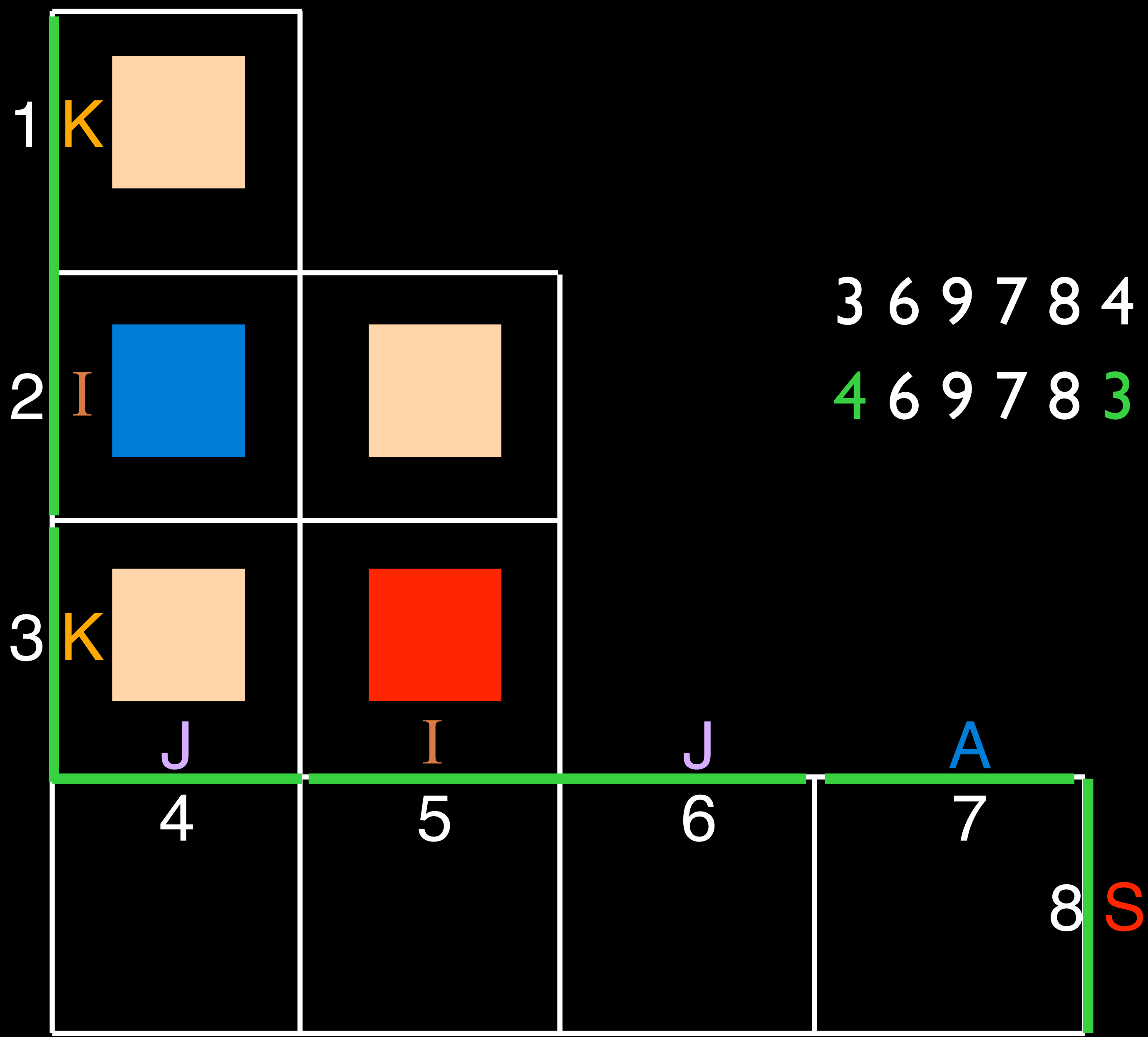
3 2 6 9 7 8 4 5 1


1 K
2 I
J
A
3
4
5 S
J
A
6
7
8 S
3 2 6 9 7 8 4 5 1
3 6 9 7 8 4 5 1



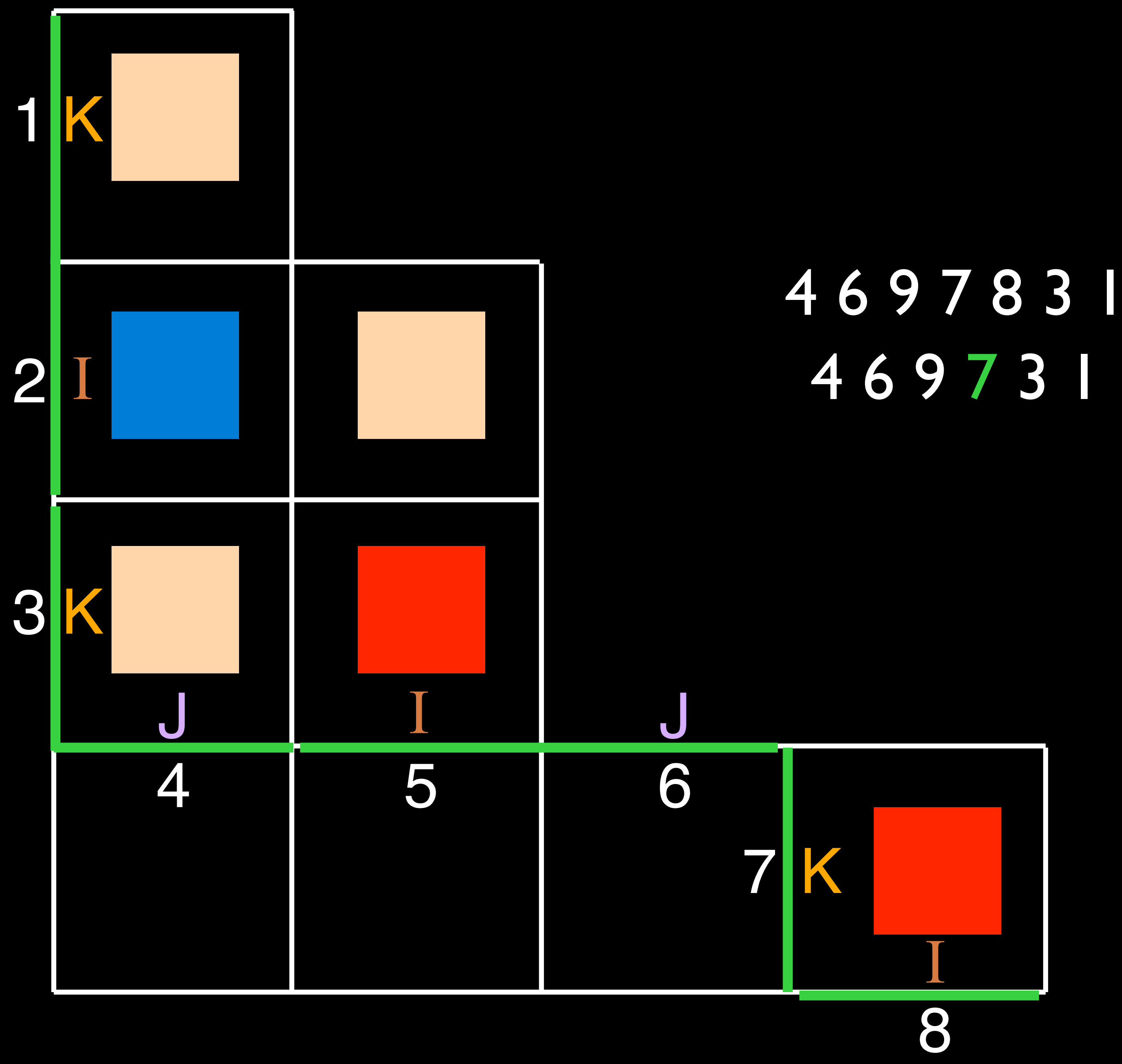
3 6 9 7 8 4 5 1

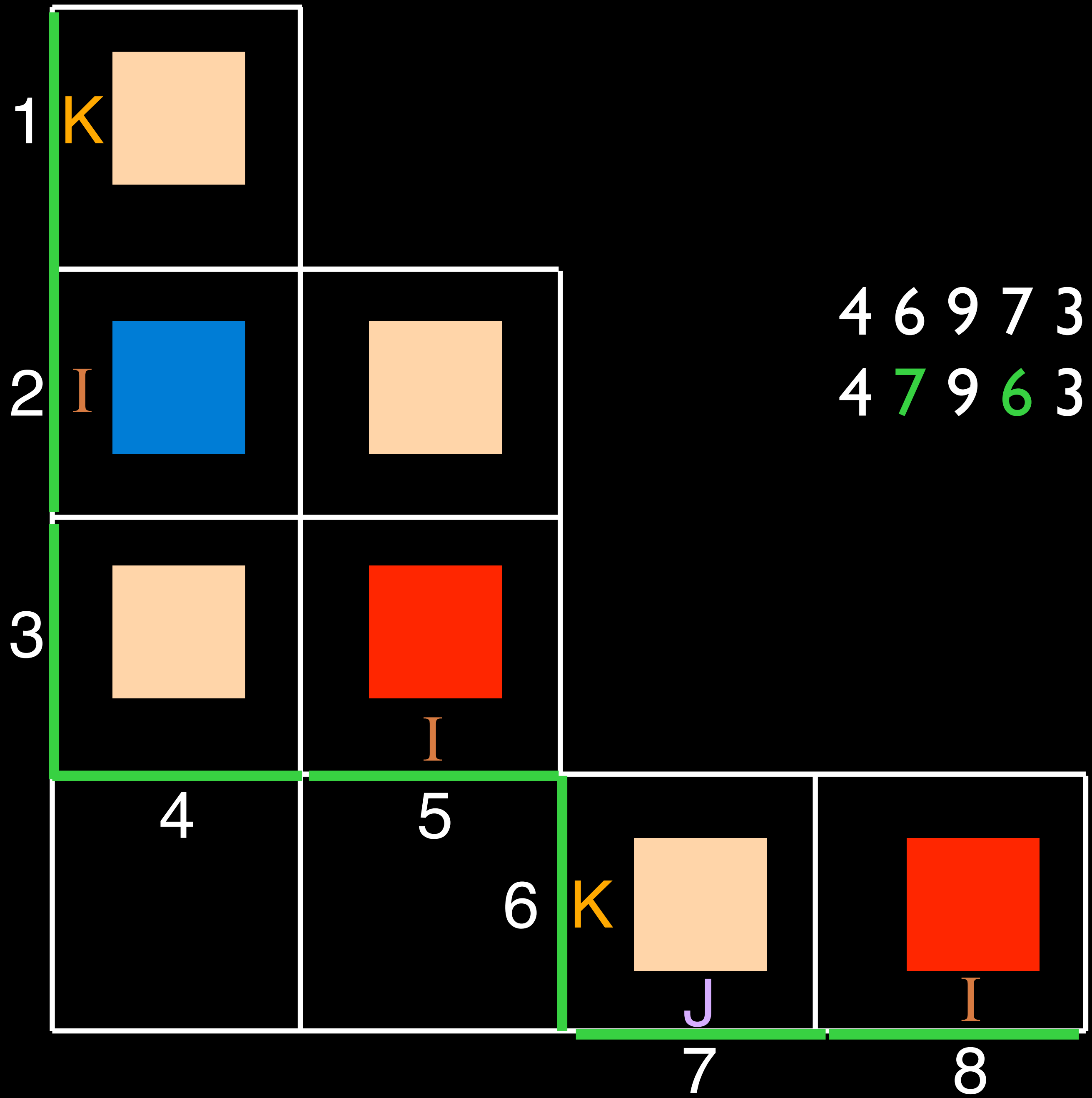
3 6 9 7 8 4 1



3 6 9 7 8 4 I

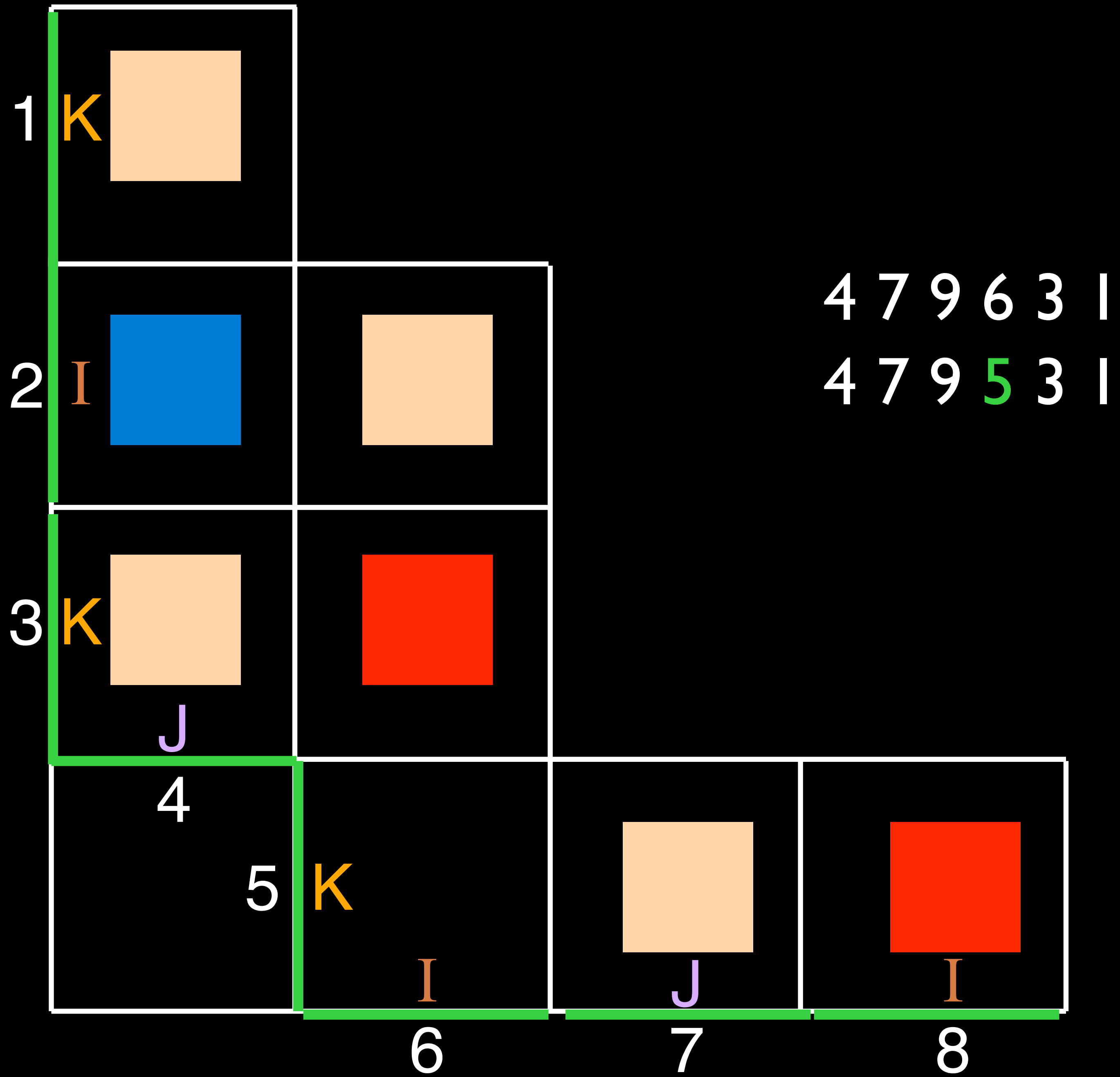
4 6 9 7 8 3 I

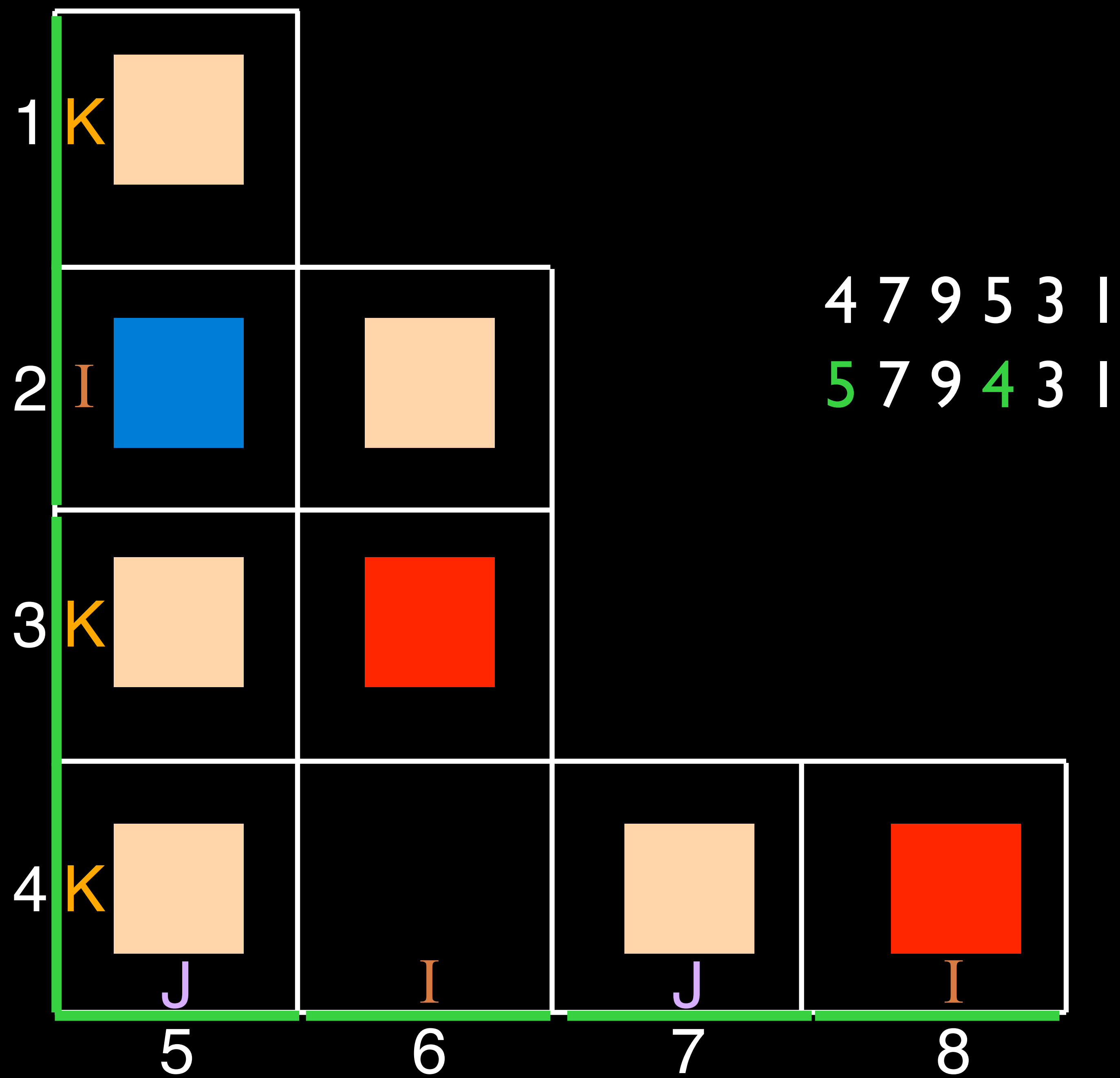

1 K
2 I
3 K
J
I
J
4
5
6
7 K
I
8
4 6 9 7 8 3 1
4 6 9 7 3 1

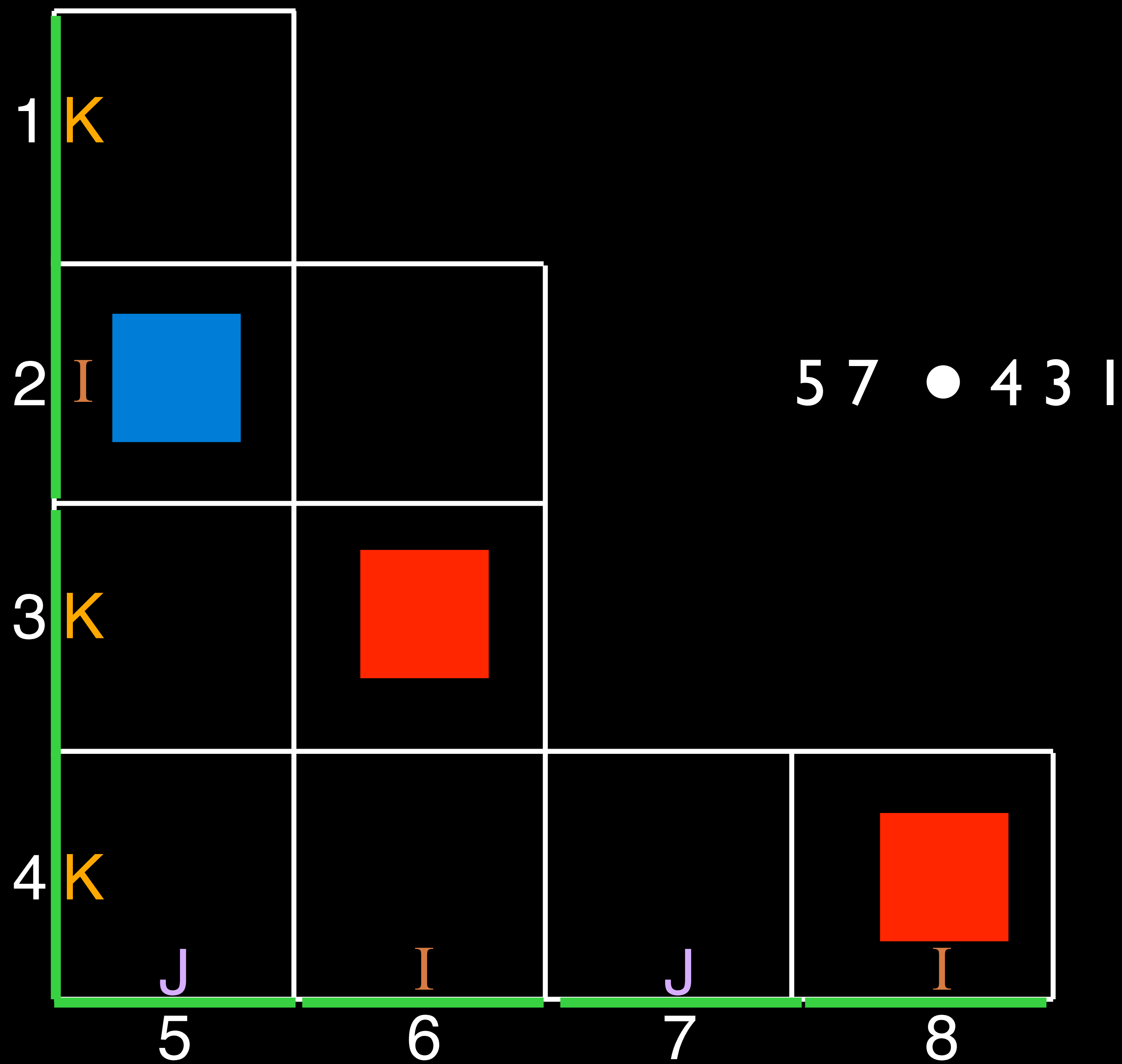


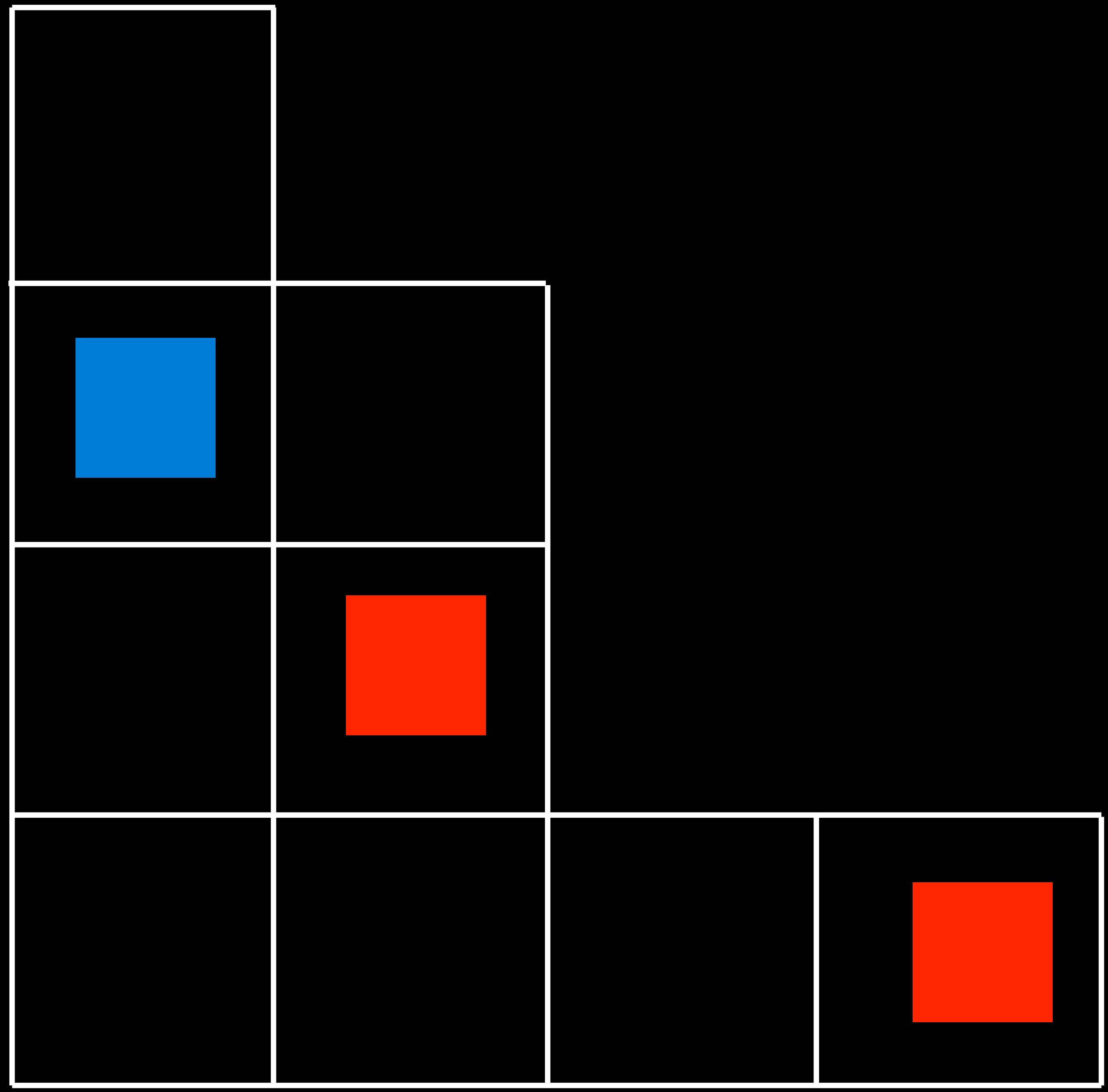
4 6 9 7 3 1

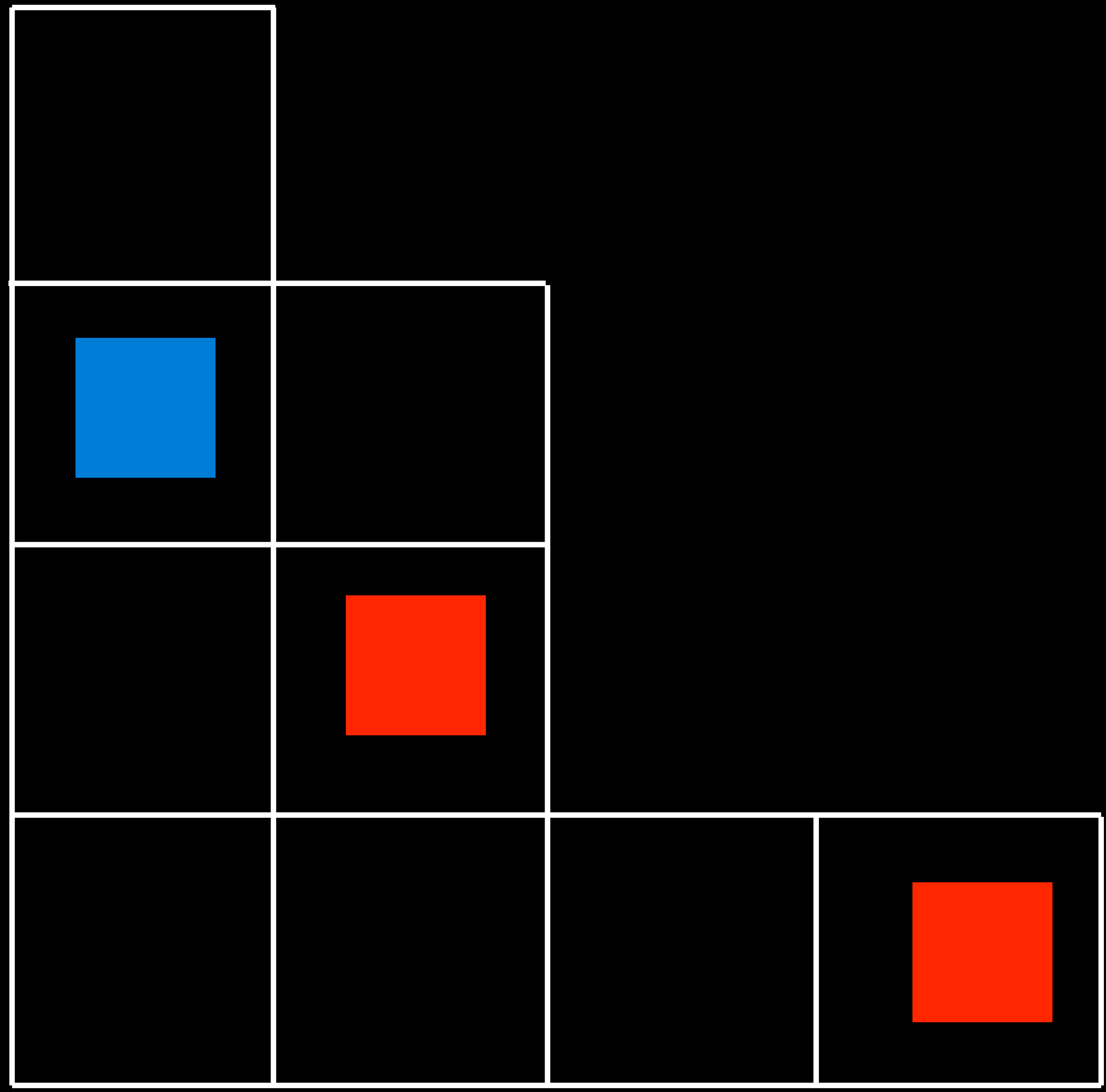
4 7 9 6 3 1

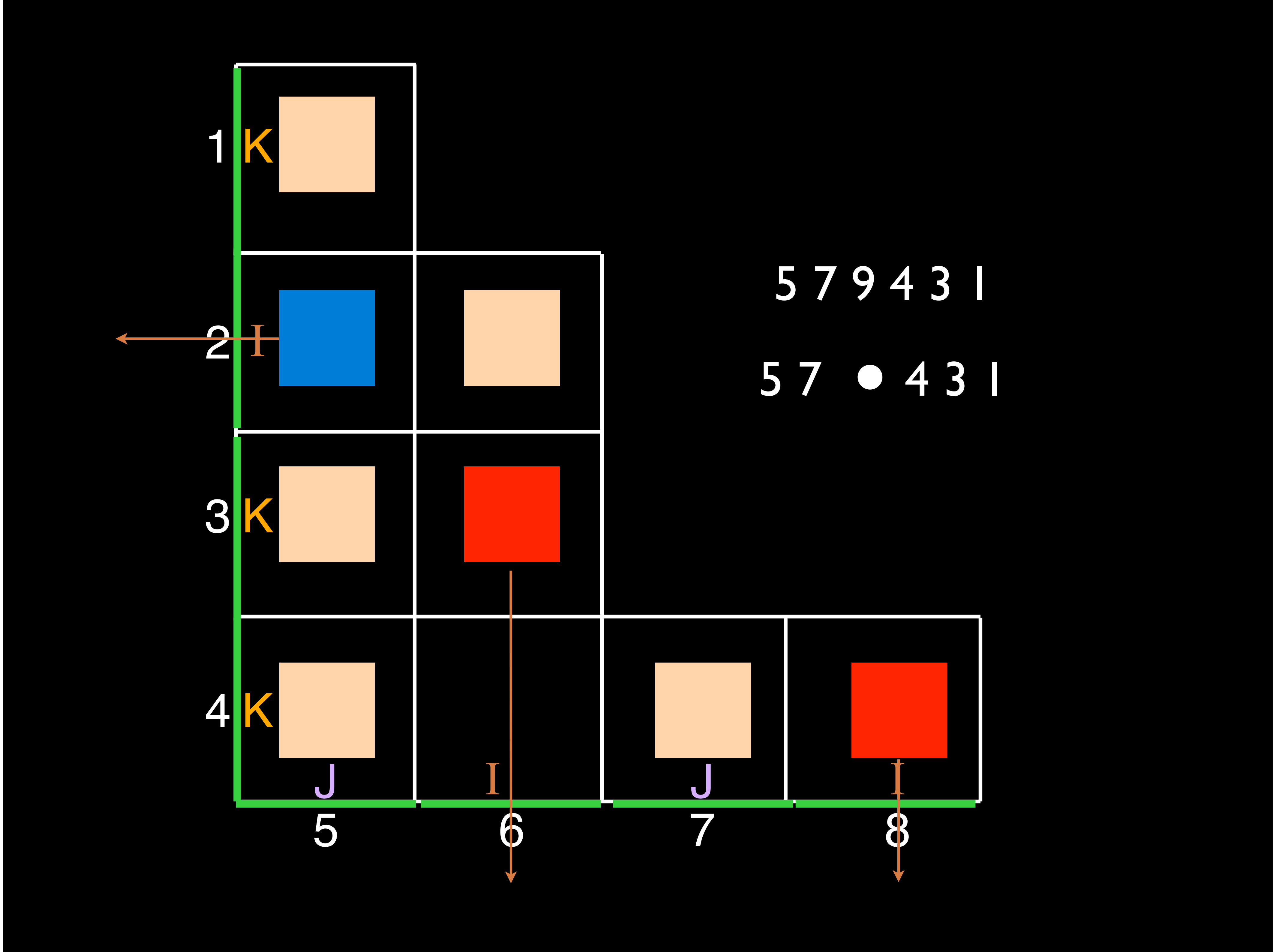

1 K
2 I
3 K
J
4
5 K
I
J
I
6
7
8
4 7 9 6 3 1
4 7 9 5 3 1

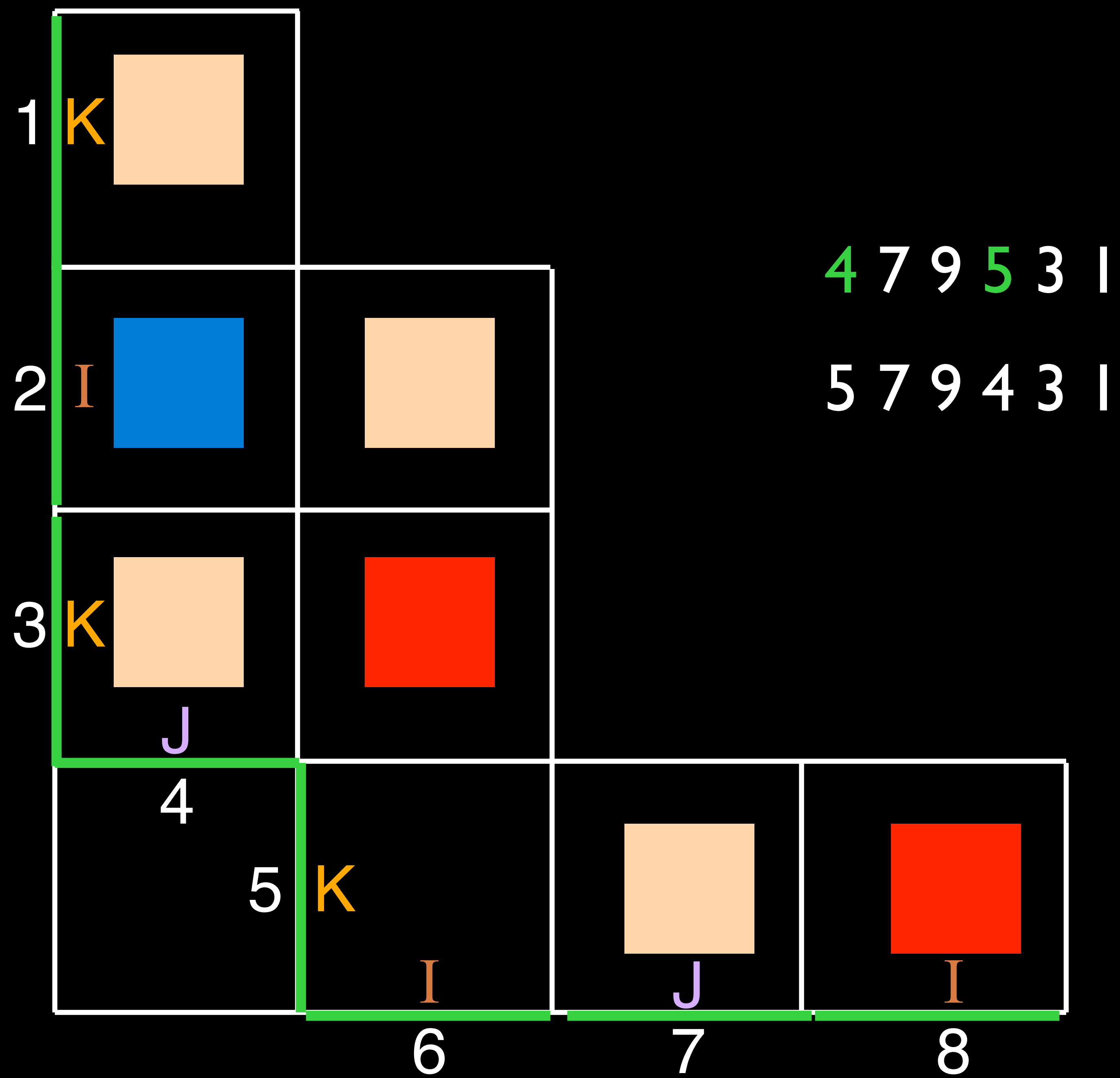

1 K
2 I
3 K
4 K
J
I
J
I
5
6
7
8
4 7 9 5 3 1
5 7 9 4 3 1

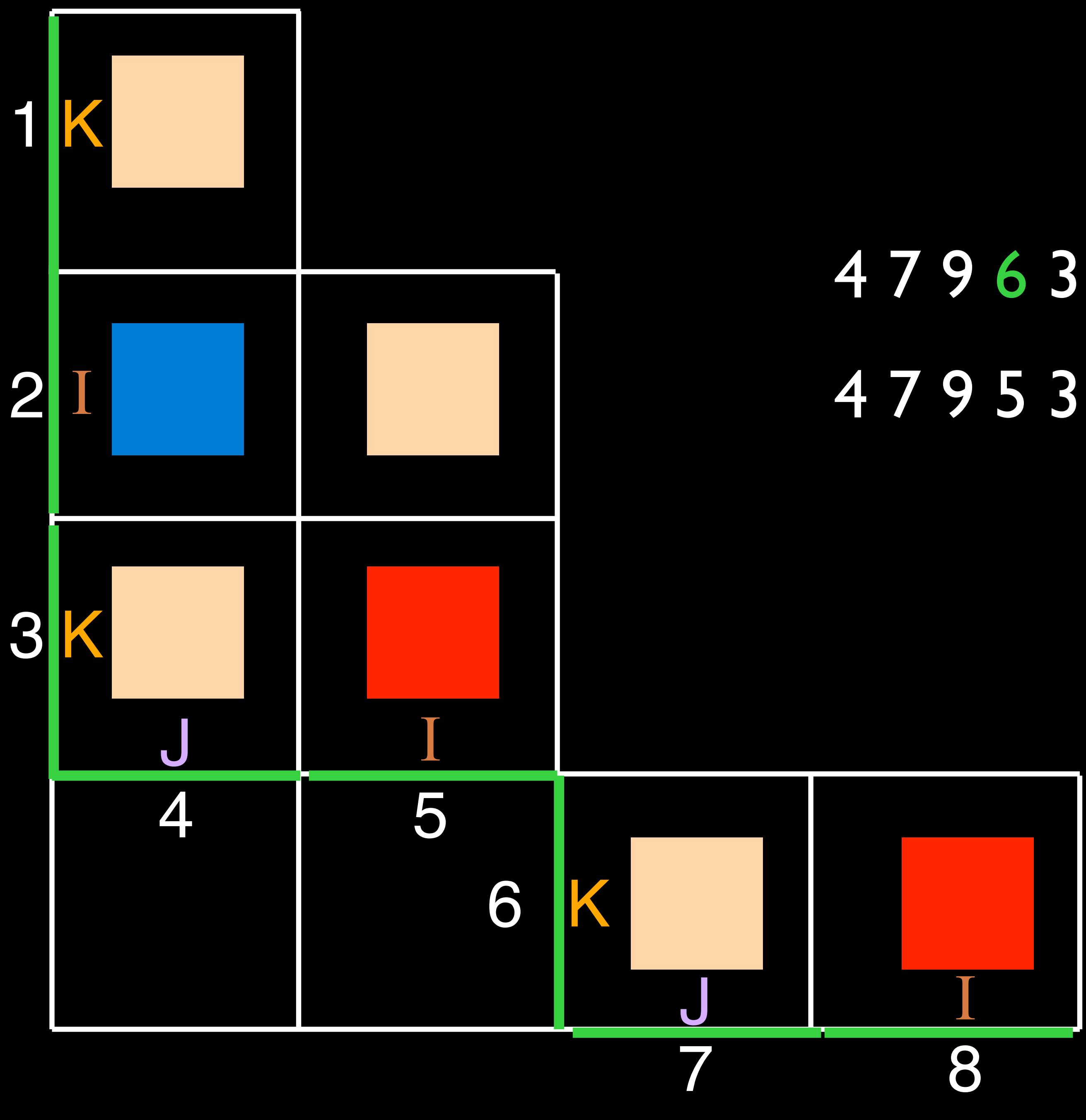

1 K
2 I
3 K
4 K
J I J I
5 6 7 8
5 7 ● 4 3 1





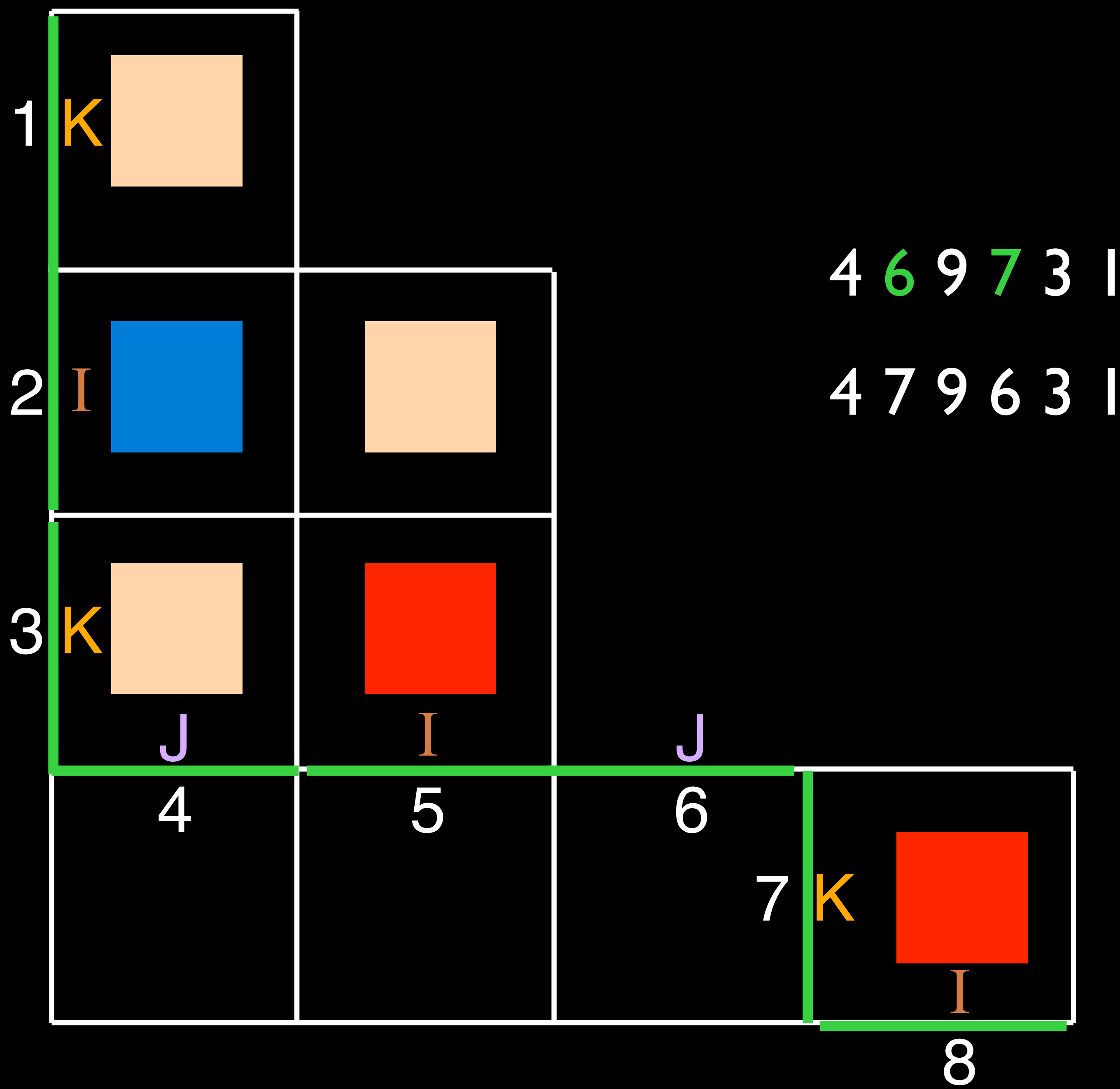

1 K
2 I
3 K
4 K
J I J I
5 6 7 8
5 7 9 4 3 1
5 7 ● 4 3 1

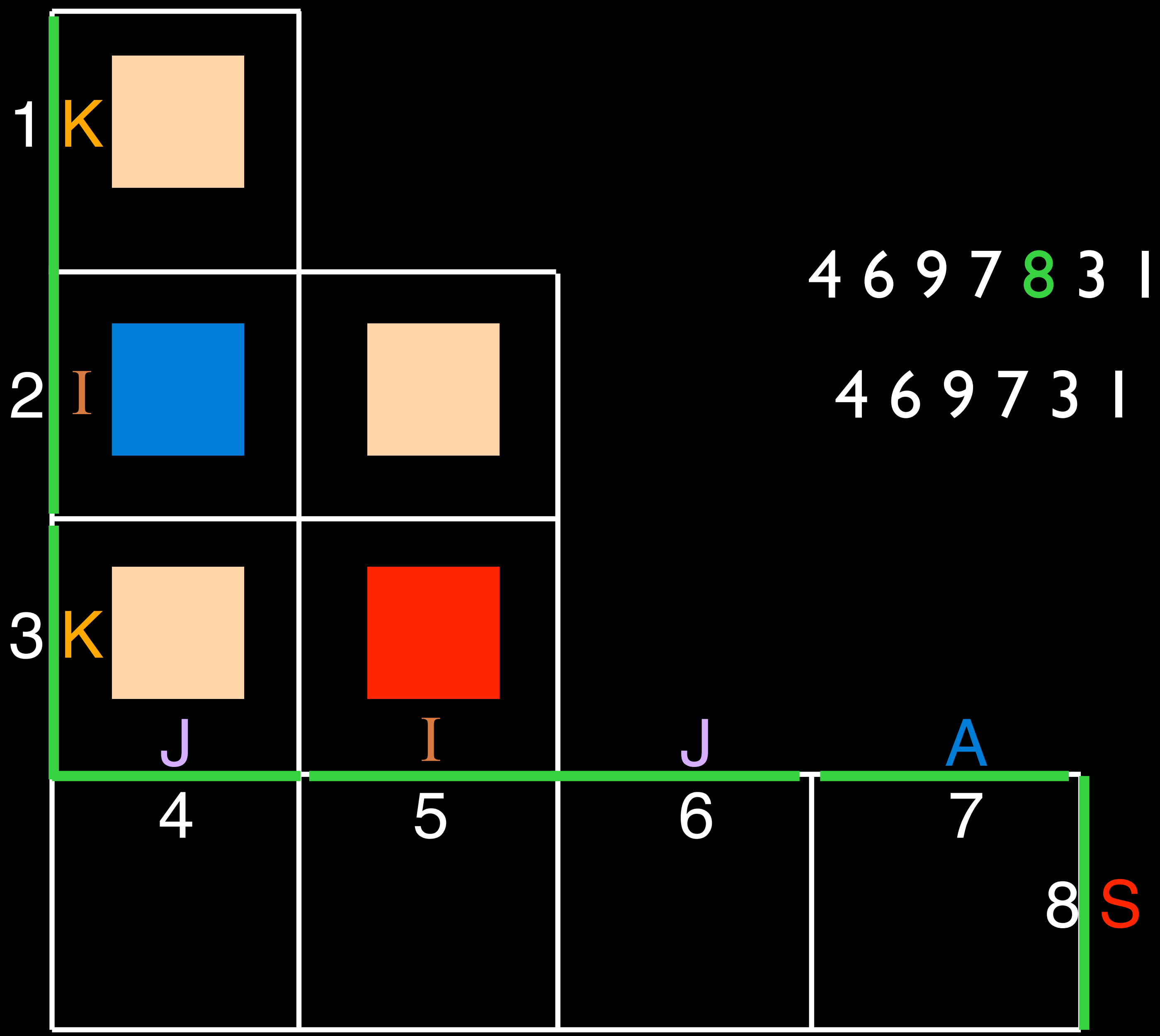

1 K
2 I
3 K
J
4
5 K
I
J
I
6
7
8
4 7 9 5 3 1
5 7 9 4 3 1

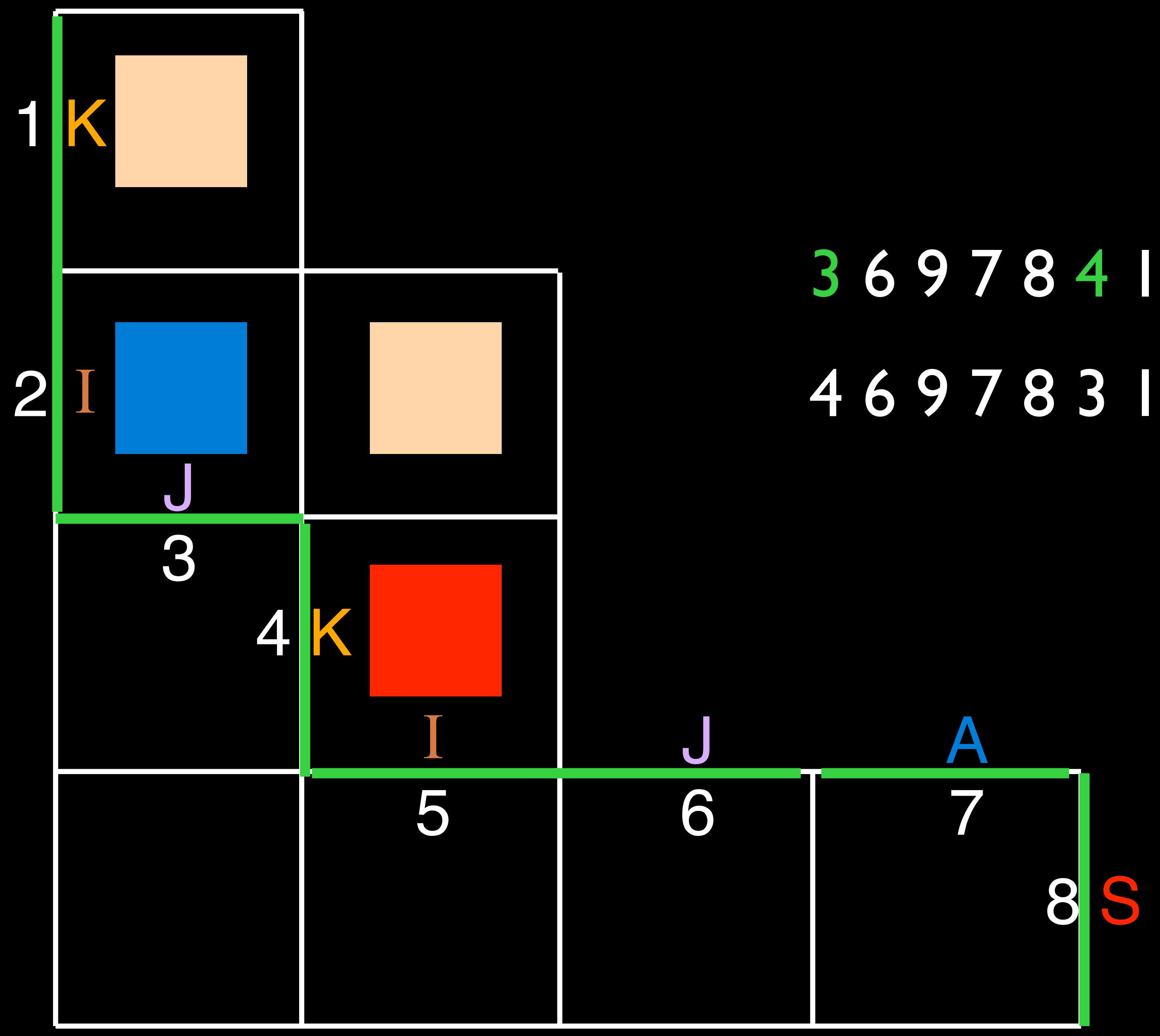


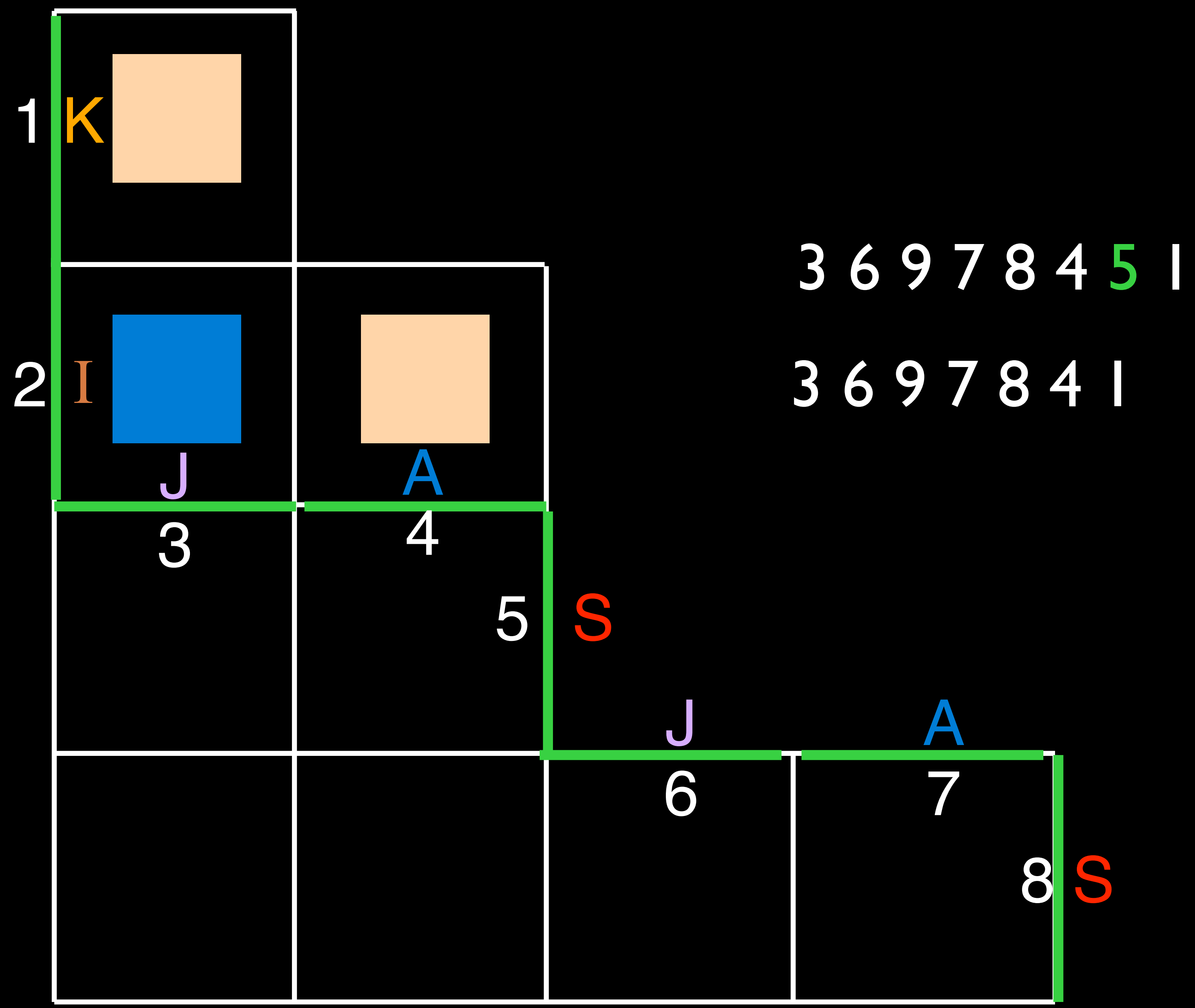
4 7 9 6 3 1

4 7 9 5 3 1

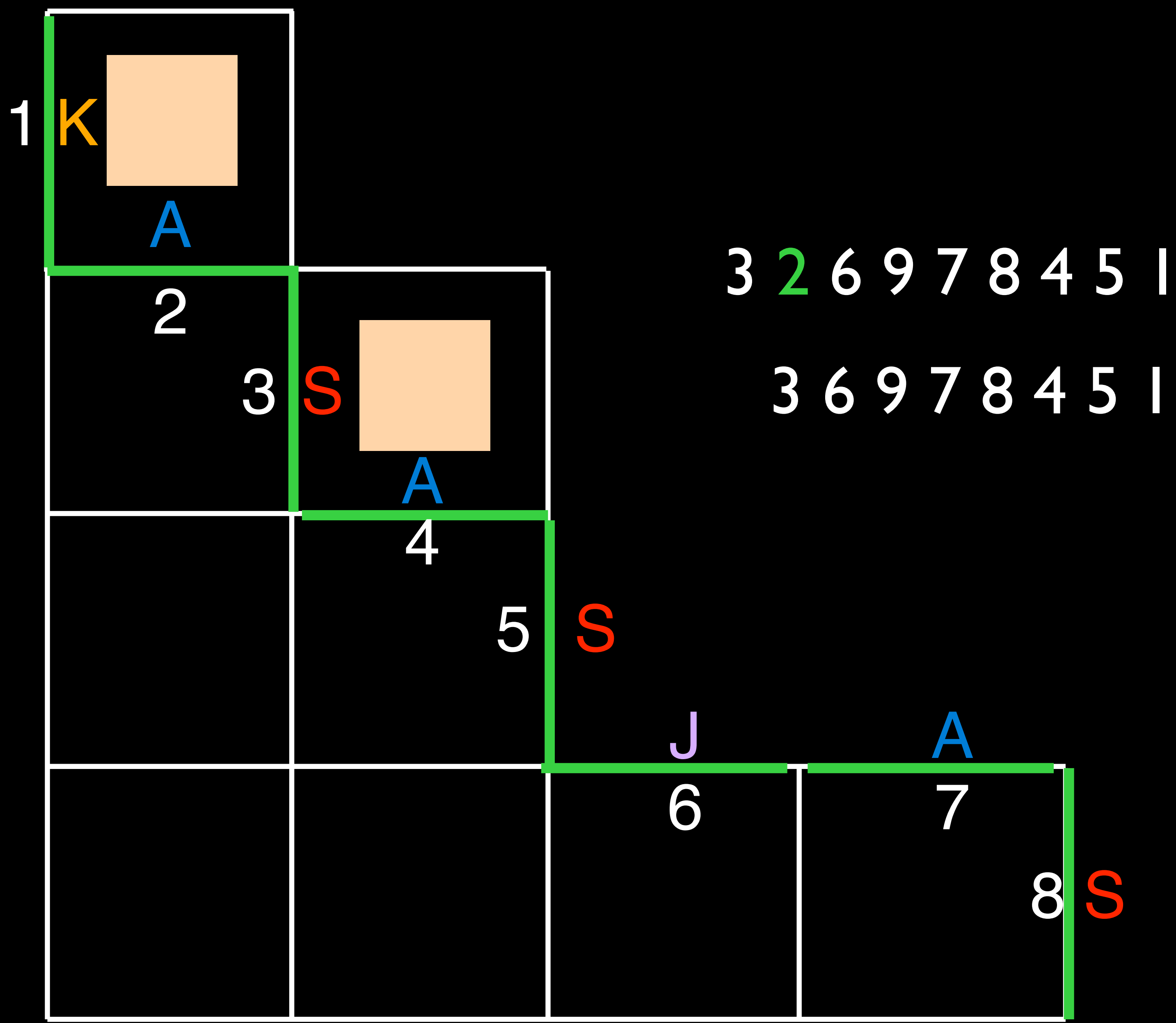

1
K
2
I
3
K
J
I
J
4
5
6
7
K
I
8
4 6 9 7 3 1
4 7 9 6 3 1


1 K
2 I
3 K
J
I
J
A
4
5
6
7
8 S
4 6 9 7 8 3 1
4 6 9 7 3 1

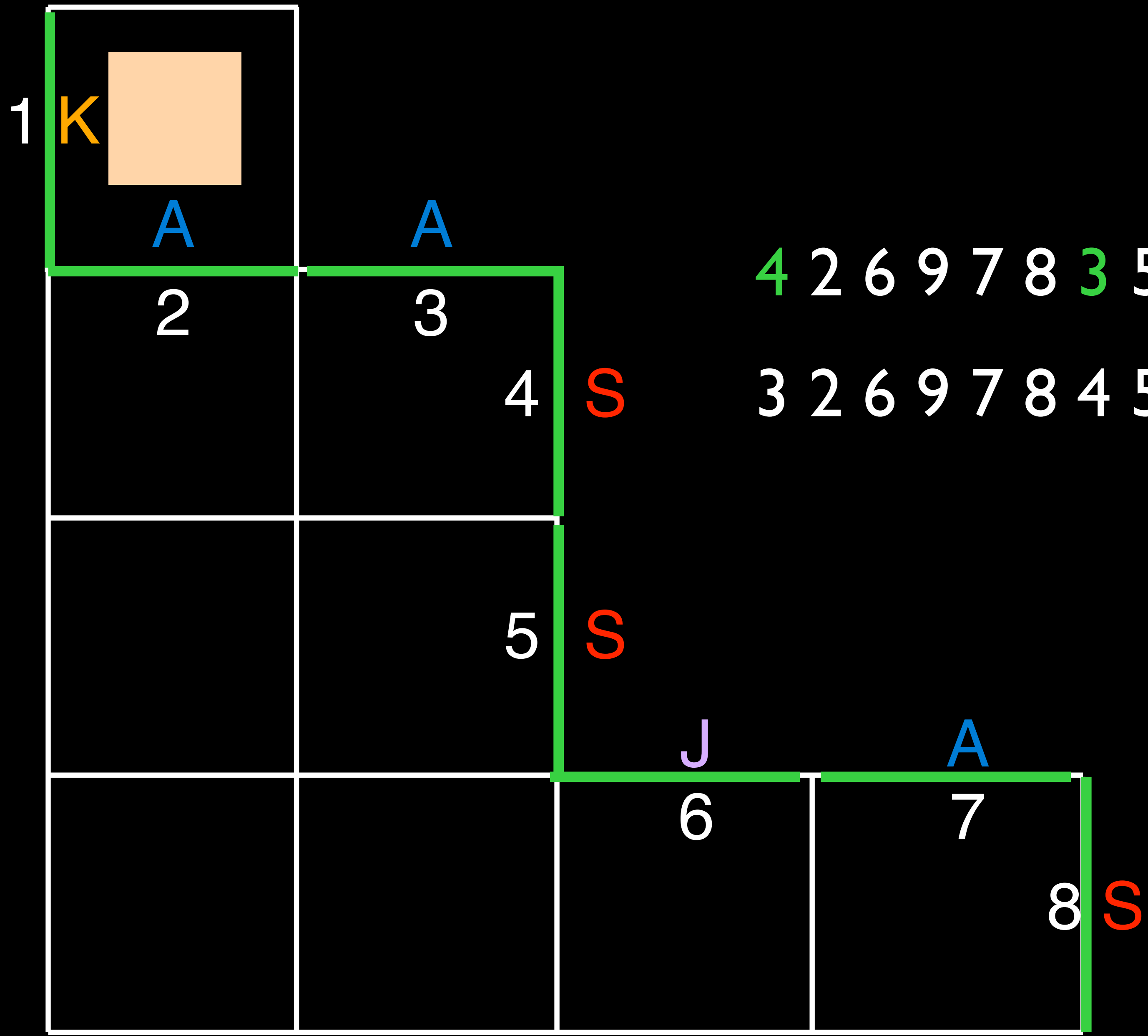

1 K
2 I
J
3
4 K
I
J
A
5
6
7
8 S
3 6 9 7 8 4 I
4 6 9 7 8 3 I



1 K
2 I
J
A
3
4
5 S
J
A
6
7
8 S
3 6 9 7 8 4 5 1
3 6 9 7 8 4 1

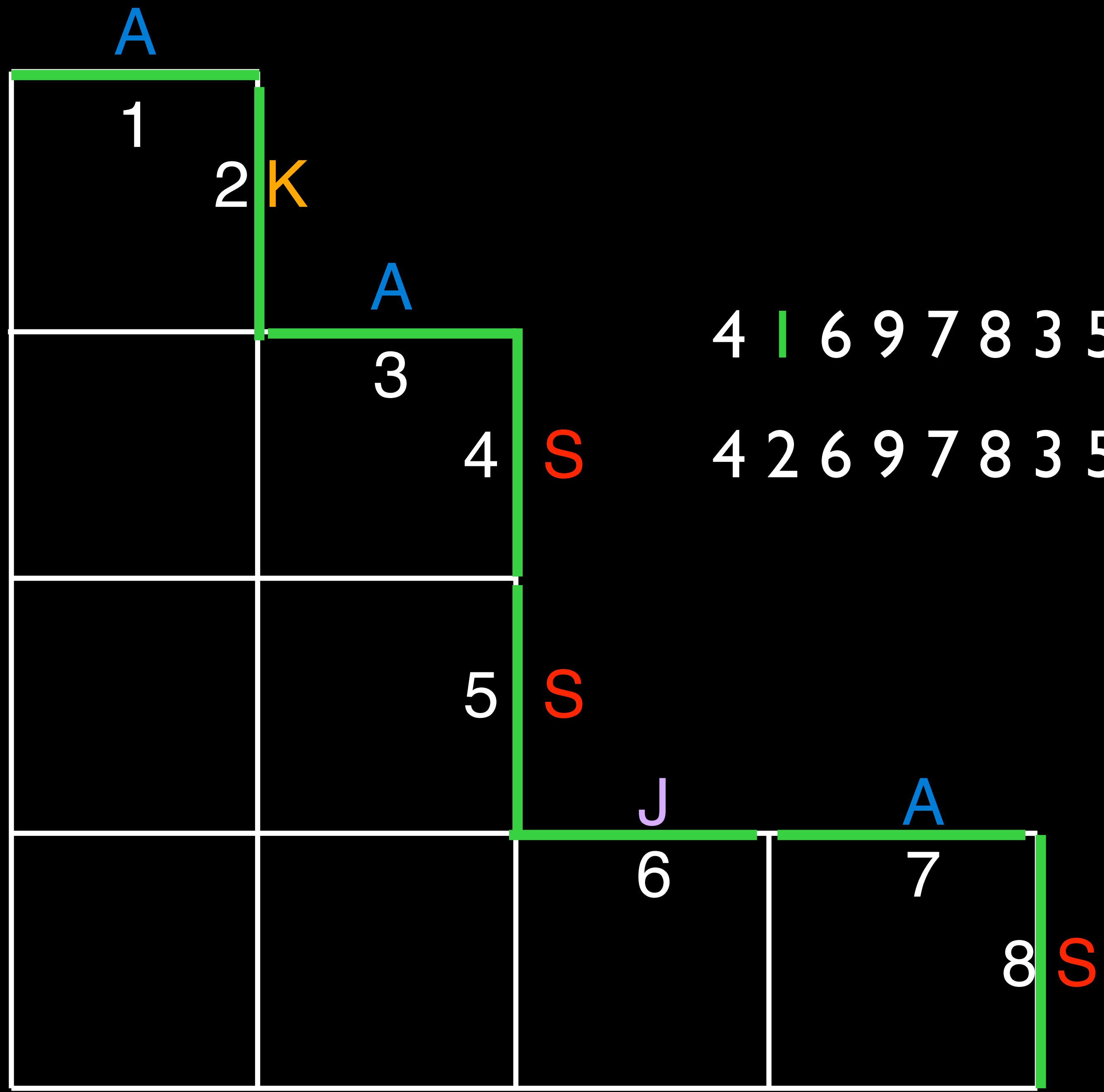


1
K
A
2
3
S
A
4
5
S
J
A
6
7
8
S
3 2 6 9 7 8 4 5 1
3 6 9 7 8 4 5 1



4 2 6 9 7 8 3 5 1

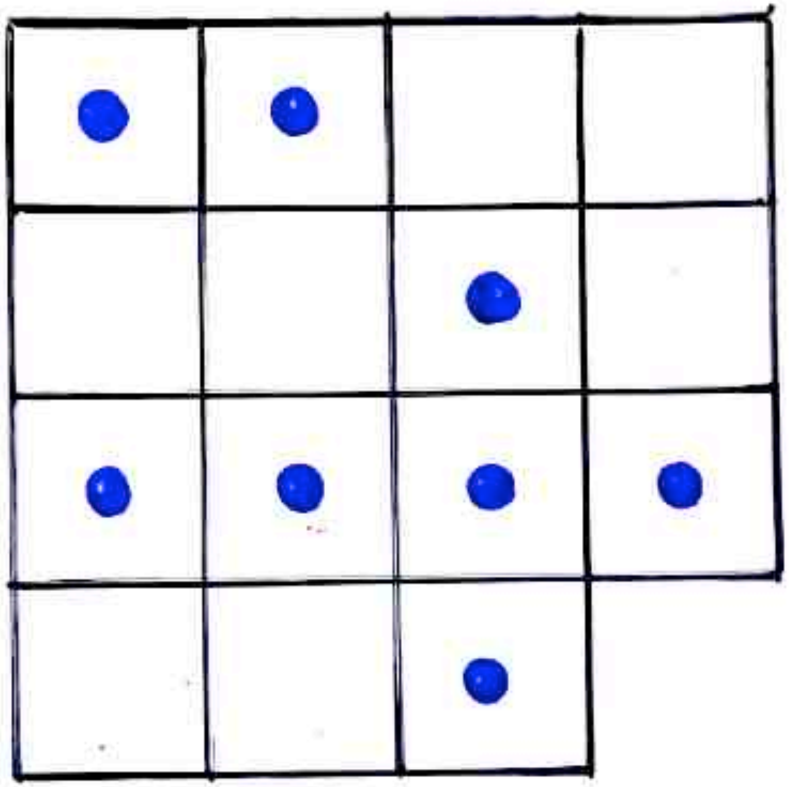
3 2 6 9 7 8 4 5 1

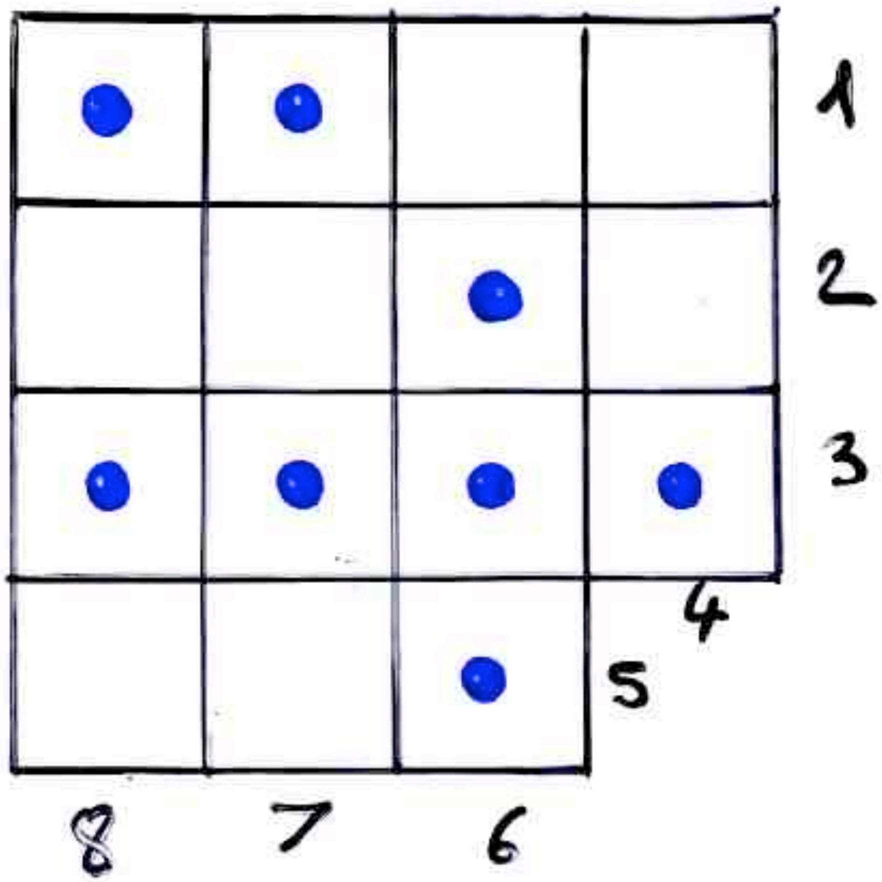


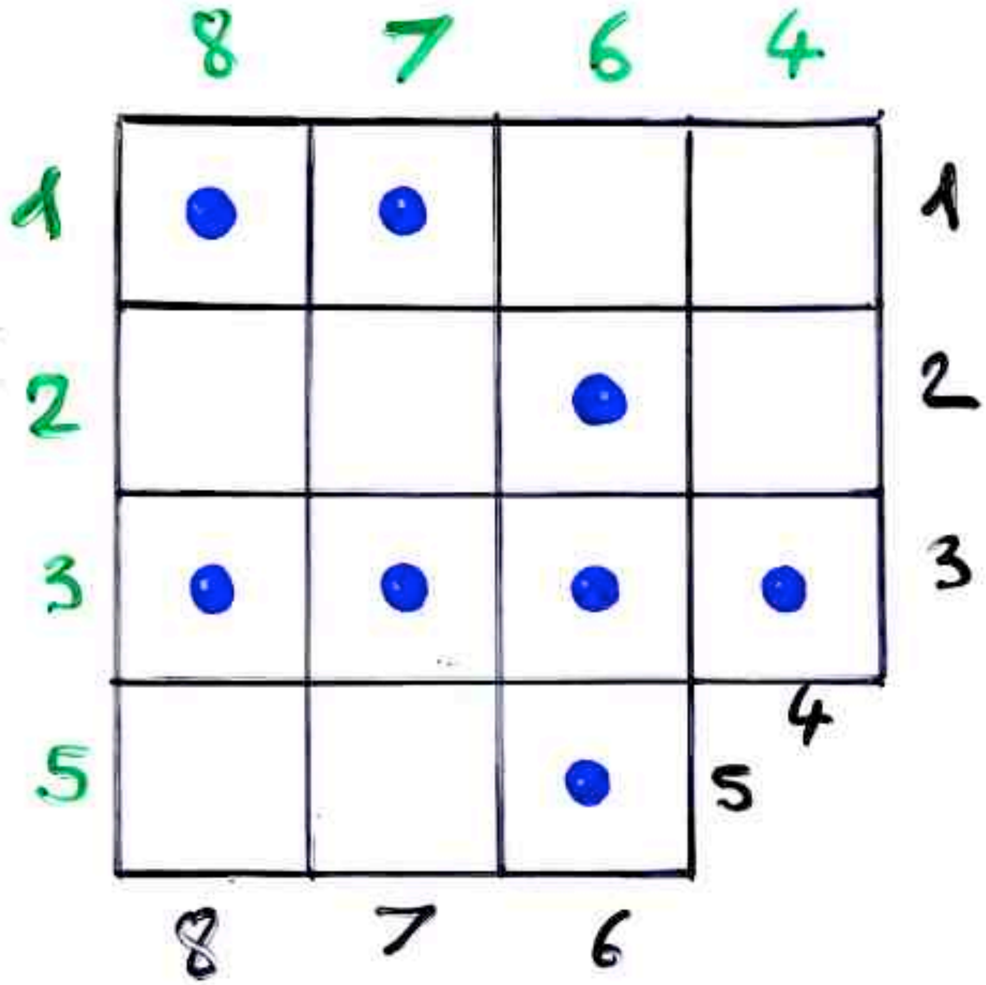
4 1 6 9 7 8 3 5 2

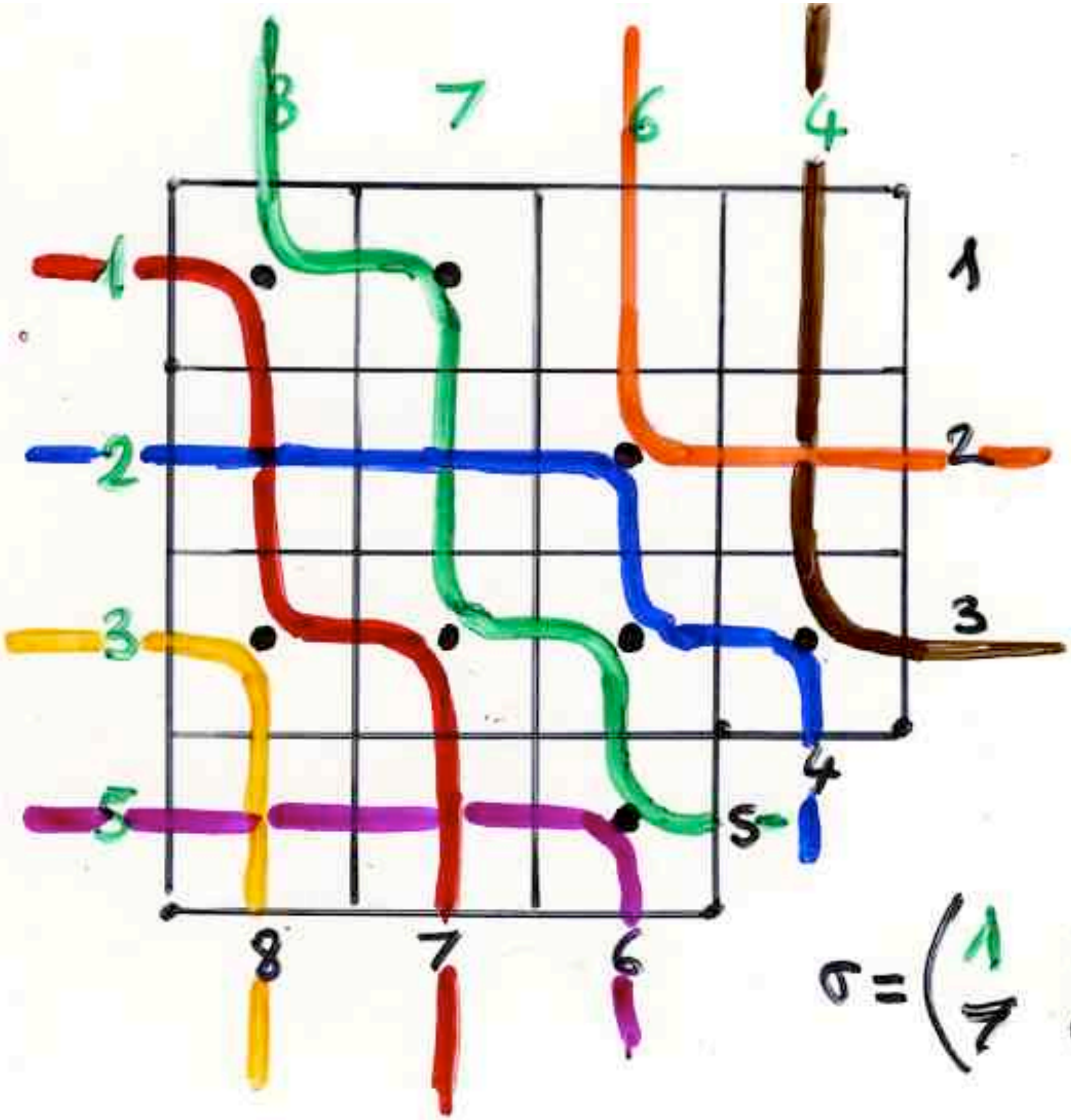
4 2 6 9 7 8 3 5 1

Postnikov bijection
permutations tableaux
permutations




1
2
3
4
5
6
7
8


8 7 6 4
1 2 3 5
1 2 3 4 5
8 7 6



$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 4 & 8 & 3 & 6 & 2 & 1 & 5 \end{pmatrix}$$

the exchange deletion algorithm

an alternative "jeu de taquin"

same example
denoted with the inverse permutation

proof of the main theorem

Postnikov bijection
permutations tableaux - permutations